

Exploring teachers' assignment of mathematical competencies to planned lessons using Epistemic Network Analysis

Benjamin Brink Allsopp¹, Jonas Dreyøe¹, Andreas Lindenskov Tamborg¹,
Brendan Eagan², Sara Tabatabai², David Williamson Shaffer^{1,2} and Morten Misfeldt¹

¹Aalborg University, Denmark, ben@learning.aau.dk;

²University of Wisconsin - Madison, USA.

In this paper, we explore teachers' use of a curriculum framework involving mathematical competencies in planning teaching activities. We have used data from a pilot learning management system and Epistemic Network Analysis to plot a visualisation of a 2D space displaying a network of teachers' associations of mathematical competencies to their teaching activities. We interpret this visualisation to create a conceptual model for how teachers understand mathematical competencies when planning classroom activities.

Keywords: Epistemic network analysis, learning management systems, mathematical competencies, teacher planning.

Introduction

There is a long history of constructing *theoretical* conceptual models in educational research. In mathematics education in Denmark, there is an elaborated understanding of what characterizes mathematical activity. During the last 15 years, the concept of mathematical competencies has been developed and elaborated (Niss & Højgaard, 2011; Niss & Højgaard, 2002). In this context, these competencies are described as "... a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Brooks, Greer & Gutwin, 2014).

The Danish Ministry of Education identifies six competencies in the mathematics curriculum for primary and lower secondary (UVM, 2014). *Problem tackling* is about being able to pose and solve mathematical problems, where a mathematical problem is a task where it is obvious that an answer must exist but the method of solution is unclear and can have different outcomes. The *modelling* competency is about being able to analyse, build and utilize mathematical models about other fields. *Reasoning* is about being able to follow and assess mathematical reasoning, and being able to devise and carry out informal and formal arguments as well as understand what constitutes a mathematical proof. Furthermore, it is about being able to think mathematically, where you know what questions could be posed and what answers are expected in mathematical contexts. The *representing* competency, on the other hand, is about being able to utilize different representations and switch between them as well as being able to choose the most meaningful representation. It also includes being able to handle mathematical symbol-language and mathematical formalism. The *communicating* competency is about being able to communicate in, with and about mathematics - to be able to understand and interpret expressions and texts as well as being able to express oneself about mathematics to different audiences. The *tools and aids* competency concerns the ability to operate and relate to tools and aids for mathematical operation, including Information Technology. It is also about being able to meaningfully reflect upon the choice of which tool or aid to use.

Although the individual competencies can be presented separately as above, they are considered to relate. For example, the so-called flower model (Brooks, Greer & Gutwin, 2014; Collins, 2004) depicts competencies as separate petals that overlap towards the centre, or as a set of dimensions, which together encompass the overall mathematical competence. However, the authors of this model recognize it as a purely theoretical development when they claim that “quite obviously it is impossible to produce scientific documentation that this is theoretically and empirically the case” (Brooks, Greer & Gutwin, 2014). In this paper we recognise the competencies framework as a central curriculum resource used to inform mathematics teachers in their practice. We are interested in exploring how a framework like this is understood and utilized by teachers when planning their lessons. And more specifically if it is possible to use quantitative data to develop conceptual models of how teachers relate the mathematical competencies. Hopefully such models would hold the potential to support the critical reflections of individual teachers.

Aims, data & methods

This paper describes a pilot project conducted in anticipation of large amounts of data generated from learning management systems, which have recently been adopted in Denmark. At the time of writing, this data set is not available, and instead, we use an inferior data set with the goal to explore a specific approach to visualizing data, and using these visualisations in hypothesis and heuristic building.

The data set used in this experiment was collected using the Goal Arrow software. The software was specifically designed to investigate the opportunities and challenges of digitally supporting teachers’ goal-setting for teaching activities. In the Goal Arrow, ca. 70 mathematics teachers associated one or more of the six mathematical competencies with their various teaching activities. Nothing was done to ensure that the teachers were representative, and the data included both empty submissions and repeated submissions due to a lack of user input validation. The latter was corrected manually, but with a total of only 182 records remaining, the data set here is not expected to support claims to the generality of a derived conceptual model.

