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CORRIGENDUM
A CURVE SELECTION LEMMA IN THE SPACE OF ARCS AND
THE IMAGE OF THE NASH MAP

ANA J. REGUERA

The purpose of this note is to correct a mistake in the article “A Curve Selection Lemma in the space of arcs and the image of the Nash map” *Compositio Math.* 142 (2006) 119-130 (cited here as [Re1]). It is due to a misunderstanding of the concept of *generically stable* subset of the space of arcs X_∞ of variety X defined over a perfect field k . We keep the notation in the original article.

In Definition 3.1 in the original article the following extra property is needed: N is not contained in $\cup_{i=1}^c (\text{Sing } X^i)_\infty$, where X^1, \dots, X^c are the irreducible components of X . This property is applied in the proof of Theorem 4.1 and its corollaries (4.4, 4.6 and 4.8). All these statements remain true while adding this condition in Definition 3.1, that is, applying the corrected version of Definition 3.1 stated below.

Theorem 4.1 has been applied mainly through its Corollary 4.8 (Curve Selection Lemma) to the sets N_E (definition 2.1 in [Re1]) where, assuming the existence of a resolution of singularities $p : Y \rightarrow X$ of X , E is an irreducible component of the exceptional locus of p . One occurrence is Theorem 5.1 in [Re1] in order to study the image of the Nash map. Theorem 5.1 is a consequence of Theorem 4.1 applied to the Nash sets $N_\alpha := N_{E_\alpha}$ where E_α is an essential component on Y (proposition 2.2 in [Re1]). These sets N_E , and hence also the N_α 's, satisfy Definition 3.1 in its corrected version.

The original definition was:

Definition 3.1. *An irreducible subset N of X_∞ will be called generically stable if there exists an open affine subscheme W_0 of X_∞ , such that $N \cap W_0$ is a nonempty closed subset of W_0 whose defining ideal is the radical of a finitely generated ideal.*

To correct the mistake in [Re1], we add the property “ N not contained in $\cup_{i=1}^c (\text{Sing } X^i)_\infty$ ” in Definition 3.1. From now on:

Definition 3.1. (corrected version). *Let X be an algebraic variety over a perfect field k and let X^1, \dots, X^c be its irreducible components. An irreducible closed subset N of X_∞ not contained in $\cup_{i=1}^c (\text{Sing } X^i)_\infty$ will be called generically stable if there exists an open affine subscheme W_0 of X_∞ , such that $N \cap W_0$ is a nonempty closed subset of W_0 whose defining ideal is the radical of a finitely generated ideal.*

Now, the main result in [Re1] (as well as its corollaries) is completely proved:

Theorem 4.1. *Let X be a variety over k and let N be an irreducible generically stable subset of X_∞ . There exists an open affine subscheme W of X_∞ such that $N \cap W$ is a nonempty closed subset of W whose ideal is finitely generated.*

Equivalently, stated for affine varieties:

C.1. ([Re1]). *Let X be an affine variety over a perfect field k . Let P be a point in X_∞ and let $Z(P)$ be the set of zeros of P in X_∞ . Suppose that $Z(P)$ is a generically stable subset of X_∞ . Then there exists $G \in \mathcal{O}_{X_\infty} \setminus P$ such that the ideal $P(\mathcal{O}_{(X_\infty)_{\text{red}}})_G$ is a finitely generated ideal of $(\mathcal{O}_{(X_\infty)_{\text{red}}})_G$.*

The point is that there exist irreducible subsets of X_∞ satisfying Definition 3.1 in the original version which are contained in $\cup_{i=1}^c (\text{Sing } X^i)_\infty$:

