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Impacts of a mathematical mistake on preservice teachers' eliciting of student thinking

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We report on a study of preservice teachers' eliciting performances in a scenario in which a student has made a mistake and, if sufficiently probed, is able to recognize the mistake and revise their work. Our findings reveal the skills and capabilities of one group of preservice teachers at the start of a teacher education program.

Keywords: Teacher education-preservice.

Student responses in teaching

Mr. Chinn, an experienced sixth grade teacher, is circulating in his sixth-grade mathematics class as students work independently on a set of problems involving operations with fractions, including mixed numbers. He pauses by Chloe and notices that she has arrived at an answer for a subtraction problem with "mixed numbers" that looks wrong. By mixed numbers, we mean numbers that combine integers and proper fractions. Chloe's work, shown in Figure 1, has resulted in an incorrect answer.

$$\begin{array}{r} 2 \quad 2/5 \\ \cancel{3} \quad \cancel{2} \\ - 1 \quad 3/5 \\ \hline 1 \quad 9/5 \end{array}$$

Figure 1: Student work, the student makes a mistake

Mr. Chinn decides that he does not want to assume what Chloe was doing and so he asks her some questions about her work: "Chloe, what did you notice when you started to work on this problem?" Chloe responds, "I was trying to take three-fifths away from two-fifths but I didn't have enough so I needed to borrow." Mr. Chinn nods and follows up, "Where did you borrow from?" Chloe responds, "I borrowed from the 3 and then the two-fifths become twelve-fifths." Mr. Chinn wonders whether Chloe is overgeneralizing "borrowing" from work with subtracting multi-digit numbers or if she has made a bookkeeping error. He asks some more questions and presses Chloe to explain the value of the "little one" in fraction notation and Chloe pauses, and then says softly, "I think I made a mistake." Mr. Chinn asks her what she thinks is a mistake and to articulate why she thought she did it. Through this interaction, he learns that Chloe believes she was just "working too quickly" and

that she forgot that she was borrowing “five-fifths.” Then, Mr. Chinn asks Chloe to solve the problem again. When Chloe changes the “two-fifths” to “seven-fifths,” Mr. Chinn asks Chloe how she knew to write seven-fifths and why the mixed number can be re-written as “three and seven-fifths.”

In this episode, Mr. Chinn encounters a regular problem of practice in mathematics teaching. A student is doing something when solving a mathematics problem that misaligns with the lesson’s mathematics goal of accurately computing the difference. The processes of teaching and learning often involve such mismatches between goals and the current status of students’ performances. For example, students might use an approach that is not familiar to a teacher. Or a student may make what Bass (personal communication) terms a “bookkeeping” error. Radatz (1980) refers to such an error as a mistake, an isolated and unrepresentative mis-executions of an algorithm (a careless move) as opposed a systematic and persistent mis-execution that reflect a conceptual or procedural misunderstanding. Chloe’s response reflects a book-keeping error. Another possibility is that student’s work might reflect a misunderstanding that results in an incorrect answer. Of course, other sorts of mismatches happen in teaching that are crucial, such as when teachers favors/expects a particular approach or route of reasoning and a student treads an alternative path. In a broader sense these mismatches illustrate a broader categorization of students’ actions/productions that are “*not what the teacher is looking for.*” This is not about anticipation as teachers may very well anticipate that students might produce some of these responses. This is more about the need for teachers to have generative responses when there is a mismatch between what the teacher prefers/hopes to see and the variability in what students produce in the course of learning mathematics.

Interactions around student responses that are not what the teacher is looking for can be powerful sites of learning for both teachers and students (Borasi, 1994; Hiebert et al., 1997). When the answer is incorrect, the joint sense-making required to interpret student thinking goes beyond the identification and correction of mistakes/errors into the conceptual analysis of why the mistake/error was made (Borasi, 1994; Kazemi, & Stipek, 2001). The information that teachers can uncover by eliciting students’ thinking can then guide their pedagogical response. This is not to say that all student responses that are different from the responses that teachers are looking for have the same potential for being a site for learning. For example, one could argue that a bookkeeping error has less potential to support learning for the teacher and the student compared to an instance in which a student’s work reflects a misunderstanding that results in an incorrect answer. While the importance of interacting with students around incorrect answers is well-documented, research suggests that practicing teachers vary greatly in their attention to and treatment of errors and mistakes in classroom discussion (Bray, 2011; Santagata, 2005; Silver, Ghouseini, Gosen, Charalambous, & Strawhun, 2005). Less is known about the ways in which preservice teachers (PSTs) elicit student thinking when they see answers that are not the answers that they want students to produce. This matters for several reasons. For one, when students do something that the teacher is not looking for, this indicates a place where the teacher could learn something about the student and/or about mathematics. Both of which merit being able to leverage the opportunity. For another, when students make what seem to be mistakes, asking questions may reveals understandings that are not evident from their written work.