In this study, we employ Quantitative Ethnography (Shaffer, 2017) to provide a method where qualitative and quantitative methods blend together completely by grounding quantitative results in a qualitative understanding of data. It aims at theoretical saturation of statistical models and using quantitative results to enable thick ethnographic descriptions (Shaffer, 2017). More specifically we used the Epistemic Network Analysis (ENA) tool which examines connections in data and uses Singular Value Decomposition (SVD) to represent it as dynamic network model that maximizes the variance in the data using a reduced number of dimensions (Shaffer, Collier & Ruis, 2016).

Quantitative Ethnography describes coding as assigning meaning to observable actions or elements of the discourse observed in one context of a specific culture. In this study, the contexts are individual teachers’ teaching activities and the coding is done by the teachers themselves when they assign relevant mathematical competencies to their individual teaching activities. In the data, each competency is represented in its own column where its absence or presence in each record of teaching activity is indicated by 0 or 1.

We imported this data into ENA and defined an *ENA set* where we specified the codes (the competencies), the conversations (individual teaching activity records) and the units (which we do not use here, but allow us to filter and group records in the visualisations). We plotted the ENA set using default settings to produce the visualisation of the competencies' interrelations depicted in Figure 1.

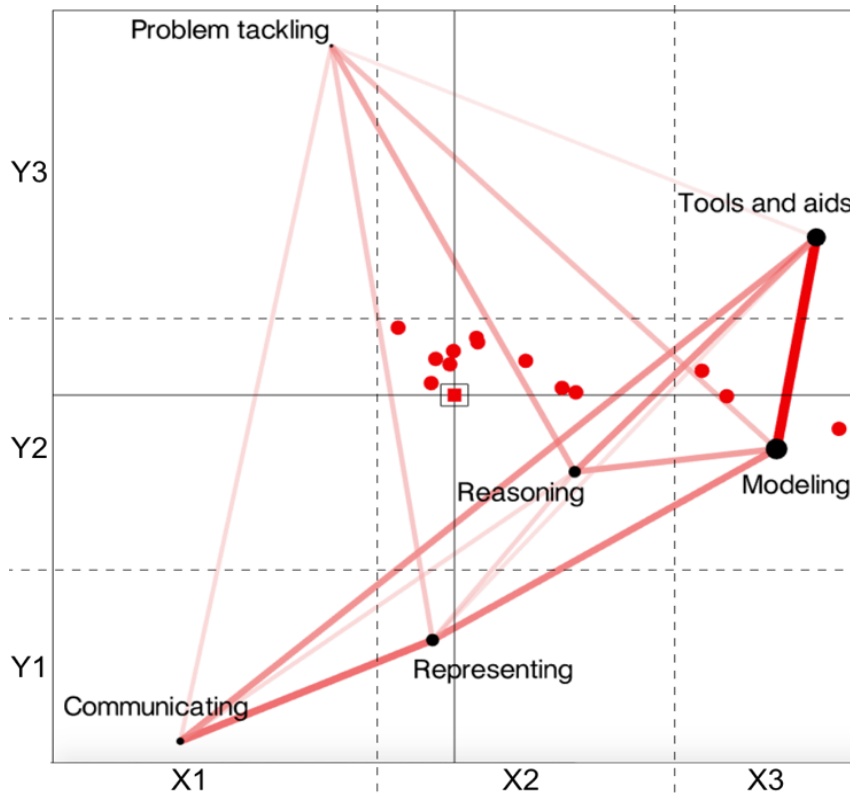


Figure 1: The space of mathematical competencies

The plotting uses SVD, which reduces the number of dimensions while still showing the maximum variance of co-occurring competencies (Shaffer, Collier and Ruis, 2016). The first dimension (the x axis) explains the most variance, and the second dimension (the y axis) explains the second most variance in the data. Individual teaching activities become the red plotted points. These seem fewer in number than the 182 records because they often overlap and should not be confused with the nodes in the network. The black plotted nodes in the network represent the individual competencies (codes). The lines between nodes represent co-occurrence of the competencies in teaching activities. The strength of the connections between competencies, determined by their frequency of co-occurrences, is represented using the thickness and saturation of the lines between nodes.