Example C.2. : Let k be a perfect field of characteristic $p > 0$,

$$X := \{x_2^p - x_3x_1^p = 0\} \subset \mathbb{A}_k^3 = \text{Spec } k[x_1, x_2, x_3]$$

which is an irreducible variety. Let $N = (\text{Sing } X)_\infty$, which is a subset of X_∞ . Then N satisfies Definition 3.1 in the original version and it is of course contained in $(\text{Sing } X)_\infty$. In fact, set $f := x_2^p + x_3x_1^p$, then, with notation as in 3.5 in [Re1], we have $\mathbf{F}_0 = \mathbf{X}_{2,0}^p + \mathbf{X}_{3,0}\mathbf{X}_{1,0}^p$, $\mathbf{F}_1 = \mathbf{X}_{3,1}\mathbf{X}_{1,0}^p$, $\mathbf{F}_p = \mathbf{X}_{2,1}^p + \mathbf{X}_{3,0}\mathbf{X}_{1,1}^p + \mathbf{X}_{3,p}\mathbf{X}_{1,0}^p$, $\mathbf{F}_{p+1} = \mathbf{X}_{3,1}\mathbf{X}_{1,1}^p + \mathbf{X}_{3,p+1}\mathbf{X}_{1,0}^p, \dots$. Hence

$$(\mathcal{O}_{X_\infty})_{X_{3,1}} \cong (k[\underline{\mathbf{X}}_0, \dots, \underline{\mathbf{X}}_n, \dots])_{X_{3,1}} / \left(\{\mathbf{X}_{1,n}^p, \mathbf{X}_{2,n}^p\}_{n \geq 0} \right).$$

But $\text{Sing } X = \{x_1 = x_2 = 0\}$, hence $(\text{Sing } X)_\infty$ is the closed irreducible subset of X_∞ given by $X_{1,n} = X_{2,n} = 0$, $n \geq 0$. Therefore

$$\overline{X_\infty \setminus (\text{Sing } X)_\infty} \subseteq \{X_{3,1} = 0\}$$

Equivalently, for the open subset of X_∞ defined by $W_0 := \{X_{3,1} \neq 0\}$, we have $N \cap W_0 = W_0$, i.e. $P(\mathcal{O}_{X_\infty})_{X_{3,1}} = \sqrt{(0)}(\mathcal{O}_{X_\infty})_{X_{3,1}}$ where P is the generic point of $N = (\text{Sing } X)_\infty$. Thus N satisfies Definition 3.1 in the original version. However, N does not satisfy Theorem 4.1. In fact,

$$P(\mathcal{O}_{X_\infty})_{X_{3,1}} = (\{X_{1,n}, X_{2,n}\}_{n \geq 0})(\mathcal{O}_{X_\infty})_{X_{3,1}}$$

is not finitely generated.

Proof of Theorem 4.1 (with the corrected version of Definition 3.1) for X irreducible: Following the proof in [Re1], one reduces to X affine. Let $J \subset \mathcal{O}_X$ be its Jacobian ideal and let $e = \text{ord}_N J$ (line 9 of the proof of lemma 4.2 in [Re1]). Then e is finite since $N \not\subseteq (\text{Sing } X)_\infty$ by the corrected version of Definition 3.1. Read $(\mathcal{O}_{(X_\infty)_{\text{red}}})_{L_e} \cong (\mathcal{O}_{(X'_\infty)_{\text{red}}})_{L_e}$ (with reduced structure) in line 10 of the proof of lemma 4.2. The proof concludes as in [Re1]. \square

Proving the main result in [Re1] requires connecting generically stable subsets with *stable* subsets in X_∞ (see (2.7) in [DL]). The term *stable* here is motivated by the foundational Denef-Loeser Stability Property in [DL]. In this note we will detail the equivalence between generically stable subsets of X_∞ and stable points of X_∞ (see proposition C.9 below). The proof of Theorem 4.1 for any variety X , not necessarily irreducible, will follow straightforwardly from this study.

Definition C. 3. (3.1 in [Re2]). *Let X be an affine irreducible variety over a perfect field k and let P be a point of X_∞ (i.e. a prime ideal of \mathcal{O}_{X_∞}). We say that P is a stable point of X_∞ if there exist $n_1 \in \mathbb{N}$, and $G \in \mathcal{O}_{X_\infty} \setminus P$, $G \in \mathcal{O}_{X_{n_1}}$ such that, for $n \geq n_1$, the map $X_{n+1} \rightarrow X_n$ induces a trivial fibration*

$$(1) \quad \overline{j_{n+1}(Z(P))} \cap (X_{n+1})_G \rightarrow \overline{j_n(Z(P))} \cap (X_n)_G$$

with fiber \mathbb{A}_k^d , where $d = \dim X$, $(X_n)_G$ is the open subset $X_n \setminus Z(G)$ of X_n and $\overline{j_n(Z(P))}$ is the closure of $j_n(Z(P))$ in X_n with the reduced structure.

