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Time Domain Full Waveform Inversion involving Discontinuous Galerkin approximation

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Time Domain Full Waveform Inversion involving Discontinuous Galerkin approximation

Waves 2019

Pierre Jacquet

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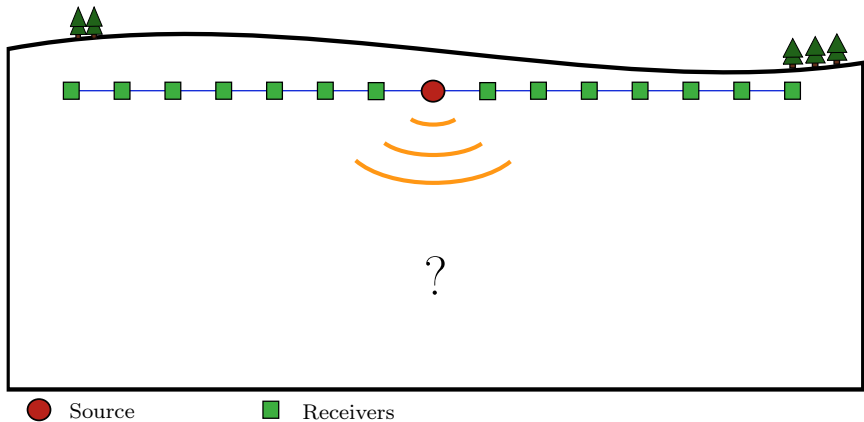
Atle Andreas, Barucq H el ene, Calandra Henri, Diaz Julien

Second year PhD Student

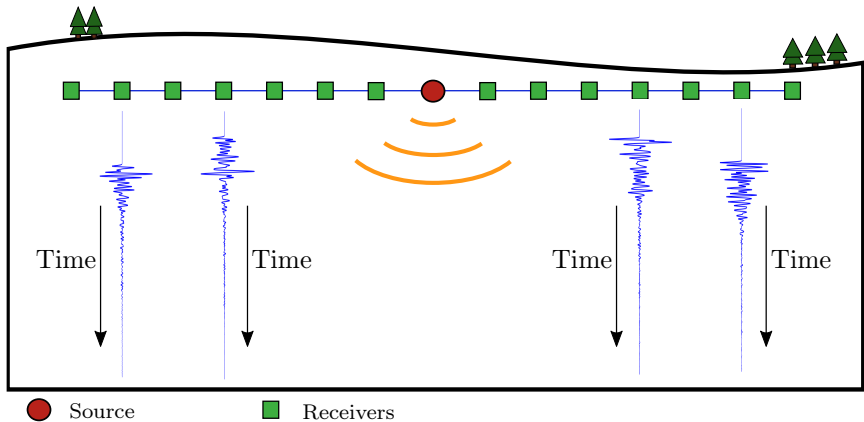
Inria - Magique 3D - DIP

Pau, FRANCE

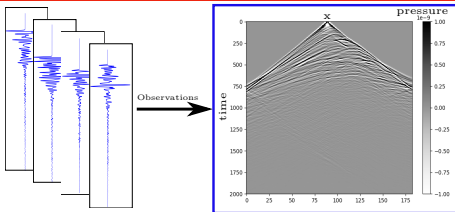




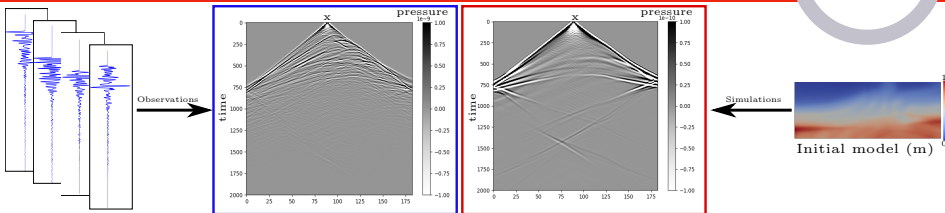
Seismic Acquisition

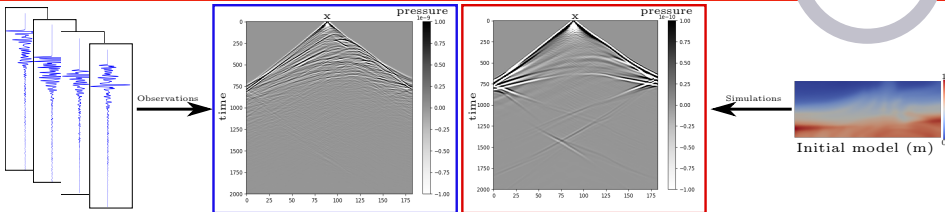


FWI Workflow



FWI Workflow





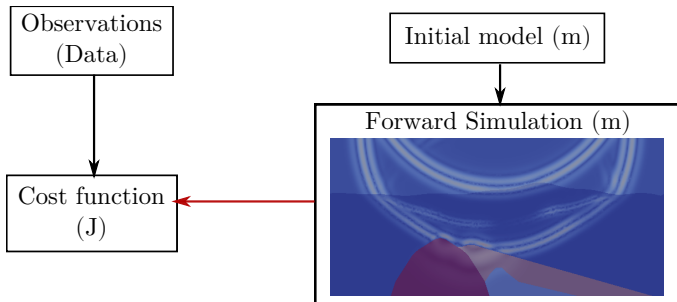
Cost function to minimize :

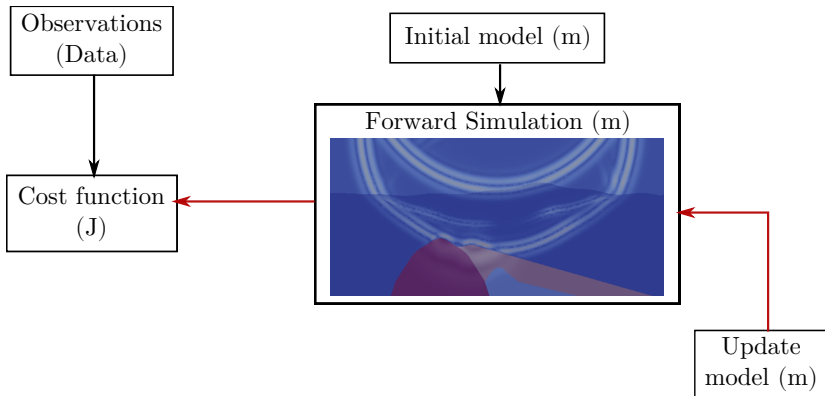
$$\mathcal{J}(\mathbf{m}) = \frac{1}{2} \|\mathbf{d}_{obs} - \mathcal{F}(\mathbf{m})\|^2$$