The positions of competencies (the nodes) in this network graph can be used to interpret the dimensions in the space and this can explain the positions of individual teaching activities (red points). This can provide a conceptual frame (Shaffer & Ruis, 2017) to interpret different combinations of mathematical competencies in teaching settings. It is our ability to do this interpretation that becomes the measure of this approach to generating empirically driven conceptual models.

Interpretation

By looking at the visualisation generated by ENA shown in Figure 1, we identified and used two different approaches to characterizing the axes.

1. For each axis, we look at the nodes with the greatest distance.
2. For each axis, we split the competencies into three groups and compare these groups.

First, we looked at the *opposites* on the y-axis which are *problem tackling* and *communicating*. From the definitions of these competencies, we understand *communicating* to be about being able to communicate in, with and about mathematics versus *problem tackling* which is about working in the nitty gritty with mathematical tasks that require the activation of more than routine skills. It is tempting to tentatively recognize the y-axis as being about “talking the talk” versus “doing actual mathematics”, or using Collins’ (2004) concepts, *interactional expertise* versus *contributory expertise*. But how well is this supported when we consider the other competencies that also vary on the y-axis?

With regards to splitting the competencies into three groups, we overlaid the visualisation with a grid that segregated the competencies, as shown with the dashed lines in Figure 1, such that the following groups on the y-axis were identified: Y1) *communicating* and *representing*, Y2) *modelling* and *reasoning* and Y3) *tools and aids*, and *problem tackling*.

By considering *communicating* and *representing* as a group (Y1) we see something different about the lower end of the y-axis than when we only considered *communicating*. *Representing* is not about saying the “right” things but more about getting the meaning across, by having something stand for something else. *Representing* is therefore concerned with getting the actual meaning across by any means and if necessary by multiple means, and not just to other people, but also to oneself. In other words, the lower end of the y-axis is not categorizable as mere *interactional expertise* but also requires *contributory expertise*. Combined with our understanding of *communicating*, the bottom end of the y-axis seems more precisely to be about describing and explaining mathematical issues, topics or problems in different ways in different situations.

We revisit the other end of the y-axis in the same way as above. By considering *problem tackling* and *tools and aids* as a group (Y3) we do not have to change our understanding of the top end of the y-axis, from when we only considered *problem tackling*. The *tools and aids* competency is about being able to utilize tools and aids in a mathematical context, and being able to choose the right tool for the job. *Tools and aids* are thus like *problem tackling* concerned with working on or with mathematical problems, tasks and so on efficiently. The top end of the y-axis seems to remain something to do with the actual doing of mathematics with a focus on pragmatic and efficient choices of means for mathematical work.

We must still consider the two remaining competencies, *reasoning* and *modelling*, which form the middle group (Y2). However, the interpretation of their positioning does not disturb the understanding of the y-axis discussed above. The position of *reasoning* near the centre of the y-axis can be explained by the perceived need for *reasoning* along the entire y-axis. It is needed both for describing and explaining as well as working with mathematical problems. The position of

modelling near the centre of the y-axis can be explained slightly differently as a necessary bridge between talking about problems from other domains, that we want to apply mathematics to as well as being able to work on those problems with mathematics.

By looking at the placement of competencies along the y-axis, firstly with a focus on the extremes and secondly in three groups, we understand the y-axis as representing a continuum from talking about, drawing, describing and explaining mathematics over *modelling* to actually doing mathematical moves, actions, and solutions with tools and aids. A continuum from describing and explaining to doing mathematics.

The x-axis does not in the same way as the y-axis have a clear pair of *opposites* to help us interpret it, but we did attempt to compare the two competencies with the most extreme distance on the x-axis; *communicating* and *tools and aids*. It was tempting to tentatively recognize the x-axis as being about psychology and cognition versus interacting with something tangible or material. But how well is this supported when we consider the other competencies that also vary on the x-axis. We approached this by splitting the competencies into, and interpreting the three groups: X1) *communicating* and *problem tackling*, X2) *representing* and *reasoning* and X3) *modelling* and *tools and aids*.