Now, recall the Stability Property in [DL]:

C.4. ([DL] lemma 4.1). Let X be an algebraic variety over k of pure dimension d . There exist $c \in \mathbb{N} \setminus \{0\}$ such that, for all $n, e \in \mathbb{N}$ with $n \geq ce$, the map

$$j_{n+1}(X_\infty) \longrightarrow j_n(X_\infty)$$

is a piecewise trivial fibration over $j_n(X_\infty \setminus j_e^{-1}((\text{Sing } X)_e))$ with fiber \mathbb{A}_k^d . That is, there exists a finite partition of $j_n(X_\infty \setminus j_e^{-1}((\text{Sing } X)_e))$ into subsets S which are locally closed in X_n such that the morphism restricted to the inverse image of S is a trivial fibration with fiber \mathbb{A}_k^d .

Applying this Stability Property we have proved:

C.5. ([Re2] proposition 3.7 (i)). If P is a stable point of X_∞ , X irreducible, then the arc $h_P : \text{Spec } \kappa(P)[[t]] \rightarrow X$ defined by P is dense; here $\kappa(P)$ is the residue field of P .

An elementary consequence of C.5 is:

Lemma C. 6. *Let X be an affine variety over a perfect field k and let P be a point of X_∞ .*

- (i) *Let U be an open affine subscheme of X . Then $P \in U_\infty$ if and only if U contains the center of P . If this holds, there exists $g \in \mathcal{O}_X$ with $G_0 \notin P$ such that*

$$(\mathcal{O}_{X_n})_{G_0} \cong (\mathcal{O}_{U_n})_{G_0} \text{ for } n \geq 0, \quad \text{and} \quad (\mathcal{O}_{X_\infty})_{G_0} \cong (\mathcal{O}_{U_\infty})_{G_0}.$$

In particular, $\mathcal{O}_{X_\infty, P} = \mathcal{O}_{U_\infty, P}$.

- (ii) *Let X^1, \dots, X^c be the irreducible components of X and suppose that there exists i , $1 \leq i \leq c$, such that P is a stable point of X_∞^i . Then there exists $G \in \mathcal{O}_{X_\infty} \setminus P$ such that*

$$(\mathcal{O}_{(X_\infty)_{\text{red}}})_G \cong (\mathcal{O}_{(X_\infty^i)_{\text{red}}})_G.$$

In particular, $\mathcal{O}_{(X_\infty)_{\text{red}}, P} = \mathcal{O}_{(X_\infty^i)_{\text{red}}, P}$.

Proof: For (i), first recall that, given $P \in X_\infty$, the center of P (or the center of h_P) is the image by the arc $h_P : \text{Spec } \kappa(P)[[t]] \rightarrow X$ of the closed point of $\text{Spec } \kappa(P)[[t]]$, and that U is an open subset of X . For the second assertion, we may suppose that U is defined by $\{g \neq 0\}$ in X , where $g \in \mathcal{O}_X$, $g \neq 0$. Then,

$$\mathcal{O}_{U_n} = (\mathcal{O}_{X_n})_{G_0} \text{ for } n \geq 0, \quad \text{and} \quad \mathcal{O}_{U_\infty} = (\mathcal{O}_{X_\infty})_{G_0}.$$

Since P is a point in U_∞ , $G_0 \notin P$ and hence (i) holds.