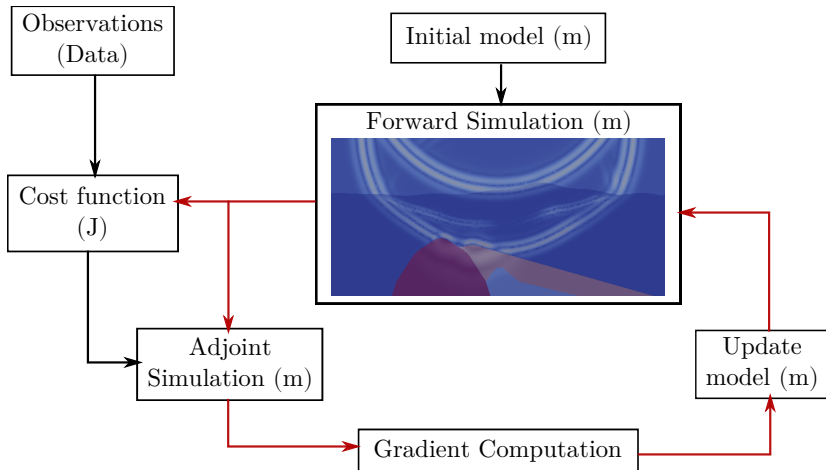
- ▶ $\mathcal{F}(\mathbf{m})$ is the restriction on the receivers of the simulated waves in the medium \mathbf{m} . (With $\mathbf{m} = \mathbf{c}, \rho, \kappa \dots$)
- ▶ FWI iterates until $\mathcal{J}(\mathbf{m}) \rightarrow 0$

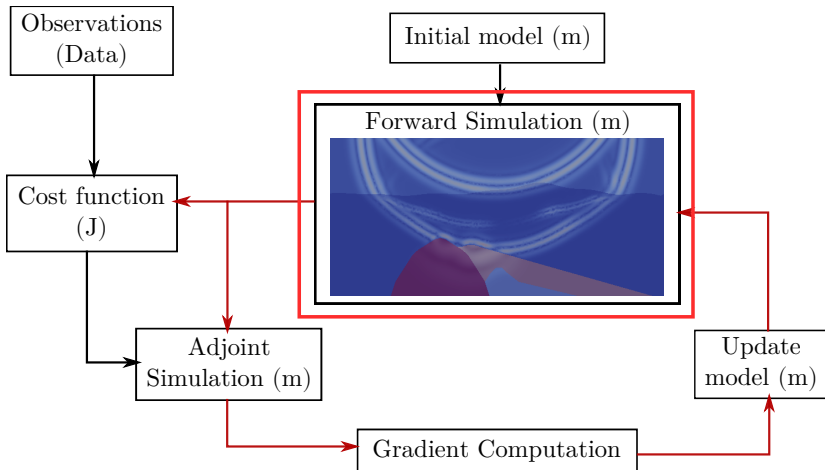
[1] Patrick Lailly
The seismic inverse problem as a sequence of before stack migrations
Conference on Inverse Scattering

[2] Albert Tarantola
Inversion of seismic reflection data in the acoustic approximation
Geophysics, Vol. 49, 1984



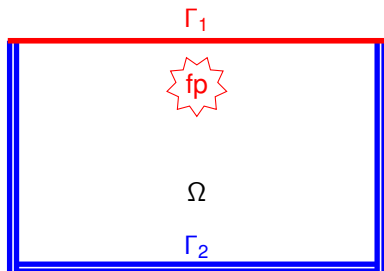






First order acoustic wave equation

$$\left\{ \begin{array}{l} \frac{1}{\rho c^2} \frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot \mathbf{v} = f_p \quad \text{on } \Omega \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{p} = 0 \quad \text{on } \Omega \\ \mathbf{p} = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \mathbf{p}}{\partial t} + \mathbf{c} \nabla \mathbf{p} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \mathbf{p}(0) = 0, \quad \mathbf{v}(0) = 0 \end{array} \right.$$



Domain with Absorbing Boundary
Conditions



Space Discretization : Discontinuous Galerkin Elements

- ▶ Nodal (Lagrangian / Jacobian)
- ▶ Modal (Bernstein-Bézier)

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Discontinuous Galerkin Elements

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Semi-discretized model :

$$\frac{\partial}{\partial t} \bar{\mathbf{U}}(t) = A\bar{\mathbf{U}}(t) + \bar{\mathbf{F}}(t)$$

with :

$$\bar{\mathbf{U}}(t) = \begin{pmatrix} \bar{\mathbf{P}}(t) \\ \bar{\mathbf{V}}(t) \end{pmatrix}$$

Space Discretization :
Discontinuous Galerkin Elements

- ▶ Nodal (Lagrangian / Jacobian)
- ▶ Modal (Bernstein-Bézier)

Time schemes :

- ▶ Runge Kutta 2/4
- ▶ Adams Bashforth 3

Semi-discretized model :

$$\frac{\partial}{\partial t} \bar{\mathbf{U}}(t) = A\bar{\mathbf{U}}(t) + \bar{\mathbf{F}}(t)$$

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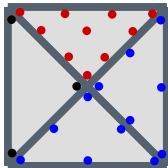


Assets of Discontinuous Galerkin Methods :

- ▶ Unstructured grid (enable to match the topography and medium irregularities)
- ▶ Robust to physical discontinuities
- ▶ hp-adaptivity
- ▶ Massively parallel performance properties



h-adaptivity



p-adaptivity with P1,
P2, P3 elements

Time Domain Full Waveform Inversion

Seismic Acquisition

FWI Workflow

Forward Discretization

Adjoint Studies

Adjoint then Discretized

Discretize then Adjoint

Some Results

1D Preliminary tests

2D Time Domain FWI Results

2D Multiscale Reconstruction

Lagrangian functional [1] :

$$\mathcal{L}(\hat{\mathbf{u}}, \hat{\boldsymbol{\lambda}}, \mathbf{m}) = \frac{1}{2} \|d_{obs} - \mathcal{R}(\hat{\mathbf{u}})\|^2 + \langle Forward_{\mathbf{m}}(\hat{\mathbf{u}}) - f_p, \hat{\boldsymbol{\lambda}} \rangle$$

If $\hat{\mathbf{u}} = \mathbf{u}$ Solution of the Direct Problem $\iff (Forward_{\mathbf{m}}(\mathbf{u}) - f_p = 0)$:

$$\mathcal{J}(\mathbf{m}) = \mathcal{L}(\mathbf{u}, \hat{\boldsymbol{\lambda}}, \mathbf{m})$$

