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Length measurement in the textbooks of German and Taiwanese primary students

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Length measurement plays a prominent role in most countries. The purpose of this study was to examine similarities and differences in Taiwanese and German elementary written textbooks concerning the treatment of length understanding, measurement, and estimation. Our analysis showed that procedural affordances dominate the conceptual learning situations in both countries, even more in Germany than in Taiwan. Although ‘units can be converted’, ‘reasoning and justification’ and ‘benchmark learning’ were the most common conceptual elements in both countries, their importance differed. The comparison of procedural elements showed that both countries mainly focused on ‘unit conversion’, ‘measuring with a ruler’, and ‘addition and subtraction of length’. The Taiwanese textbooks concentrated more on concrete actions, whereas the German textbooks focused on more abstract and mental procedures.

Keywords: length measurement, textbook analysis, binational comparison, primary school

Introduction

Length measurement plays a prominent role in measurement education in most countries (Buys & de Moor, 2008, Clements & Bright, 2003). Nevertheless, researchers all over the world continuously report poor learning outcomes especially concerning the underlying measurement concepts behind the measurement procedures (Smith III, van den Heuvel-Panhuizen, & Treppo, 2011). Although there seems to be a great consistency about the aspects of those concepts (Barrett, Sarama, & Clements, 2017; Lehrer, 2003; Clarke, Cheeseman, McDonough, & Clarke, 2003), differences in the curricula and teaching practice are assumed as well (Lee & Smith III, 2011).

Being interested in cross-cultural differences and similarities in length estimation, we’re looking closer to Taiwan and Germany as first examples, motivated by two aspects: Taiwan and Germany are two examples of very different cultural backgrounds—one Asian and one European country, and both with an official language different from English—and the learning outcomes in both countries differ enormously at 4th grade.¹ In contrast to the international results, the preliminary findings of our pilot study on length estimation did not show these differences in the overall estimation abilities. Looking closer to the answers we considered specific differences, which, perhaps, may be explained by different learning opportunities on length learning. Therefore, this textbook analysis serves as a basis for looking to the learning opportunities on length learning in both countries.

¹ Although TIMSS 2015 reported significant above average scores for German fourth-graders (overall score: 522, s.e. 2.0; ‘geometric shapes and measures’ score: 531, s.e. 2.5), Taiwanese fourth-graders performed much better (overall score: 597, s.e. 1.9; ‘geometric shapes and measures’ score: 597, s.e. 3.0)

[for details see: <https://nces.ed.gov/timss/timss2015/> [last check: 21-11-2018]]

Theoretical background

Core concept of lengths and its measurement

Length understanding, length measurement, and length estimation are thought to be a complex concept with different conceptual underpinnings as well as concrete actions. According to Lehrer (2003), Clarke et al. (2003), Stephan and Clements (2003), and others the most important foundations are

- (1) Understanding of the attribute and its relation to the units used to measure with.
A length must be understood as the distance between two points in the space. The length of an object can be found by quantifying the distance between its endpoints.
- (2) Logical operations: conservation and transitivity.
The length of a given object isn't dependent on its location, nor is it variant under special transformations, whereas other transformations don't conserve the length. The transitivity of the equivalence and order relation are one basis for the comparisons of lengths.
- (3) Mental partitioning into parts.
Being able to partition the length of an object mentally and being sure that adding the measures of the parts will give the length of the whole.
- (4) Iterative tiling with identical units and counting them.
The measuring process consists of using a unit (standardized or non-standardized) iteratively by placing it end to end without gaps or overlaps. Whether a subdivision of the unit is necessary depends on the precision the measurer looks for.
- (5) Numerical interpretation of the iteration process.
Either every iteration-step has to be counted during the process, or the number of different identical units can be counted at the end of the tiling-process. The counted number has to be understood as the measure.
- (6) Understanding of measurement tools.
The children need not only the procedural knowledge how to measure and draw with a ruler, but they need to understand the scaling itself. Crucial is the difference between a point on the scale and the length measured, being asserted to the distance between two scale-points.
- (7) Measures as the relation between unit and number.
The relation between the unit and the number of units needed to tile a length is inversely proportional. Besides this children have to learn the conventional conversions between different standardized units, including fractions and decimals.

