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Interweaving Mathematical-News-Snapshots as a facilitator for the development of Mathematical Knowledge for Teaching

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The study described in this paper accompanies a three-year R&D project that is currently in its final year. Participants are 25 high school mathematics teachers, of whom 14 have started their 2nd year and 11 are currently in their 3rd year of integrating Mathematical-News-Snapshots in their classes. This study examines the effect of integrating these snapshots in the ordinary high school curriculum on the development of several components of Mathematical Knowledge for Teaching. The findings indicate that teachers perceive the integration of Snapshots of contemporary mathematics as contributing to the development of their Specialized Content Knowledge, Knowledge of Content and Students, and Horizon Content Knowledge.

Keywords: Specialized Content Knowledge, Knowledge of Content and Students, Horizon Content Knowledge, Mathematical News Snapshots (MNSs).

Theoretical background

Knowledge of mathematics teachers

Over the past few decades, the mathematics education research community has become interested in characterizing the required knowledge for mathematics teaching. Shulman (1986) recognized that the required teacher's knowledge is unique and suggested two main types of knowledge teachers required, not necessarily in mathematics: Subject Matter Knowledge (SMK) – a deep understanding of the subject matter, and Pedagogical Content Knowledge (PCK) – a repertoire of representations, analogies, models, explanations, and demonstrations.

Relying on Shulman's work, Ball & Bass (2003) attempted to distill the knowledge required from mathematics teachers. They defined the term "Mathematical Knowledge for Teaching" (MKT) as knowledge that crosses areas and levels of school mathematics, knowledge that supports connected ideas of the mathematics teacher, and emphasizes the ability to plan, evaluate, integrate and manage appropriate mathematical content for teaching. Following this work, Ball, Thames & Phelps (2008), presented six different components of mathematical knowledge for teaching. Three of them are particularly relevant to this study:

- Specialized Content Knowledge (SCK). This component includes the mathematical knowledge and skills unique to teaching that is not typically needed for purposes other than teaching, such as searching for student errors, intelligent integration of examples during instruction;
- Knowledge of Content and Students (KCS). This component combines knowing about students and knowing about mathematics, namely, knowing how students think, what may confuse them, what kind of mathematical task may be easy or difficult for them, and more. This kind of knowledge requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking;

- Horizon Content Knowledge (HCK). “Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (ibid p. 403).

Ball & Bass (2009) suggested a more detailed definition of HCK:

“knowledge as an awareness – more as an experienced and appreciative tourist than as a tour guide – of the large mathematical landscape in which the present experience and instruction is situated. It engages those aspects of the mathematics that, while perhaps not contained in the curriculum, are nonetheless useful to pupils’ present learning, that illuminate and confer a comprehensible sense of the larger significance of what may be only partially revealed in the mathematics of the moment.” (Ibid ,2009, p. 5).

They present four components of HCK: A sense of the mathematical environment surrounding the current “location” in instruction; major disciplinary ideas and structures; key mathematical practices; core mathematical values and sensibilities.

There is a strong connection between MKT components and mathematical quality of instruction (Hill et al., 2008; Ma, 1999). Teachers with a strong MKT can present their students with rich mathematical links, representations and examples beyond what is given in most textbooks. These teachers are familiar with the curriculum and with ways to relate students to various subjects. In contrast, teachers with a poor MKT have a limited arsenal of examples to cater to diverse student needs.

The Mathematical News Snapshots project

Mathematics is a dynamic field that is developing constantly. The current number of publications in the mathematical reviews database is presently over three million, and over 80,000 additional reviewed papers are added to the database each year (American Mathematical Society, 2018). However, school mathematics curricula generally do not reflect this dynamic nature nor the constant struggle of mathematicians to discover new results. As a result, high school graduates often graduate while erroneously perceiving mathematics as a field in which all questions and answers are known in advance, and there is little space for further research (Amit & Movshovitz-Hadar, 2011). In some countries, this situation has led to a decline in enrollment to mathematically intensive academic studies, and even developed into an economic, social, and national concern (Noyes & Adkins, 2017).

The Mathematical News Snapshots (MNSs) project, in which the current study is situated, was set up to provide a better perspective of the nature of contemporary mathematics by introducing high-school students to current frontiers in the rapidly growing area of mathematics (Movshovitz-Hadar, 2008). The chosen method was to prepare high school teachers to interweave snapshots of mathematical news in their teaching on a regular basis. Each MNS is developed in the form of a PowerPoint presentation, with three main characteristics (Amit & Movshovitz-Hadar, 2011): (i) A new mathematical result published in the professional literature in the past few decades; (ii) Exposition of about 25 minutes focusing on the new result, elaborating on its history, the main underlying ideas, and the people involved in the field; and (iii) Taking into account students’ limited background, preferably linking it to some topics in the mandatory curriculum, without harming the progress in their teaching.