[1] Plessix R-E

A review of the adjoint-state method for computing the gradient of a functional with geophysical applications
Geophysical Journal International, Volume 167, Issue 2, 2006

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Let us choose $\hat{\boldsymbol{\lambda}} = \boldsymbol{\lambda}$ such as $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = 0$

$$(\mathcal{R}^* d_{obs} - \mathbf{u}) + Forward_{\mathbf{m}}^*(\boldsymbol{\lambda}) = 0$$

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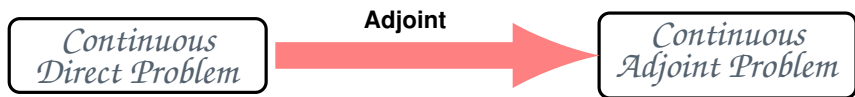
For $Forward_{\mathbf{m}}(\mathbf{u}) - f_p = 0$:

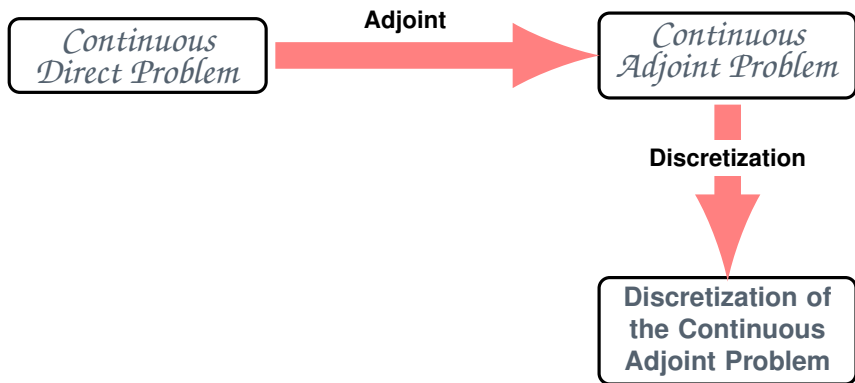
$$\partial_{\mathbf{m}_i} \mathcal{J}(\mathbf{m}) = \partial_{\mathbf{m}_i} \mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}, \mathbf{m}) = \partial_{\mathbf{m}_i} \langle Forward_{\mathbf{m}}(\mathbf{u}), \boldsymbol{\lambda} \rangle$$

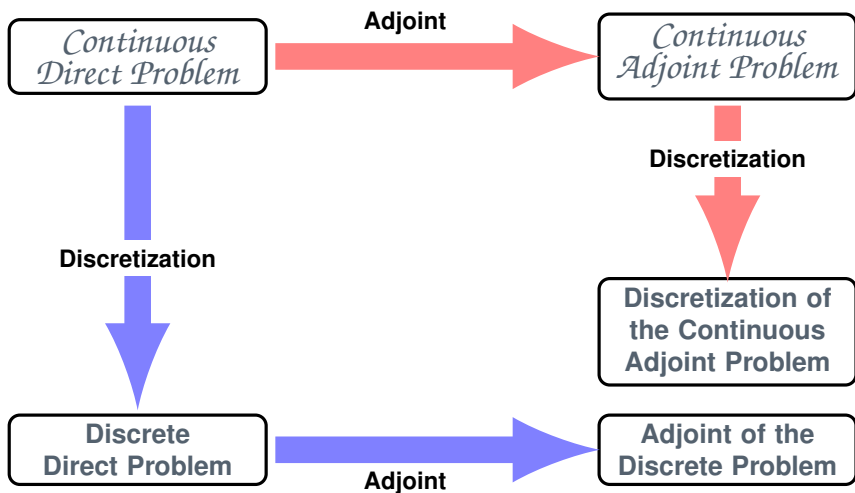
[1] Plessix R-E

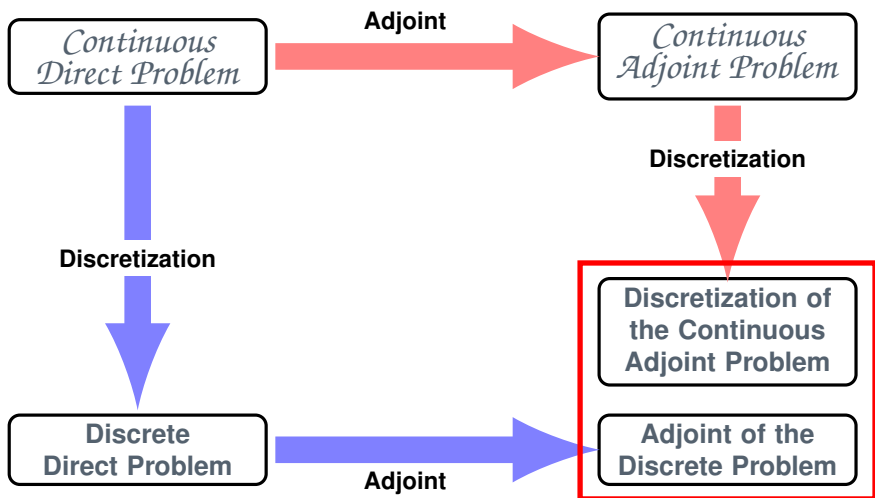
A review of the adjoint-state method for computing the gradient of a functional with geophysical applications
Geophysical Journal International, Volume 167, Issue 2, 2006

*Continuous
Direct Problem*









$$\mathcal{J}(\mathbf{p}) = \frac{1}{2} \|d_{obs} - R\mathbf{p}\|^2$$

$$\left\{ \begin{array}{l} \frac{1}{\rho c^2} \frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot \mathbf{v} = f_p \quad \text{on } \Omega \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{p} = 0 \quad \text{on } \Omega \\ \mathbf{p} = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \mathbf{p}}{\partial t} + \mathbf{c} \nabla \mathbf{p} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \mathbf{p}(0) = 0, \quad \mathbf{v}(0) = 0 \end{array} \right.$$

$$t \in [0, T]$$

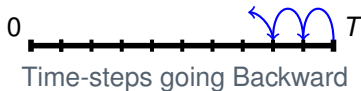
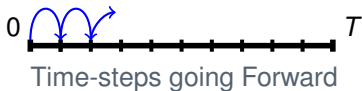
$$\left\{ \begin{array}{l} \frac{1}{\rho c^2} \frac{\partial \lambda_1}{\partial t} + \nabla \cdot \lambda_2 = \frac{\partial \mathcal{J}}{\partial \mathbf{p}} \quad \text{on } \Omega \\ \rho \frac{\partial \lambda_2}{\partial t} + \nabla \lambda_1 = 0 \quad \text{on } \Omega \\ \lambda_1 = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial \lambda_1}{\partial t} - \mathbf{c} \nabla \lambda_1 \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_2 \\ \lambda_1(T) = 0, \quad \lambda_2(T) = 0 \end{array} \right.$$