For (ii), there exists $g \in \mathcal{O}_X$, $g \neq 0$, such that $X^j \subset \{g = 0\}$ for $j \neq i$, $1 \leq j \leq c$ and $X^i \not\subset \{g = 0\}$. Then, by C.5, there exists n such that $G_n \notin P$. Let us consider the induced surjective morphism

$$(\mathcal{O}_{(X_\infty)_{\text{red}}})_{G_n} \longrightarrow (\mathcal{O}_{(X_\infty^i)_{\text{red}}})_{G_n}.$$

Let us show that it induces a bijection on spectra; then, since both rings are reduced, we conclude that the morphism is an isomorphism. In fact, given a prime ideal Q of $(\mathcal{O}_{(X_\infty)_{\text{red}}})_{G_n}$, it induces a morphism $\varphi_Q : \text{Spec } A \rightarrow X_\infty$, where $A := (\mathcal{O}_{(X_\infty)_{\text{red}}})_{G_n} / Q$, whose image is not contained in the space of arcs of $\{g = 0\}$. Equivalently, we have an A -arc $h_Q : \text{Spec } A[[t]] \rightarrow X$, whose image is not in $\{g = 0\}$. Since A , hence also $A[[t]]$, is a domain, the image of h_Q is irreducible. Therefore it is contained in X^i . We conclude then that φ_Q factors through X^i , hence, Q is a point of X_∞^i . This concludes the proof. \square

Definition C. 7. (Paragraph before 3.8 in [Re2]). Let X be an irreducible variety over a perfect field k and let P be a point of X_∞ . Let X_0 be any irreducible open affine subscheme of X which contains the center of P . We say that P is a stable point of X_∞ if P is a stable point of $(X_0)_\infty$.

Now, let X be any variety over k and let X^1, \dots, X^c be its irreducible components. A point P of X_∞ is said to be a stable point of X_∞ if there exists i , $1 \leq i \leq c$, such that P is a stable point of X_∞^i .

Remark C.8. This definition does not depend on the choice of X_0 : this follows from (i) in lemma C.6. Note also that, for X not irreducible, if X^i is an irreducible component of X and P is a stable point of X_∞^i then, by C.5, X^i is the unique irreducible component of X such that $(X_\infty^i)_{\text{red}}$ contains $Z(P)$, or equivalently, X^i contains the image of h_P . That is, the set of stable points of X_∞ is the disjoint union of the sets of stable points of the spaces of arcs of the irreducible components of X (proposition 3.8 in [Re2]).

Relating the stable points of X_∞ with the generically stable subsets of X_∞ (following Definition 3.1 in its corrected version) we have:

Proposition C. 9. Let X be a variety over a perfect field and let P be a point in X_∞ . The following are equivalent:

- (i) P is a stable point of X_∞ .
- (ii) $Z(P)$ is a generically stable subset of X_∞ .

Proof: We may assume without loss of generality that X is affine. We will first prove the proposition for X irreducible. Then we will deduce the general case from this irreducible case, applying C.5 and lemma C.6. In (ii) \Rightarrow (i) for X irreducible we will apply the proof of Theorem 4.1 in [Re1] for the irreducible case.

Suppose that X is irreducible, let $d = \dim X$. With notation as in section 4 of [Re1], for $n \geq 0$, let \mathcal{P}_n be the prime ideal of $\mathcal{O}_{\overline{j_n(X_\infty)}}$ defining $\overline{j_n(Z(P))}$. For (i) \Rightarrow (ii), let P be a stable point of X_∞ . Then, there exists $n_1 \in \mathbb{N}$ and $G \in \mathcal{O}_{X_\infty} \setminus \mathcal{P}$, $G \in \mathcal{O}_{X_{n_1}}$ such that, for $n \geq n_1$, the map (1) is a trivial fibration with fiber \mathbb{A}_k^d . This implies that $N \not\subset (\text{Sing } X)_\infty$ (see C.5) and hence, there exists $e \in \mathbb{N}$ such that $N \subset X_\infty \setminus j_e^{-1}((\text{Sing } X)_e)$. By lemma 4.1 in [DL] (see C.4), there exist a constant c and a locally closed subset S of X_{n_0} , where $n_0 = \sup\{n_1, ce\}$, such that the generic point of $j_{n_0}(Z(P))$ belongs to S and, for $n \geq n_0$,