In this study, we examine PSTs' skills around asking questions to learn about student thinking (eliciting student thinking) when a student has an incorrect answer. We explored a case in which the student makes a bookkeeping error, a mistake, in their arithmetic process and will recognize the mistake if asked specific questions about their reasoning. The context is challenging for many PSTs. It has significant implications for students because teachers' understanding the basis of student's mistakes is necessary for responsive instruction. In other words, teachers need to know the degree to which a mistake reflects underlying understandings in order to ascertain how to move a student's understanding forward. Specifically, we sought to analyze the extent to which PSTs' (a) elicited the full process used; (b) elicited the student's understanding of the process, and (c) elicited the student's mistake, including the reason for the mistake and their revised process. We illustrate how knowledge of this kind of skill can be gathered through the use of a simulation in which a PST interacts with a teacher educator who is taking on the role of a student, what such findings reveal about PSTs in one teacher education program, and the implications of these findings.

Focusing on PSTs' skills at the beginning of teacher preparation

There has been increasing focus on preparing PSTs for the work of teaching by focusing on specific instructional practices (e.g., Ball, & Forzani, 2009; Ball, Sleep, Boerst, & Bass, 2009; McDonald, Kazemi, & Kavanagh, 2013). In "practice-based teacher education," learning goals for PSTs are tied to developing skills in carrying out specific teaching tasks. To help PSTs develop skill, it would be useful to know what they bring with them to teacher education, so as to be able to design the program in ways that capitalizes to their prior ways of acting and interacting. Prior research provides knowledge of some of the orientations and assumptions that PSTs bring to teacher preparation and their knowledge of subject-matter content. Research on particular teaching practices often focuses on the utility of the approach, composite parts of the teaching practice, and/or challenges in learning to enact particular teaching practices. Such knowledge is useful for the design of teacher education; however, it is insufficient. Needed is knowledge of the skill with which PSTs can enact particular teaching practices upon entry to a teacher education program.

We build on a prior study which examined the skills with eliciting student thinking at the start of a teacher education program (Shaughnessy, & Boerst, 2018a). PSTs elicited the thinking of a student who arrived at a correct answer to a multi-digit addition problem using a method that was likely to be unfamiliar to PSTs. That study revealed skills with eliciting student thinking that could be built on (e.g., eliciting the student's process), needed to be learned (e.g., eliciting the student's understanding), and needed to be unlearned (e.g., directing the student to use a different process). However, the case of eliciting when a student has made a mistake is different. We wondered whether PSTs would learn that the student had made a mistake and what understandings underlie the mistake.

The practice of eliciting student thinking

In teaching, "teachers pose questions or tasks that provoke or allow students to share their thinking about specific academic content in order to evaluate student understanding, guide instructional decisions, and surface ideas that will benefit other students" (TeachingWorks, 2011). Eliciting student thinking makes the nature of students' current knowledge available to the teacher. Such

information is essential for engaging students' preconceptions and building on their existing knowledge (Bransford, Brown, & Cocking, 2000). In actual practice, eliciting student thinking is often done in conjunction with interpreting student thinking and responding to student thinking in ways that support students in building on their current understandings (Jacobs, Lamb, & Philipp, 2010).

Because of the crucial nature of the practice and the need to teach PSTs to do this work, it is necessary to specify the work involved in eliciting student thinking. We conceive of the work of eliciting student thinking in the context of a mathematics problem as asking questions to bring out the student's process and the student's understanding of key ideas underlying the process (Shaughnessy, & Boerst, 2018b). Students are at the center of this work. It is their thinking which is sought and intended to be understood, and the work is situated in mathematical context that focus dialog, shape interaction, and influence follow-up questions. Throughout this paper, we use "skill with eliciting student thinking" to refer to the degree to which PSTs are able to engage in this work.