$$t \in [T, 0]$$

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2} \|\mathbf{d}_{obs} - R\mathbf{p}\|^2$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{\mathbf{U}}^n}{\partial t} = A\bar{\mathbf{U}}^n + \bar{\mathbf{F}}^n \\ \text{With : } \bar{\mathbf{U}}^n = \begin{pmatrix} \bar{\mathbf{P}}^n \\ \bar{\mathbf{V}}^n \end{pmatrix} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{\boldsymbol{\Lambda}}^n}{\partial t} = A\bar{\boldsymbol{\Lambda}}^n + R^*(R\bar{\mathbf{U}}^n - \mathbf{d}_{obs}) \\ \text{With : } \bar{\boldsymbol{\Lambda}}^n = \begin{pmatrix} \bar{\boldsymbol{\Lambda}}_1^n \\ \bar{\boldsymbol{\Lambda}}_2^n \end{pmatrix} \end{array} \right.$$



DtA : Discretize then Adjoint Strategy

Example With RK4



All time scheme can be summed-up such as :

$$L\bar{U} = E\bar{F}$$

RK4 time-scheme leads to :

$$\bar{U}^{n+1} = B\bar{U}^n + C_0\bar{F}^n + C_{\frac{1}{2}}\bar{F}^{n+\frac{1}{2}} + C_1\bar{F}^{n+1}$$

$$L\bar{U} = E\bar{F} = \bar{G}$$

$$\begin{pmatrix} I & & & & & \\ -B & I & & & & \\ & -B & I & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & -B & I \end{pmatrix} \begin{pmatrix} \bar{U}^0 \\ \bar{U}^1 \\ \bar{U}^2 \\ \vdots \\ \bar{U}^n \end{pmatrix} = \begin{pmatrix} \bar{G}^0 \\ \bar{G}^1 \\ \bar{G}^2 \\ \vdots \\ \bar{G}^n \end{pmatrix}$$

All time scheme can be summed-up such as :

$$\mathbf{L}\bar{\mathbf{U}} = \mathbf{E}\bar{\mathbf{F}}$$

We are looking for a Discrete Adjoint state satisfying :

$$\mathbf{L}^*\bar{\boldsymbol{\Lambda}} = -\mathbf{R}^*(d_{obs} - \mathbf{R}\bar{\mathbf{U}})$$

With the adjoint operator \mathbf{L}^* satisfying :

$$\langle \mathbf{L}\bar{\mathbf{U}}, \bar{\boldsymbol{\Lambda}} \rangle = \langle \bar{\mathbf{U}}, \mathbf{L}^*\bar{\boldsymbol{\Lambda}} \rangle$$

All time scheme can be summed-up such as :

$$L\bar{U} = E\bar{F} = \bar{G}$$

We are looking for a Discrete Adjoint state satisfying :

$$L^*\bar{\Lambda} = -R^*(d_{obs} - R\bar{U}) = \bar{D}$$

With the adjoint operator L^* satisfying :

$$\langle L\bar{U}, \bar{\Lambda} \rangle = \langle \bar{U}, L^*\bar{\Lambda} \rangle$$

$$\langle \bar{G}, \bar{\Lambda} \rangle = \langle \bar{U}, \bar{D} \rangle \quad (\text{Adjoint Test})$$

Adjoint test succeeds \iff operator L^* well established

DtA : Discretize then Adjoint Strategy

Example with RK4



RK4 time-scheme leads to :

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So :

$$L^* = \begin{pmatrix} I & -B^* & & & \\ & I & -B^* & & \\ & & \ddots & \ddots & \\ & & & I & -B^* \\ & & & & I \end{pmatrix}$$

Adjoint Then Discretize

- + Physical approach
- + Same discrete operators for Forward and Backward
- - Approximate gradient [1]
- Consistent with the discretization

Discretize then Adjoint

- + Numerical approach
- + Has an Adjoint Test
- Tremendous work to develop the adjoint operators
- Non-consistency of the adjoint state [2]

[1] Sirkes, Ziv and Tziperman, Eli
Finite Difference of Adjoint or Adjoint of Finite Difference ?
1997

[2] Sei Alain and Symes William
A Note on Consistency and Adjointness for Numerical Schemes
1997

Adjoint Then Discretize

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- Consistent with the discretization

Discretize then Adjoint

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- + / - Has an Adjoint Test (**in theory**)
- Tremendous work to develop the adjoint operators
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Seismic Acquisition

FWI Workflow

Forward Discretization

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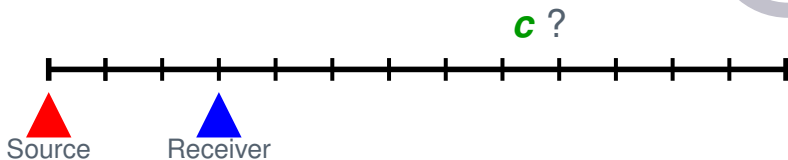
Some Results

1D Preliminary tests

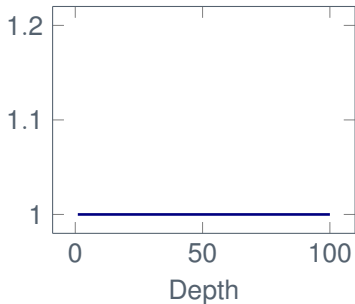
2D Time Domain FWI Results

2D Multiscale Reconstruction

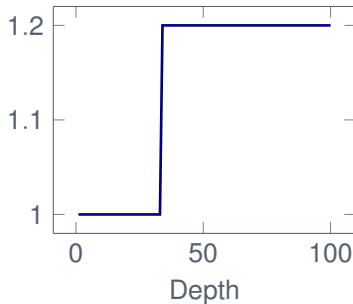
1D Preliminary tests



Initial c Model



Target c Model



1D FWI :

- ▶ Lagrange / B-Bézier Operators
- ▶ RK4 / AB3 time-schemes

Gradient expression :

$$\nabla_{\mathbf{c}} \mathcal{J} = - \int_0^T \int_{\Omega} \frac{2}{\rho \mathbf{c}^3} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt$$

1D FWI :

- ▶ Lagrange / B-Bézier Operators
- ▶ RK4 / AB3 time-schemes

Adjoint test passed with :

- ▶ With a canonical space inner-product
 $\langle u, v \rangle_X = \sum_i u_i v_i$
- ▶ With a M-space inner product
 $\langle u, v \rangle_X^M = \langle Mu, v \rangle_X$