$$j_{n+1}(X_\infty) \longrightarrow j_{n_0}(X_\infty)$$

induces a trivial fibration over S with fiber $\mathbb{A}_k^{d(n+1-n_0)}$. Note that, for $n \geq n_0$,

$$\overline{j_{n+1}(Z(P))} \cap (X_{n+1})_G \rightarrow \overline{j_{n_0}(Z(P))} \cap (X_n)_G$$

is also a trivial fibration with fiber $\mathbb{A}_k^{d(n+1-n_0)}$. By restricting the open subset $(X_n)_G$, we may suppose that $\overline{j_{n_0}(Z(P))} \cap (X_n)_G$ is contained in S . It follows that the defining ideal of $\overline{j_{n+1}(Z(P))} \cap (X_{n+1})_G$ is the radical of $\mathcal{P}_{n_0}(\mathcal{O}_{X_{n+1}})_G$. Hence the defining ideal of $Z(P) \cap (X_\infty)_G$ is the radical of $\mathcal{P}_{n_0}(\mathcal{O}_{X_\infty})_G$ and (ii) holds.

For (ii) \Rightarrow (i), let $P \in X_\infty$ such that $Z(P)$ is a generically stable subset of X_∞ which is not contained in $(\text{Sing } X)_\infty$. Arguing as in lemma 4.1 in [DL] and Theorem 4.1, or more precisely lemma 4.2, in [Re1], we reduce to the case where X is a complete intersection (not necessarily irreducible). Then, (10) in the proof of lemma 4.2 in [Re1] asserts that there exists $G \in \mathcal{O}_{X_\infty} \setminus \mathcal{P}$ and there exists $n_1 \in \mathbb{N}$ such that, for $n \geq n_1$, we have

$$(\mathcal{O}_{\overline{j_{n+1}(X_\infty)}})_G / \mathcal{P}_{n+1} \cong \left((\mathcal{O}_{\overline{j_n(X_\infty)}})_G / \mathcal{P}_n \right) [\mathbf{X}_{m-d+1, n+1}, \dots, \mathbf{X}_{m, n+1}]$$

that is, $(\mathcal{O}_{\frac{j_{n+1}(Z(P))}{j_n(Z(P))}})_G \cong (\mathcal{O}_{\frac{j_n(Z(P))}{j_{n-1}(Z(P))}})_G[\mathbf{X}_{m-d+1, n+1}, \dots, \mathbf{X}_{m, n+1}]$. Hence (i) holds.

Now, if X is not irreducible, let X^1, \dots, X^c be its irreducible components. For (i) \Rightarrow (ii), let P be a stable point of X_∞ ; then there is i , $1 \leq i \leq c$, such that P is a stable point of X_∞^i . Therefore there exists $G \in \mathcal{O}_{X_\infty} \setminus P$ such that $(\mathcal{O}_{(X_\infty)_{\text{red}}})_G \cong (\mathcal{O}_{(X_\infty^i)_{\text{red}}})_G$ (lemma C.6 (ii)). On the other hand, since P is a stable point of X_∞^i and X^i is irreducible, $Z(P)$ is a generically stable subset of X_∞^i . Hence there exists $G' \in \mathcal{O}_{X_\infty} \setminus P$ such that $P(\mathcal{O}_{(X_\infty^i)_{\text{red}}})_{G'}$ is the radical of a finitely generated ideal (we identify G' with its image by the surjective morphism $\mathcal{O}_{X_\infty} \rightarrow \mathcal{O}_{X_\infty^i}$). Then $GG' \in \mathcal{O}_{X_\infty} \setminus P$ and $P(\mathcal{O}_{(X_\infty)_{\text{red}}})_{GG'}$ is the radical of a finitely generated ideal. Since $Z(P) \not\subseteq \cup_{j=1}^c (\text{Sing } X^j)_\infty$ by C.5, we conclude (ii).