Conceptual and procedural knowledge in length measurement

In the literature, it has been stated since a long time that measurement learning is much more concentrated on superficial procedural aspects than on the underlying concepts (e.g. Lehrer, 2003; Stephan & Clements, 2003). In sharpening the meaning of conceptual and procedural knowledge in measurement learning, the STEM research group at Michigan State University (e.g. Lee & Smith III,

2011, Smith III, Males, Dietiker, Lee, & Mosier, 2013) developed iteratively a coding scheme with conceptual, procedural, and conventional codes. They included all measurement actions children are asked to carry out into the term ‘procedures’ and ‘procedural knowledge’, whereas they use ‘concepts’ and ‘conceptual knowledge’ “to designate the general principles that underlie and justify procedures” (Smith et al., 2013, 399). The third category, ‘conventional knowledge’ was used to code aspects, which are contingent and culturally defined, like the metric of units or aspects of rulers etc.

Their analysis of three US and one Singapore textbook series showed strong emphasis on procedural aspects in all grades and both countries (Lee & Smith III, 2011). If concepts are focused and explicitly addressed, they are not in line with the procedures but appear much later in the curriculum (Smith III et al., 2013).

Length estimation

In the sense of Bright (1976) we consider length estimation as being a mental process of determining a length for an attribute without the aid of measurement tools. Although children’s estimates in length are more accurate than in other measurement areas (Joram, Subrahmanyam, & Gelman, 1998), children are even worse in estimating than in measuring lengths. Researchers stress the importance of strategies (Jones, Taylor, & Broadwell, 2009; Huang, 2015) and their conjectures to the underlying measurement concepts and procedures. Different learning environments based on different curricula might lead to different knowledge (e.g., different estimation strategies). Since most studies on estimation focus on one country with the underlying assumption that the curriculum for measurement estimation will be similar across countries (Jones et al., 2009; Ruwisch, Heid, & Weiher, 2017), we try to get deeper insights into similarities and differences in the lengths curriculum and its focus on learning length estimation. The study of the STEM-project only referred to length estimation at the edge. Nevertheless, they reported great differences in the frequencies among the three US textbook series, and concluded that “despite the frequent calls to estimate lengths, little attention was given in any curriculum to specify the estimation process” (Smith III et al., 2013, 416).

Research questions

The purpose of this study was to examine similarities and differences in Taiwanese and German elementary written textbooks concerning the treatment of length understanding, measurement, and estimation. Specifically, we were interested in those aspects which may lead to different understandings of length estimation.

Q1: On a coarse and organizational level: Are there differences in the main syllabus—number of units, instructional time—on length learning between Taiwan and Germany?

Q2: Do procedural aspects also dominate the Taiwanese and the German curriculum as it has been reported from the USA and Singapore?

Q3: Which concrete differences in the opportunities to learn length understanding, measurement, and estimation can be observed between Taiwan and Germany?

Method

Choice of written curricula and scope of analysis

In both countries, elementary school mathematics textbooks were developed on the curriculum guidelines for compulsory mathematics education mandated by the responsible ministry that has them licensed (Lan 2005 in Chinese for the Taiwanese procedure; Stöber 2010 in German for the German procedure). We examined four German elementary textbook series and three from Taiwan which were chosen by the ranking of publishers (see Ruwisch 2017).

A primary textbook series in both countries normally includes a package for each grade. Since the additional materials differ from series to series and are optional for a teacher, the only materials included in our analysis are the main textbooks for each grade.

Locating the length content

For the purpose of this comparison, we focused on those pages of the textbooks that were designated as length measurement units by the authors, excluding the units involving perimeters of shapes. So our analysis was a first step to come to a deeper analysis as such of Lee and Smith III (2011).

Coding process of the length content

For coding the different aspects of length content we adopted the coding scheme of Lee and Smith III (2011, 689). We also differentiated between conceptual and procedural knowledge. The analysis presented here is coarser than the very fine one of Lee and Smith III in one sense and broader in another one: The coding unit was not the sentence or question but normally the task, which could contain more than one sentence. Sometimes, there were two different requests in only one sentence. As a consequence, some tasks got more than one code, although normally we tried to decide which element is more important in a special situation. Therefore, we restricted ourselves to tendencies here. In applying the scheme to our data, we also needed to extend the coding scheme. *BL* was added as a conceptual element for tasks that ask students for exploration and learning the measures of benchmarks or personal reference objects for estimation. *CS* is a conceptual code for the definitions of curve versus straight line. A category for reasoning and justification was added (*RJ*), which we think is a linking category between conception, procedure, metacognition, and language.