Gradient expression :

$$\nabla_{\mathbf{c}} \mathcal{J} = - \int_0^T \int_{\Omega} \frac{2}{\rho \mathbf{c}^3} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt$$

```
./run  
--- Adjoint test ---  
inner product U/D 553123.57586755091  
inner product G/Q 553123.57586756046
```

1D FWI :

- ▶ Lagrange / B-Bézier Operators
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Adjoint test passed with :

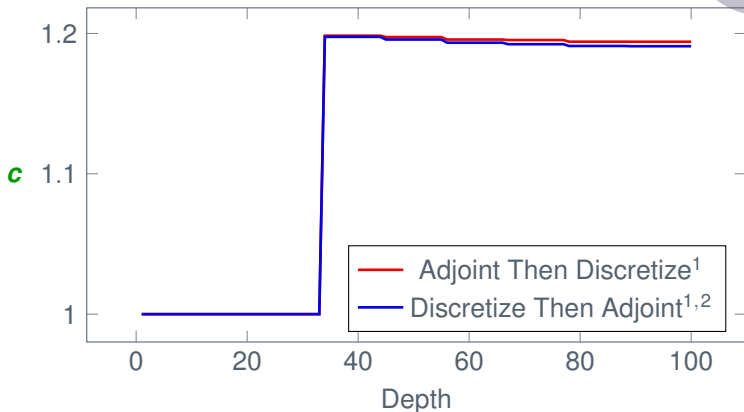
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($\langle u, v \rangle_X = \sum_i u_i v_i$)
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```
./run
--- Adjoint test ---
inner product U/D 553123.57586755091
inner product G/Q 553123.57586756046
./run
--- Adjoint test ---
inner product U/D -75077.332007383695
inner product G/Q -75077.332007386358
./run
--- Adjoint test ---
inner product U/D 125669.89223600870
inner product G/Q 125669.89223600952
```

1D Velocity Model Reconstructions



c Model at the 100th FWI iteration

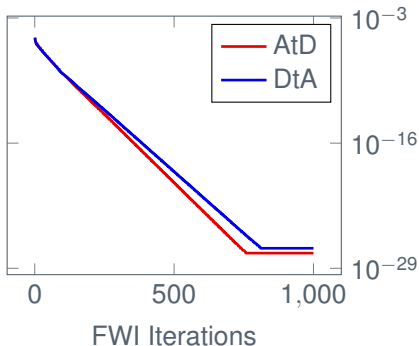
¹With Bernstein-Bézier elements and AB3 time scheme

²With canonical scalar product

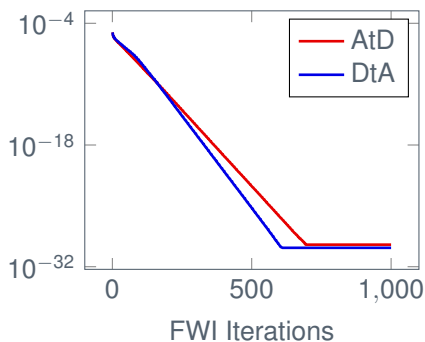
1D Velocity Model Reconstructions



With RK4 :



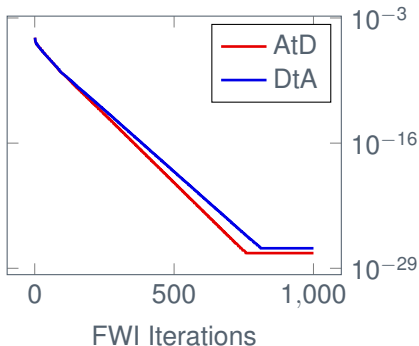
With AB3 :



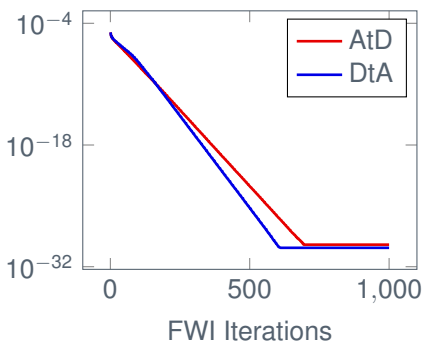
1D Velocity Model Reconstructions



With RK4 :



With AB3 :



- ▶ For RK4 scheme : AtD is slightly better than DtA
- ▶ For AB3 scheme : DtA is slightly better than AtD
- ▶ No predominant behaviour

2D FWI :

- ▶ Developed in Total environnement (DIP³)
- ▶ Nodal Space Operators (Lagrangian/Jacobian)
- ▶ Modal Space Operators (Bernstein-Bézier)
- ▶ Runge Kutta 2/4 and Adams Bashforth 3 time-schemes

Discretize Then Adjoint strategy not implemented :

- ▶ Tremendous task in a complex industrial code