Studying skill with eliciting student thinking

Since 2011, we have used teaching simulations to study PSTs' skill with eliciting student thinking. A simulation serves as an "approximation of practice" (Grossman, Hammerness, & McDonald, 2009). Simulations have been used in many professional fields such as medicine and more recently in the preparation of teachers (Self, 2016). In these simulations, a PST interacts with a "standardized student" (a teacher educator taking on the role of a student using a well-defined set of rules for responding) around a specific piece of written work. This form of assessment has advantages over field-based interviews to assess skill with eliciting student thinking (Shaughnessy, Boerst, & Farmer, 2018). Field-based interviews leave the particular student thinking being elicited to chance and can be highly variable. The teaching simulation uses highly specified protocols that enable us to control key aspects of the student's process, understanding, and demeanor. This leads to comparable eliciting contexts for all of our PSTs and facilitates fair and more nuanced assessment of PSTs' eliciting skill.

We design teaching simulations to have a consistent three-part format (Shaughnessy, & Boerst, 2018b). First, PSTs are provided with student work on a problem and given 10 minutes to prepare for an interaction. Second, PSTs have five minutes to interact with the standardized student, eliciting the student's thinking to understand the steps they took, why they performed particular steps, and their understanding of the key mathematical ideas involved. Third, PSTs are interviewed about their interpretations of the student's thinking. In total, the assessment takes approximately 25 minutes.

We designed this simulation to be one in which the student uses non-standard process (meaning a process other than the standard US algorithm) for solving multi-digit subtraction problems and makes a mistake. The process we selected is sometimes referred to as "Expand and Trade" (see the student work in Figure 2). The process involves writing the value of the minuend and subtrahend in expanded form and making any necessary trades. When used correctly, the user would then subtract the numbers place-by-place in expanded form. This student correctly applies the expanding and trading process, but mistakenly adds values by place instead of subtracting. This student has

conceptual understanding of expanded form, the meanings of addition and subtraction, and when, how, and why to make trades. In this instance, however, the student loses sight of the fact that they are solving a subtraction problem due to the addition symbols in the expanded form. This is a bookkeeping error. During the interaction, the student will change their mind when pressed to: make and evaluate an estimate for the original problem, represent their process with a picture, talk about the meaning of the operation, explain why trades were made, solve another multi-digit subtraction problem, or re-solve the original problem. The student is trained to only reveal the mistake if pressed by the PST on one of these specific points.

$$\begin{array}{r}
 583 \rightarrow 500 + 80 + 3 \\
 - 295 \quad 200 + 90 + 5 \\
 \hline
 600 + 260 + 18 = 878
 \end{array}$$

Figure 2: Student work, the student makes a mistake

Methods

Thirty PSTs enrolled in a university-based teacher education program in the United States participated during the first week of the teacher education program. The simulations were video-recorded. In this paper, we focus our analysis on: (a) eliciting the student’s original process, (b) eliciting the student’s understanding of the process, (c) eliciting the student’s mistake, including the reason for the mistake and the revised process and solution. For each of these components, we identified specific “moves” and tracked their presence or absence in each performance. We used the software package Studiocode© to parse the video into talk turns. Then, we identified “instances,” which contain a question posed by a PST and the student’s response to that question. Two independent coders applied all of the relevant codes to each instance. Disagreements were resolved through discussion and by referencing a code book.

Findings

Eliciting the student’s original process

The student’s process had five steps: expanding both the minuend and subtrahend, comparing the numbers in each place, trading, adding (rather than subtracting) numbers by place, and adding the partial sums to arrive at the answer. The highest rates of eliciting occurred around the expansion (70%), the comparison of the numbers in each place (90%), and the trading steps (80%). In fact, 65% of PSTs elicited all three of these steps and 90% of the PSTs elicited two or more. However, only 53% of the PSTs elicited that the student added numbers by place in the expanded form after trading. This was surprising given that it was the point where the mistake occurred. The smallest percentage of eliciting occurred around the adding of the partial sums (10%). This was not surprising given that (a) it occurred after the point where the student made the mistake and may have not been relevant if the PST had asked questions that resulted in the student revealing that they had made a mistake, and (b) it could be easily inferred from the written work. Given that these PSTs elicited some, but not all, of the student’s steps, we concluded that they brought skills relevant to

eliciting a student's process that may be leveraged and built upon in the teacher education program; however, there was also a need for these PSTs to work on identifying and actively asking about steps that are particularly important to understand in the context of the problem.

