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Web of problem threads (WPT) — a theoretical frame and task design tool for inquiry-based learning mathematics

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Goal, Context and Methodology

The poster presented the basic concepts and a sample of the web of problem threads (WPT), the core element of a theoretical framework under development, and a tool for task-design in mathematics education, based on the qualitative analysis of the Pósa-method for inquiry-based learning mathematics. The WPT is the result of the research aiming at, on the one hand, theorizing the ‘intuitively’ developed Pósa method, a ‘good practice’ as it is widely used in Hungarian talent care education, and on the other hand, providing theoretical background for the development and reconstruction of the method to be applied in public education.

The term Pósa method refers to the nationally well-reputed three-decade-long teaching practice of Lajos Pósa, in Hungarian out-of-school weekend mathematics camps for highly talented 12-18 years old students, who form a study group for 6 years. They solve connected mathematical problems (tasks) of various kinds (regarding content area), but the focus is on discovering and discussing the mathematical ideas connecting the problems and the corresponding ‘ways of thinking’, or kernels, according to our WPT framework. The Pósa-method was developed “based in teaching as craft knowledge” (Watson & Ohtani, 2015, p. 5), lacking the construction of, or building on any theoretical framework. There is also a demand (by the Hungarian Academy of Sciences) for the re-design of the method to be applied in public education, based on a theoretical background to be built. The main goal of the research is to subsequently (re)construct the theoretical frame and the tools of the task-design of the Pósa method. The term reverse didactic engineering is suggested for this research methodology (also based on the discussion in TWG 17 of CERME11).

Theoretical Background, Steps of Theorizing and some Results

Based on mathematical content analysis, the web of problem threads has been theorized as the first step of theorizing, focusing on specific kinds of connections, common features (called the kernels) between the mathematical tasks (problems). According to this theorization, a set of connected tasks, in a partially fixed order, creates a thread of the problems. Kernel is the manifestation of a kind of connection (common feature) that creates the thread (that can actually have multiple kernels). As some problems belong to several threads, threads cross each other, forming a web, the WPT. The problems are selected and created (partially) for giving birth to the kernels, and not (usually) for their own sake.

In the 2nd chapter of the 22nd ICMI study (Watson & Ohtani, 2015), C. Kieran, M. Doorman and M. Ohtani categorize theoretical frames into grand, intermediate-level, and domain-specific frames.
Our study aims at (re)constructing an intermediate-level framework, which we call the Theory of WPTs (TWPT), and links this to already established frameworks, such as the anthropological theory of the didactic (ATD) and its application, known as study and research paths (SRP) (Chevallard, 2007; Bosch & Gascón, 2014; Watson & Ohtani, 2015, pp. 260–272). The first step of theorizing will in a second step be analyzed through the lenses of ATD, where WPT is considered as part of the technology and theory elements of the studied praxeologies (Bosch & Gascón, 2014), which, similarly to SRPs, focuses on the connections between tasks (or questions). Therefore, considering ‘distribution as a dilemma’ in the categorization of design elements of tasks along 5 dilemmas by P. Sullivan, L. Knott and Y. Yang in the 3rd chapter of the 22nd ICMI study, both task-design approaches favour creating “doing mathematics” tasks (Watson & Ohtani, 2015, pp. 91–94).

The poster is to present a sample of a WPT (see Figure 1 below) linked preliminarily to the ATD, with tasks, solutions, and the analysis of the highlighted ‘kernels’ yieldingness and invariant (quantities). In Task B (Can you tell a power of 3 that ends with 127?), we do not need to consider divisions by $10^3$, it is enough (yieldingness) by it’s divisor, 8, as any power of 3 divided by 8 gives 1 or 3 as the remainder.

![Figure 1: A sample of the Pósa WPT](image)

Some other kernels in the TWPT are experimentation, bounds (upper and lower), recursion (recursive thinking), induction, and proof of impossibility. The collection and analysis of a (more) complete set of kernels is one of the main future goals of the present research.

References