³<http://dip.inria.fr/>

2D FWI :

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- ▶ Nodal Space Operators (Lagrangian/Jacobian)
- ▶ Modal Space Operators (Bernstein-Bézier)
- ▶ Runge Kutta 2/4 and Adams Bashforth 3 time-schemes

Gradient expression :

$$\nabla_{\frac{1}{\kappa}} \mathcal{J} = \int_0^T \int_{\Omega} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt \quad \text{with : } \kappa = \rho \mathbf{c}^2$$

\mathbf{c} , ρ and κ Constant per elements

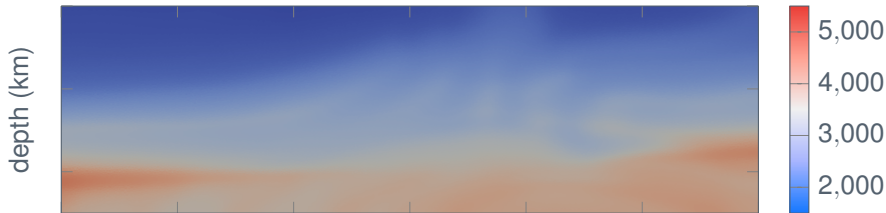
³<http://dip.inria.fr/>

2D Time Domain FWI Reconstructions

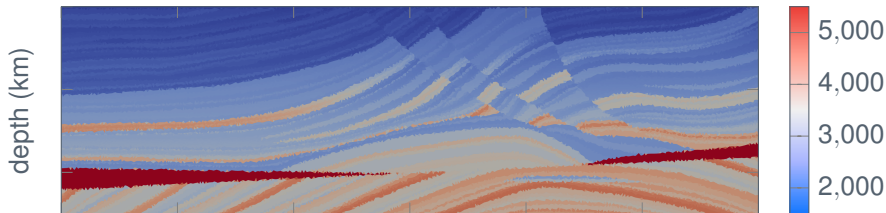
Time-schemes comparison



Initial **c** Model



Target **c** Model

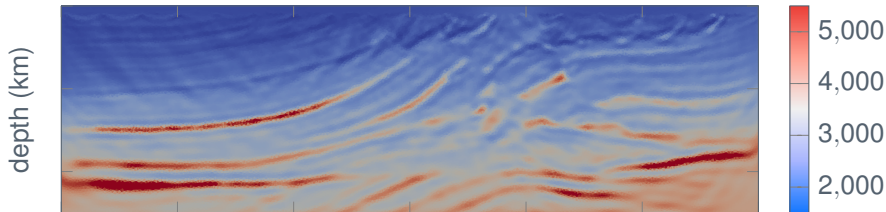


2D Time Domain FWI Reconstructions

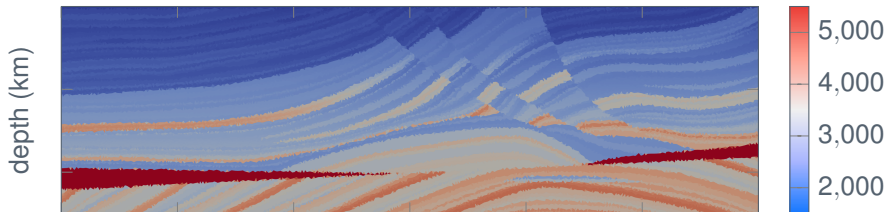
Time-schemes comparison



RK2 Reconstructed **c** Model (30 iterations)



Target **c** Model

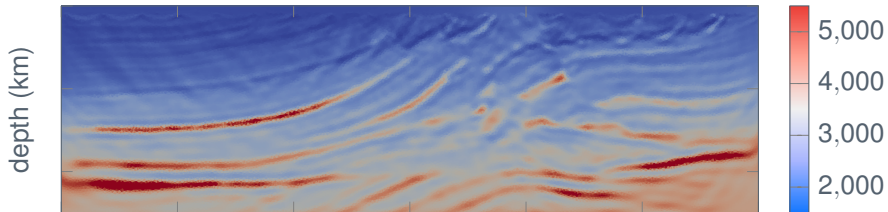


2D Time Domain FWI Reconstructions

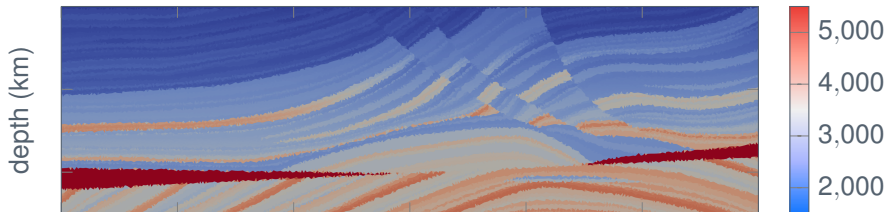
Time-schemes comparison



RK4 Reconstructed **c** Model (30 iterations)



Target **c** Model



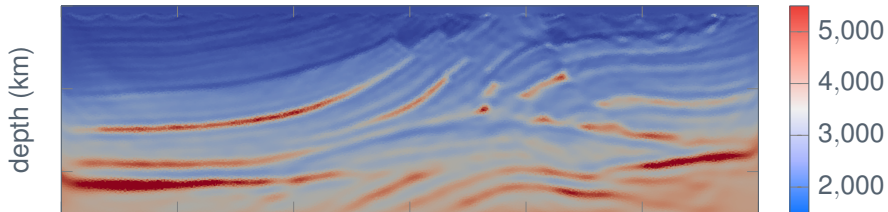
2D Time Domain FWI Reconstructions

Time-schemes comparison



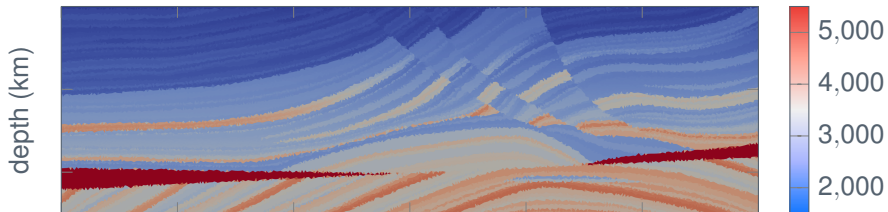
AB3 Reconstructed **c** Model (30 iterations)

$m \cdot s^{-1}$



Target **c** Model

$m \cdot s^{-1}$



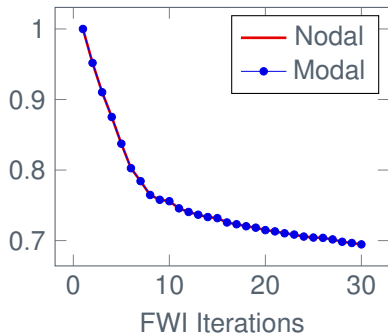
2D Time Domain FWI Reconstructions

Nodal/Modal Comparison



- ▶ 47k P1 elements
- ▶ Time Scheme : AB3
- ▶ Constant ρ model ($\rho = 1$)
- ▶ 19 sources / 181 Receivers
- ▶ Noise : SNR=10
- ▶ 30 iterations
- ▶ 120 cores
- ▶ Nodal computation time : 5h10
- ▶ Modal computation time : 7h10^[1]

Cost function evolution :



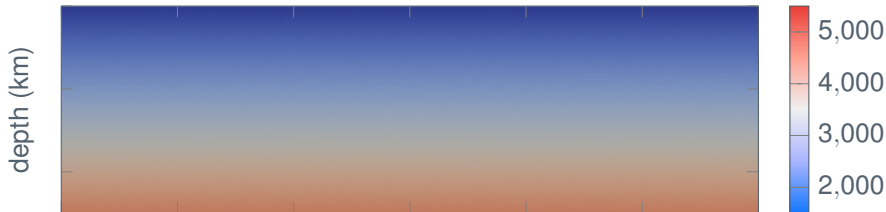
[1] Chan J. and Warburton T.