For (ii) \Rightarrow (i) note first that, for $1 \leq i \leq c$, any generically stable subset of X_∞ which is contained in X_∞^i is also a generically stable subset of X_∞^i . In fact, since $\mathcal{O}_{X_\infty^i}$ is a quotient of \mathcal{O}_{X_∞} , this follows from the definition. Now, let P be such that $Z(P)$ is a generically stable subset of X_∞ , and let i , $1 \leq i \leq c$, be such that $Z(P)$ is contained in X_∞^i . Then $Z(P)$ is a generically stable subset of X_∞^i and X^i is irreducible, hence P is a stable point of X_∞^i and we conclude (i). \square

Corollary C. 10. *Given a point P in X_∞ , we have that $Z(P)$ is a generically stable subset of X_∞ if and only if there exists an irreducible component X^i of X such that $Z(P)$ is a generically stable subset of X_∞^i . Moreover, the arc h_P is dense in X^i , therefore the irreducible component X^i is uniquely determined.*

Proof of Theorem 4.1 (with the corrected version of Definition 3.1) for any variety X : We may assume that X is an affine variety. Let $P \in \mathcal{O}_{X_\infty}$ be the generic point of the generically stable subset N of X_∞ . Then there exists an irreducible component X^i of X such that N is a generically stable subset of X_∞^i . By Theorem 4.1 applied to the irreducible variety X^i , there exists $G \in \mathcal{O}_{X_\infty} \setminus P$ such that the ideal $P(\mathcal{O}_{(X_\infty^i)_{\text{red}}})_G$ is a finitely generated ideal of $(\mathcal{O}_{(X_\infty^i)_{\text{red}}})_G$. By proposition C.9 and lemma C.6, there exists $G' \in \mathcal{O}_{X_\infty} \setminus P$ such that $(\mathcal{O}_{(X_\infty)_{\text{red}}})_{G'} \cong (\mathcal{O}_{(X_\infty^i)_{\text{red}}})_{G'}$. Then, $GG' \in \mathcal{O}_{X_\infty} \setminus P$ and $P(\mathcal{O}_{(X_\infty)_{\text{red}}})_{GG'}$ is a finitely generated ideal of $(\mathcal{O}_{(X_\infty)_{\text{red}}})_{GG'}$. This concludes the proof. \square

Finally we will show that the closed subsets of X_∞ satisfying Definition 3.1 in the original version which are not generically stable subsets of X_∞ , are generically stable subsets of Y_∞ for some irreducible subvariety Y of X :

Proposition C. 11. *Let X be an algebraic variety over a perfect field and let X^1, \dots, X^c be its irreducible components. Let N be an irreducible closed subset of X_∞ satisfying Definition 3.1 in the original version such that $N \subseteq \cup_{i=0}^c (\text{Sing } X^i)_\infty$. Let $P \in X_\infty$ be its generic point. Then there exists i , $1 \leq i \leq c$, and an irreducible subvariety Y of $\text{Sing } X^i$ such that $N \subset Y_\infty$ and P is a stable point of Y_∞ .*

Proof: As at the end of the proof of proposition C.9, note that, for any subvariety Y of X , an irreducible closed subset of X_∞ satisfying Definition 3.1 in the original version for X_∞ which is contained in Y_∞ , satisfies also Definition 3.1 in the original version for Y_∞ . Now, let N be an irreducible closed subset of X_∞ satisfying Definition 3.1 in the original version for X_∞ such that $N \subseteq \cup_{i=0}^c (\text{Sing } X^i)_\infty$. Let i , $1 \leq i \leq c$, with $N \subseteq (\text{Sing } X^i)_\infty$ and set $Y_1 := \text{Sing } X^i$. Then N satisfies Definition 3.1 in the original version for $(Y_1)_\infty$. If $N \not\subseteq \cup_{i=0}^{c_1} (\text{Sing } Y_1^i)_\infty$, where $Y_1^1, \dots, Y_1^{c_1}$ are the irreducible components of Y_1 , then N is a generically stable subset of $(Y_1)_\infty$. Applying proposition C.9, we conclude the result. If not, let i_1 , $1 \leq i_1 \leq c_1$, be such that $N \subseteq (\text{Sing } Y_1^{i_1})_\infty$ and set $Y_2 := \text{Sing } Y_1^{i_1}$, so that $N \subseteq (Y_2)_\infty$, and argue by induction. We conclude the result after at most $\dim X$ steps. \square

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