Considering the competencies *tools and aids* and *modelling* (X3) we first identified for each of these a new concept that a) seemed to relate to its respective competency and b) seemed to relate to the other concept. The definition of *tools and aids* clearly defines it to be very much about technical tools and not about mental tools (e.g. about straight edges and not about theorems). This gave us the concept “materiality”. We interpreted the definition of *modelling* to be an especially map-like portrayal in the sense that it closely resembles (although simplifies) the object being studied. Together, the concepts materiality and map-like suggested something being tangible or close to the world at the right end of the x-axis. Of course, both competencies are not physical objects in the world, they are psychological and cognitive in nature, but there seems to be an immediacy of attention to things outside of a purely mental realm; to material or non-abstract things. Loosely speaking, to things in the world.

The above immediacy of attention to things outside of a purely mental realm seemed much less necessary when we considered the two competencies *problem tackling* and *communicating* (X1). Of course, both *problem tackling* and *communicating* may relate to something in the world beyond mental or linguistic gymnastics, but the necessary immediacy does not seem to be there because *communicating* and *problem tackling* does not to the same degree have to relate to something concrete but can remain in the abstract. Although all competencies are in virtue of being competencies psychological and cognitive in nature, the competencies on the left end of the axis can be more symbolic or interpretive, and less immediately connected to the world. Loosely speaking, to things in the mind.

The remaining two competencies, *representing* and *reasoning*, which make up the middle group (X2) of the x-axis also do not (like the middle group of the y-axis) seem to disrupt our interpretation of the x-axis. *Reasoning*'s position near the middle of the x-axis can (like its position near the centre of the y-axis) be explained in terms of it being needed across the whole axis. regardless of how

directly one engages in the world, mathematical thinking requires reasoning. *Representing's* position near the middle of the x-axis can (like *modelling's* position near the centre of the y-axis) be explained as a necessary bridge between the two ends of the x-axis. To be able to apply abstract mathematical thinking or communicate to the world we need to portray the world, but those portrayals need not be as map-like as those used in *modelling*. Other representations are also needed. One of these representations is natural language explanations, which must be considered an important part of the *communicating* competency.

By looking at the placement of competencies along the x-axis in three groups, we understand the x-axis as representing a perceived continuum from working in the abstract to working more directly in the world. We have summarized this as a continuum from “in-the-mind” to “in-the-world”.

Together these considerations allow us to identify the dimension of greatest variance (the x-axis) as a continuum between “in the mind” and “in the world”, and the dimension with the second greatest variance (the y-axis) as a continuum between “describing and explaining” and “doing mathematics”.

Limitations & discussion

We have discussed earlier that the available data is inferior. The small number of teachers and registered teaching activities, the fact that no effort was made to ensure that the involved teachers were representative, and the uncertainty due to the lack of user input validation, prevent us from drawing confident conclusions.

This paper and our use of Goal Arrow data is a pilot application of the method we have described. The emphasis in this paper is on exploring an approach, and especially the extent to which we can meaningfully interpret the visualisations. Our interpretation of the current data has allowed us (as a hypothesis or heuristics building exercise) to define a space of mathematical competences. In this space, we define the dimension of greatest variance (the x-axis) as a continuum between “in the mind” and “in the world”, and the dimension with the second greatest variance (the y-axis) as a continuum between “describing and explaining mathematics” and “doing mathematics”.

What is most important is that we expect to be able to use ENA for visualizing massively more of the same sort of data when made available from the learning management systems currently being implemented in Denmark. We now have reason to believe that this data and their visualisation in ENA can be interpreted meaningfully. We must recognize that these interpretations are based on what teachers have expressed about their lessons, which may differ from what they have actually planned, and to a greater degree from what is actually enacted in class. However, the process of associating lessons with competencies is well established in Denmark.