GPU-Accelerated Bernstein Bézier Discontinuous Galerkin Methods for Wave Problems
SIAM Journal on Scientific Computing 2017

2D Multiscale Reconstructions

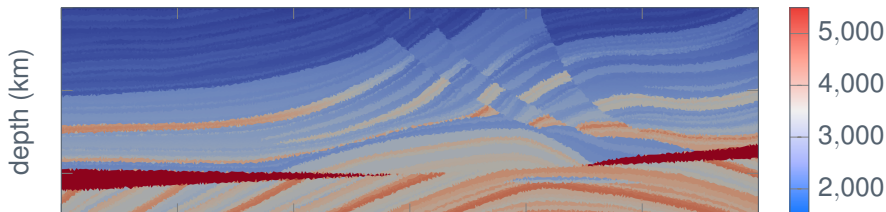
Reconstruction with an initial smooth model



Initial **c** Model



Target **c** Model



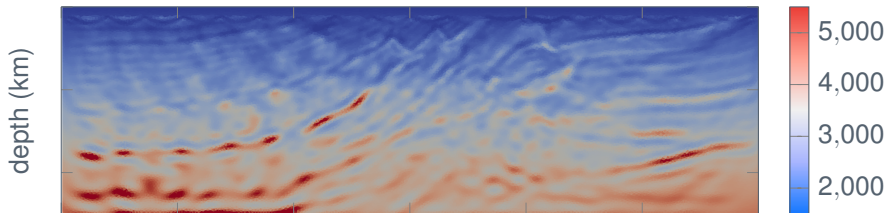
2D Multiscale Reconstructions

Reconstruction with an initial smooth model



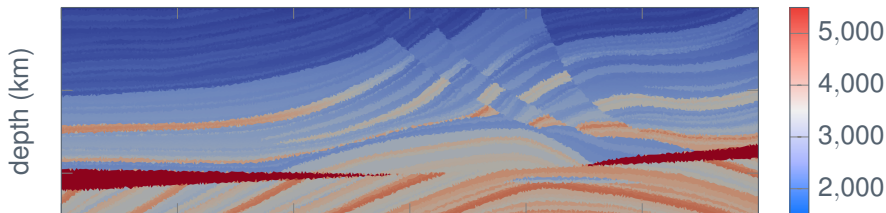
Reconstructed model **c** Model (30 iterations AB3)

$m \cdot s^{-1}$



Target **c** Model

$m \cdot s^{-1}$



2D Multiscale Reconstructions

Multiscale Principle [1]

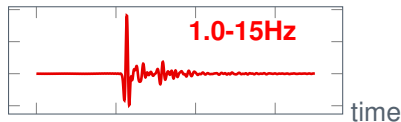
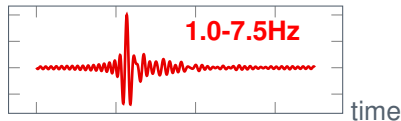
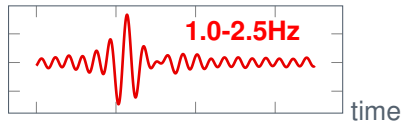


Low Frequencies
↔
Reconstruct **coarse** structures

High Frequencies
↔
Reconstruct **small** structures

Filtered Traces :

p



[1] C. Bunks, F. M. Saleck, S. Zaleski, and G. Chavent
Multiscale seismic waveform inversion
GEOPHYSICS, Vol. 60, No. 5, 1995

2D Multiscale Reconstructions

Multiscale Principle [1]



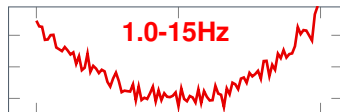
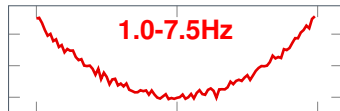
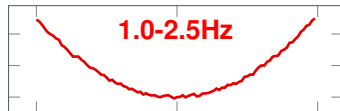
Heuristic Illustration :

\mathcal{J}

Low Frequencies



Reconstruct **coarse** structures



High Frequencies



Reconstruct **small** structures

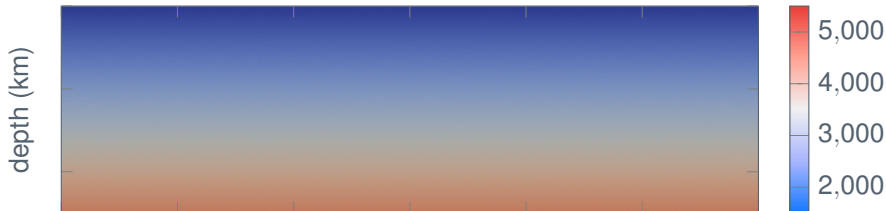
[1] C. Bunks, F. M. Saleck, S. Zaleski, and G. Chavent
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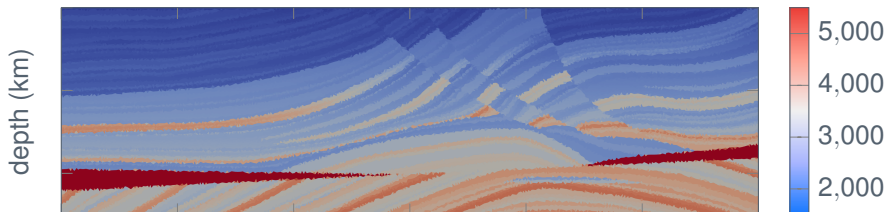
Reconstruction with an initial smooth model



Initial **c** Model



Target **c** Model



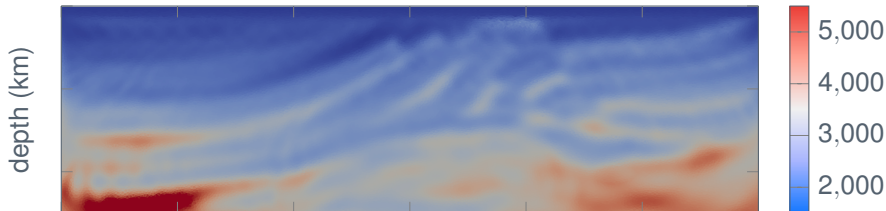
2D Multiscale Reconstructions

Reconstruction with an initial smooth model



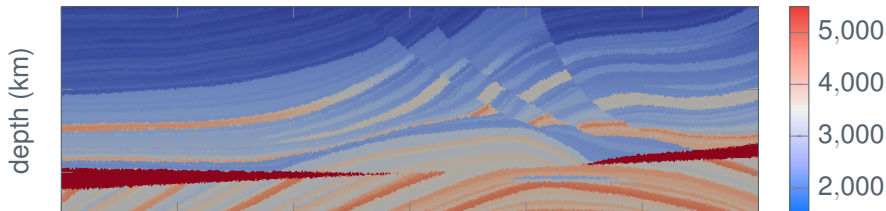
Reconstructed c Model with 1.0-2.5Hz filter