Exemplary for our differentiation between the procedure and the underlying concept we look closer to “unit conversion”. If the task asked the child to “write the measures given in mm”, this was coded with the procedural code “unit conversion”. If a short description or definition of conversion was given—“Sometimes the unit centimeter is too rough. We then need the finer unit millimeter. 1 cm = 10 mm; 1 m = 1,000 mm.” — we coded this with the conceptual code “units can be converted”.

Results

Syllabus, units and time

In Taiwan, the learning of length measurement starts at the first semester of grade 1 in all textbook series. In every grade, length measurement units are included in the textbooks. Whereas in grades 1

and 2, two length measurement units are allocated in every textbook series—one per semester—, only one unit is included in all series for grade 3 and 4, respectively.

Most schools in Germany start length learning in second grade. Only *Das Zahlenbuch* gives an opportunity for learning length measurement in grade one, but only by one page in a book of 132 pages. Nearly every textbook contains one special unit about length learning per grade—with the exception of *Das Zahlenbuch*, which offers two units in grade three and none in grade four.

Table 1 gives detailed information about both countries. On the one hand, the total numbers of pages on length measurement in the Taiwanese textbooks are more than three times of those contained in the German textbooks. On the other hand, comparing the intended instructional time in both countries does not reflect this great difference. German teachers seemed to be asked to refer for a longer period of teaching time to the same page of the textbook.

Grade	German textbook series				Taiwanese textbook series		
	Denken und Rechnen	Flex & Flo	Welt der Zahl	Zahlenbuch	Han-Lin	Kuang-Hsuan	Nan-I
1	—	—	—	1/132 = 0.75% (no time given)	14/224 = 6.25% 6 (240 min.)	15/218 = 6.9% 7 (280 min.)	15/238 = 6.3% 6 (240 min.)
2	10/133 = 7.5% ~10 (450 min.)	6/148 = 4% ~10 (450 min.)	2/133 = 1.5% ~5 (225 min.)	3/132 = 2.3% (no time given)	22/256 = 8.6% 10 (400 min.)	19/219 = 8.7% 10 (400 min.)	21/246 = 8.5% 10 (400 min.)
3	7/117 = 6% ~ 8 (360 min.)	8/156 = 5% ~ 8 (360 min.)	9/125 = 7.2% ~ 8 (360 min.)	7/132 = 5.3% (no time given)	13/268 = 4.85% 5 (200 min.)	8/260 = 3.1% 4 (160 min.)	9/258 = 3.5% 5 (200 min.)
4	3/117 = 2.6% ~ 8 (360 min.)	5/144 = 3.5% ~ 4 (180 min.)	4/125 = 3.2% ~ 8 (360 min.)	—	7/276 = 2.5% 2 (80 min.)	9/256 = 3.5% 5 (200 min.)	11/278 = 4% 6 (240 min.)
Total	20/500 = 4% ~ 26 (1170 min.)	19/596 = 3.2% ~ 22 (990 min.)	15/416 = 3.6% ~ 21 (945 min.)	11/528 = 2.1% (no time given)	56/1024 = 5.5% 23 (920 min.)	51/953 = 5.4% 26 (1040 min.)	56/1020 = 5.5% 27 (1080 min.)

Note: The first row gives the proportion by the number of pages: Pages of the length measurement unit in proportion to the whole number of pages. The second row gives the intended teaching time for the units.

Table 1: The opportunities for learning length measurement

In both countries, most time for learning length is spent in second grade. The comparison between both countries suggests that length learning in Germany seems to be more concentrated than in Taiwan: starting in second grade and offering only one unit per grade.

Conceptual versus procedural affordances

In the German textbooks, the total numbers of conceptual codes were nearly the same in all four series, whereas the number of procedural codes differed. *Das Zahlenbuch* and *Welt der Zahl* got twice as many procedural codes than conceptual ones, *Denken und Rechnen* and *Flex & Flo* got three times as many procedural as conceptual codes.

The Taiwanese textbook series showed little differences in the total numbers of procedural elements, but differed in the number of conceptual codes. Whereas *Kuang-Hsuan* and *Nan-I* were about close in the number of conceptual codes, the third series, *Han-Lin*, only contained about 60 % of the number of *Kuang-Hsuan*, the series with the most conceptual elements. Therefore, this textbook series (*Han-Lin*) contained more than twice as many procedural than conceptual elements. The other two have about one third more procedural than conceptual elements.