Eliciting the student's understanding of the original process

We coded whether PSTs elicited the student's understanding of six mathematical ideas underlying the process. The highest percentages of eliciting of understanding occurred around the operation in the problem (27%), why the student expanded (37%), and why the student traded (27%). The lowest percentages occurred around the equivalence of expanded form and the "original" number (7%), the reasonableness of the answer (7%) and what trading means (0%). This suggests that limited eliciting of understanding of mathematical ideas occurred. However, when we looked across the set of ideas, we found that 67% of PSTs elicited the student's understanding of one or more idea. Thus, we concluded that this 67% of PSTs brought skills relevant to eliciting the student's understanding in this scenario, but note that their eliciting does not show discernment of which understandings are more important to focus on and/or more relevant to subsequent instruction.

Eliciting the student's mistake, including the reason for the mistake and the revised process

We coded the extent to which the PSTs elicited the student's mistake, including their understanding of why they made the mistake. We found three foci in PSTs' performances: eliciting the student's realization that a mistake had been made (47% of PSTs), pointing out the mistake and getting the student to admit a mistake without first eliciting the mistake from the student (20% of PSTs), and only asking questions that were not focused on the mistake (33% of PSTs). In other words, 67% of the PSTs uncovered the mistake either by eliciting it or asking the student to confirm that they made a mistake; 33% percent did not learn about the mistake through questioning.

We now turn to the 47% of the PSTs who elicited the student's realization that a mistake had been made. These 14 PSTs elicited the mistake in different ways. The most common way, used by 11 PSTs, was to ask about the operation involved in the problem and to press on how the operation in the problem (subtraction) differed from the operation that the student had used (addition). Another two PSTs posed another problem for the student to solve or asked the student to redo the original problem. The student profile specifies that the student should correctly apply the Expand and Trade method if prompted to solve another problem. A third approach to eliciting the mistake was to ask about why the student had made the trades. One PST leveraged this to ask why the student had added.

We next turn briefly to the 20% of the PSTs uncovered the student's mistake by pointing it out themselves and getting the student to admit the mistake. Often, PSTs made statements about what it appeared the student had done based on the written work. For example, one PST stated her impression that that expansion and trades were executed correctly and that the student had added instead of subtracting. This exchange took place after the PST had elicited the steps of the student's original process through the trading step. In the exchange, the idea that the student mistakenly added instead of subtracted is introduced by the PST, not elicited from the student. The student merely confirms what the PST has stated. We see this approach to uncovering the mistake as qualitatively distinct from eliciting the mistake through asking questions.

To explore what PSTs did after learning the student had made a mistake, we considered the 20 cases in which PSTs uncovered the student's mistake either by eliciting the student's realization that a mistake had been made or pointing out the mistake. Twelve of the 20 PSTs did not ask any further questions about the mistake. These PSTs need to learn to ask additional questions once a student recognizes a mistake. Of the 8 PSTs who followed up about the mistake, 5 made statements based on their own inferences about how and/or why the student made the mistake and got the student to agree. The remaining 3 PSTs asked questions to learn about how and/or why the mistake was made. These findings suggest that these 8 PSTs bring skills relevant for eliciting a student's mistake, but may need to learn reasons and strategies for eliciting how and why a mistake was made.

Discussion

Understanding the skills with teaching practices that PSTs bring to teacher education is key to designing experiences that are responsive to the needs of PSTs (Shaughnessy, & Boerst, 2018a). This study examined the ways in which PSTs at the beginning of a teacher preparation program elicited the thinking of a student who made a mistake when solving a subtraction problem. Learning about the reason for a mistake is important for teachers to accurately assess and respond to student thinking. For example, a student who has made a mistake due to misunderstanding about ideas underlying the process would need different instruction than a student who has made a bookkeeping error when carrying out a well-understood process. However, at the start of their teacher education program, these PSTs focused more on eliciting the revised method and/or solution than asking about why the mistake was made. We hypothesize that there might be multiple reasons that this pattern occurred. Some PSTs might refrain from directly asking the student why they made the mistake because they are fearful of embarrassing the student by asking them to talk about it. For other PSTs, limitations in their content knowledge for teaching might impede being able to identify important mathematical ideas related to the process. Still other PSTs might not have been able to formulate a question that could be used to elicit evidence of the student's understanding.

The data suggest several directions for continuing research. First, these PSTs were in their first week of a teacher education program. In what ways do their skills with eliciting student thinking develop over time? Second, what course activities might effectively cultivate an inclination to elicit how and why students make mistakes? Third, the assessment itself involved a mistake where the student was using a "non-standard" approach to solve the problem. Anecdotally, we have reason to think that some PSTs were discounting the student's reasoning because they believed that the student should be using a different method to solve the subtraction problem. A future study could compare skills in eliciting around a mistake with a "standard" and an "alternative" algorithm.

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