$m \cdot s^{-1}$



Target c Model

$m \cdot s^{-1}$



2D Multiscale Reconstructions

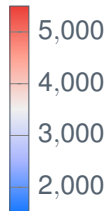
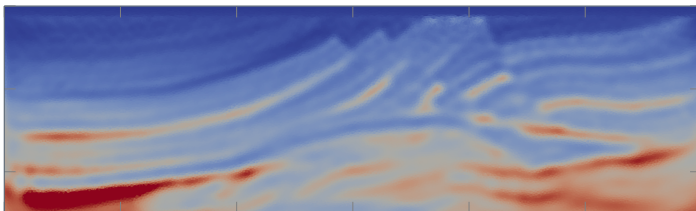
Reconstruction with an initial smooth model



Reconstructed **c** Model with 1.0-7.5Hz filter

$m \cdot s^{-1}$

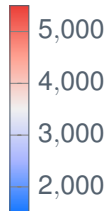
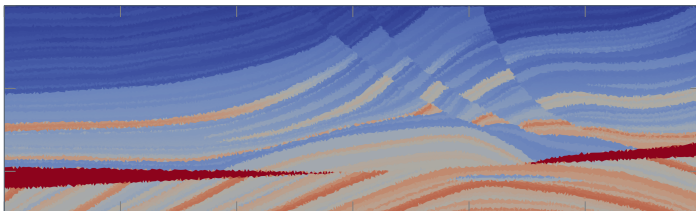
depth (km)



Target **c** Model

$m \cdot s^{-1}$

depth (km)



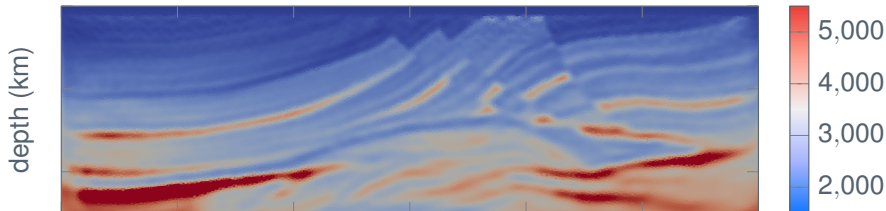
2D Multiscale Reconstructions

Reconstruction with an initial smooth model



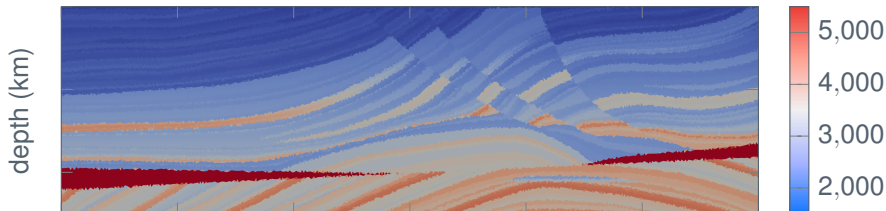
Reconstructed **c** Model with 1.0-10Hz filter

$m \cdot s^{-1}$



Target **c** Model

$m \cdot s^{-1}$



2D Multiscale Reconstructions

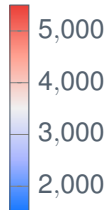
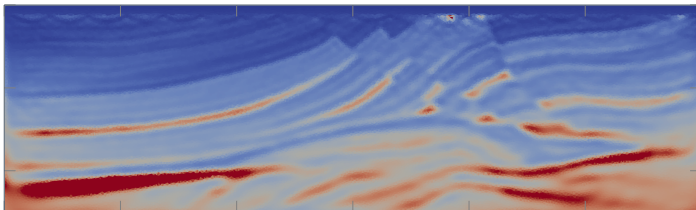
Reconstruction with an initial smooth model



Reconstructed **c** Model with 1.0-15Hz filter

$m \cdot s^{-1}$

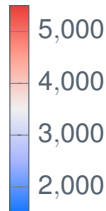
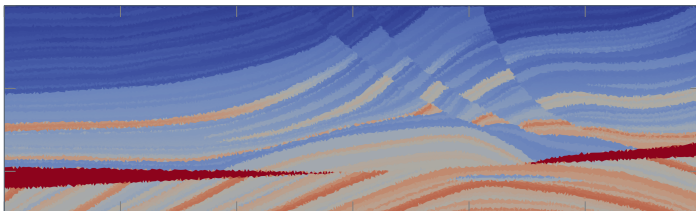
depth (km)



Target **c** Model

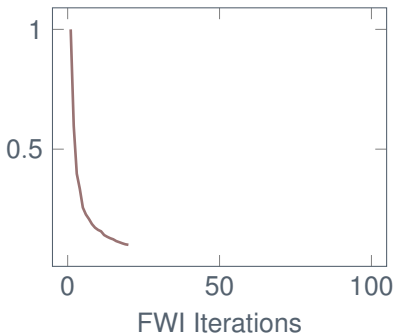
$m \cdot s^{-1}$

depth (km)



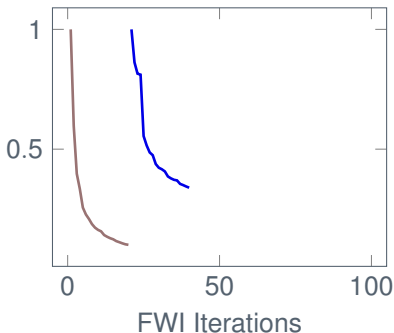
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Cost function evolution :



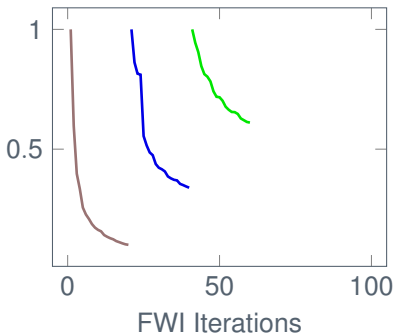
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- ▶ Frequencies : 1-2.5Hz,
1-5.0Hz

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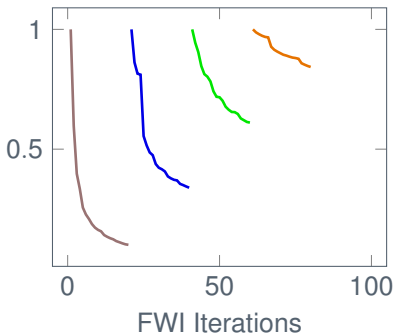
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Cost function evolution :



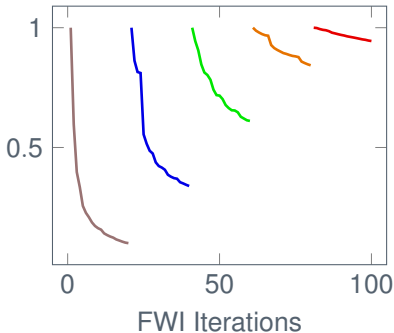
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1-15Hz

Cost function evolution :



Main Results :

- ▶ Comparison between adjoint formulations (**AtD** and **DtA**)
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Thank you.