Defined spaces like the one above can be thought of as adding user friendliness or simplifying an existing understanding of mathematical competencies; we are looking at a 2D space instead of a six-columned table. However, there is also a sense in which the defined space is a development of an understanding of mathematical competencies in its own right. When the teachers relate the competencies in individual teaching activities, they are adding information. They are pointing at how mathematical competencies are likely to be engaged together. They are also negotiating

between the formal breakdown of the competencies and their own understanding of what mathematics requires. Because of this, we can think of these defined spaces as new conceptual models of mathematical competencies developed using leaning analytics.

We can think of these conceptual models as a complement to the competency flower model. Most obviously, they give us another way to think about the competencies. When planning lessons, a space such as the one presented in this paper may work as a tool to support teachers' reflection and discussion on the direction of their teaching activities. Supported interactively, we can imagine teachers aiming their teaching activities at locations in the space, registering teaching activities by associating these locations to teaching activities and compare their teaching activities to those of other teachers' or their own over time. Such empirically driven conceptual models could also potentially support research and policy. For example, they could direct the attention to challenges or differences in the progression of mathematics education. Specifically, it can raise flags for individual grades, that seem to lack certain areas or have significantly different networks.

Conclusion

In this paper, we consider the context in which mathematical competencies are used in Denmark, and use data collected in the Goal Arrow software, where teachers associate competencies to their teaching activities. We describe the creating of an ENA set and the plotting of a visualisation comprising a 2D space displaying a network based on the co-occurrence of mathematical competencies in teaching activities. Notwithstanding limitations on the data, we attempt a thorough interpretation of the visualisation which allows us to explore the approach as a way of understanding how teachers use the competencies framework in their lesson planning.

References

- Brooks, C., Greer, J., & Gutwin, C. (2014). The data-assisted approach to building intelligent technology-enhanced learning environments. In B. White, & J.A. Larusson (eds.). *Learning analytics: From research to practice* (123-156). New York: Springer.
- Collins, H. (2004). Interactional expertise as a third kind of knowledge. *Phenomenology and the cognitive sciences*, 3(2), 125–143. <https://doi.org/10.1023/B:PHEN.0000040824.89221.1a>
- Johnson, L., Adams Becker, S., & Hall, C. (2015). *2015 NMC Technology outlook for Scandinavian schools: A horizon project regional report*. Austin, Texas: The New Media Consortium.
- Niss, M. A., & Højgaard, T. (2011). *Competencies and mathematical learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde: Roskilde University, Department of Science, Systems and Models.
- Niss, M., & Højgaard Jensen, T. (2002). *Kompetencer og matematiklæring - Ideer og inspiration til udvikling af matematikundervisning i Danmark* (Uddannelsesstyrelsens temahæfteserie, Undervisningsministeriet 2002 (18)). Copenhagen: Undervisningsministeriet.
- Shaffer, D. W. (2017). *Quantitative ethnography*. Madison, Wis.: Cathcart Press.

- Shaffer, D. W., Collier, W., & Ruis, A. R. (2016). A tutorial on Epistemic Network Analysis: Analyzing the structure of connections in cognitive, social, and interaction data. *Journal of Learning Analytics*, 3(3), 9–45. <https://doi.org/10.18608/jla.2016.33.3>
- Shaffer, D. W., & Ruis, A. (2017). Epistemic Network Analysis: A worked example of theory-based learning analytics. In C. Lang, G. Siemens, A. Wise, & D. Gašević (Eds). *Handbook of learning analytics* (pp. 175–187). SOLAR. <https://doi.org/10.18608/hla17.015>
- UVM (Ministry of Education). (2014). Fælles Mål (Common Goals) located at <https://www.emu.dk/omraade/gsk-lærer/ffm/matematik>, November 23rd., 2018.
- White, B., & Larusson, J. A. (2014). Identifying points for pedagogical intervention based on student writing: Two case studies for the “Point of Originality”. In B. White & J. A. Larusson (eds.). *Learning analytics* (pp. 157–190). New York, NY: Springer. https://doi.org/10.1007/978-1-4614-3305-7_8