Thus, in the German and the Taiwanese curriculum also the procedural affordances dominate, a tendency which is stronger for the German than the Taiwanese textbook series.

Main categories of conceptual and procedural affordances

The main concepts in all German textbook series are *benchmark learning* and *reasoning and justification*. The latter normally is combined with benchmark learning (more conceptual) or with proportional reasoning as in distance-time-relationships and scale (more procedural). The third most common conceptual category is *units can be converted*, normally at the top of a page, on which the students are asked to convert units in the following tasks.

The main concept involved in all Taiwanese textbook series is *units can be converted*. It occurs about twice as often as *reasoning and justification*, which is the second important conceptual element. Although *benchmark learning* occurs as the third often coded element of conceptual categories in Taiwanese textbooks, it is much rarer than the other two.

Looking to the procedural elements, the most common one in all series of both countries is *conversion of units*.

We differentiated between those procedures that ask for a concrete measurement action, and more abstract procedures, which are presented on the symbolic level. In German textbooks only about one third of the tasks asks for concrete procedures, another third for the *conversion of units*, and the last third for other symbolically presented procedures. In Taiwan nearly one half of the coded tasks ask for concrete and the other half for more abstract procedures, if *conversion of units* is included in the latter one.

Concerning the concrete procedures the most common one in both countries was *measure with a ruler*, when the object is shorter than the ruler. In Germany *draw with a ruler* (also objects shorter than the ruler) and *visual estimation* were the second and third common ones, whereas in the Taiwanese textbooks *measuring with sufficient non-standard units* and *visual estimation* and different kinds of *direct and visual comparisons* were found more often than drawing activities. No *direct comparison* was coded in the German textbooks.

The most common abstract procedures in the Taiwanese textbook series were generating the *sum and differences of length given through word descriptions and representations*, and doing so *given word descriptions only*. In Germany, *word problems with lengths*, sometimes *with the aid of a representation*, also dominate the abstract procedures—besides the conversion of units—, followed by *the comparison or order of length which were given in a symbolic*.

Discussion and conclusion

The analysis showed similarities and differences between the countries as well as between the textbook series concerning the opportunities for learning length understanding, measurement, and estimation.

Although all textbooks show a dominance of procedural aspects over conceptual elements, this tendency is stronger in the German textbooks. The Taiwanese textbooks focus much more on

conceptual elements than the German did. If this difference in the frequency of the opportunities to understand length measurement is crucial for better results in international comparative studies, needs further investigation. As a first step, our analysis itself can be deepened insofar that we will look, if and how the procedural and underlying conceptual knowledge elements are temporally interlocked or decoupled like in the US textbooks (Smith III et al., 2013).

Although *units can be converted, reasoning and justification* and *benchmark learning* are the most common conceptual elements in both countries, their importance differs. The Taiwanese textbooks mainly stress the idea of conversion, whereas the German textbooks focus more on benchmark learning. Although there were only a few problems involving ‘visual estimation’ as a procedure, we did not yet deepen our analysis in this point to get to know, if the content of this request differs in both countries: Is the procedure itself specified? And if so, how? Is a specific strategy shown? Benchmark knowledge and estimation strategies are seen as crucial elements for good length estimation (Jones et al., 2009; Joram et al., 1998; Huang, 2015).

The comparison of procedural elements showed that both countries very often focus on unit conversion, measuring with a ruler, and addition and subtraction of length. In the Taiwanese textbooks more tasks ask for direct comparison and measurement with nonstandard units, whereas the German textbooks stress visual estimation. Overall, the Taiwanese textbooks ask much more for concrete actions, whereas the German textbooks stress more abstract and mental procedures. How are these results connected to length understanding and estimation abilities?

In concluding these preliminary findings we have to point out that we ourselves are examples of cultural differences. Up to now, every person coded the textbooks of her own country. So the coding procedure has to be done vice versa or perhaps with a person who can translate Chinese directly to German. We also want to broaden and deepen our analysis to get a better understanding of the differences. If we take a broader corpus besides the textbooks, we may get a deeper insight in the differences how teachers and students work with the textbooks in both countries, and how the denser curriculum in the German textbooks is implemented differently from the Taiwanese in classrooms. A much deeper analysis with regard to the coding scheme of Lee and Smith III (2011) as well as qualitative analyses may help to get a better understanding of the so far superficial suggestion that German length learning is more abstract and mental than the Taiwanese one and may result in other length estimation abilities.

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