The current study focuses on one question – *how do high-school mathematics teachers perceive the contribution of integrating MNSs (in their mathematics lessons) to the development of their Mathematical Knowledge for Teaching?* This study is part of a larger research program that examines the experiences of teachers as well as of students participating in the MNS project.

Method

Setting: Mathematics teachers participating in the MNS project attend a dedicated three-year professional development program. This program involves both face-to-face meetings and online meetings in which teachers are getting acquainted with seven MNSs each school year (Grades 10-12) and hold discussions about the mathematical, historical, and innovative contents of the MNSs. Following the implementation of each MNS in their classes, teachers are required to document their experiences (e.g., their own reflections and their students' responses) into an online database via a dedicated proprietary software called "Ramzor" (lit. Traffic Light). These reports often entail information regarding students' comments and classroom atmosphere, as well as suggested accompanying activities, recommended videos, and questions regarding possible mistakes in the presentation. The project staff provides ongoing one-on-one support for each participating teacher. To date, we have completed two and a half (out of three) years of our longitudinal study.

Sample: The participants are 25 high school mathematics teachers who volunteered to participate in the MNS project: 14 teachers have integrated MNSs for one year ("beginning teachers" – Group A), and 11 teachers have integrated MNSs for two consecutive years with the same students ("continuing teachers" – Group B). (They are continuing this year, but the data is restricted to the 2nd year).

Research instruments: Three data collection instruments are used in this study: Teacher questionnaires, teacher interviews, and field notes.

Teacher questionnaires. At the beginning of the first year (prior to the students' first introduction to the MNSs) and at the end of each school year, each participating teacher is asked to complete a questionnaire. The questions involve three types of items: 5-level Likert-type scale items, multiple-choice items, and open-ended items. The intention was to trace the development of teachers' MKT following the integration of MNSs in their lessons. The questionnaires were constructed with, and validated by, a team of mathematics education experts.

Participants' interviews. We conducted an informal semi-structured interview with each teacher at the end of each school year, in the form of a friendly conversation conducted online via video chat. The interviews dealt with the considerations that guided the teacher in integrating the MNSs.

Field notes – Analysis included all (and only) comments made by teachers on the proprietary software and during conversations with the project staff.

Data analysis: We utilized a qualitative research paradigm, implementing a process of open and axial coding, to identify the main categories and sub-categories (Corbin & Strauss, 2008). At the first stage, the teachers' responses to the questionnaires were analysed, to generate preliminary categories. In the second stage, the data analysed from the observations and interviews in order to identify and map the initial categories. We identified a correlation between the reports received from the final evaluation questionnaire and the knowledge obtained from the interviews that

supports the findings (Miles & Huberman, 1994). In the third phase of the analysis, while reviewing the findings of the first and second stages, we focused on the data related to the development of key components of MKT.

Findings

The integration of the MNSs introduced teachers to diverse and innovative mathematics contents, in a systematic and structured manner. The teachers' experience in integrating the MNSs led to the development of various teachers' knowledge components, as they perceived it. Some of which are presented below.

Teachers' perceptions about Integrating MNSs as a lever for the development of their mathematical knowledge

In each end-of-year questionnaire the teachers were asked to relate to the contribution of the integration of the MNSs to their mathematical knowledge, both in the context of school curriculum and beyond the curriculum. Table 1 presents the distribution of teachers' degree of agreement with the statement: To what extent did the integration of MNSs in your mathematics lessons contribute to the expansion of your mathematical knowledge?

| Teachers | | Degree of Agreement | | | | |
|----------------|--------------------|---------------------|-------|--------|--------|------------|
| | | Very Large | Large | Medium | Little | Not at all |
| Group A (N=14) | Curricular context | 14.29 | 14.29 | 7.14 | 57.14 | 7.14 |
| | Extracurricular | 28.57 | 28.57 | 28.57 | 7.14 | 7.14 |
| Group B (N=11) | Curricular context | 9.09 | 0 | 36.36 | 45.46 | 9.09 |
| | Extracurricular | 18.18 | 27.27 | 36.37 | 18.18 | |

Table 1: Teachers' grading of their mathematical knowledge development, according to groups and type of knowledge (in percentage)

Here are a few quotes (translated from Hebrew to English) from group A teachers' testimonials about the expansion of their mathematical knowledge: "*The MNSs helped me understand, and present in class the distinction between proofs and refutation*"; "*I realized that there are proofs of impossibilities, such as the impossibility to trisect an angle*"; "*I learned about the existence of notions such as Mersenne number*". As can be seen from Table1, both groups perceived the contribution of integrating the MNSs to their mathematical knowledge beyond the curriculum was greater than its contribution to knowledge related to the curriculum itself. Group A teachers maintained that: "*I was exposed to mathematics theorems such as Fermat's last theorem and Kepler's theorem*"; "*I became familiar with interesting mathematical subjects that I did not know, like the connection between guarding an art gallery and mathematics*". Similar to Group A, Group B teachers reported that: "*My mathematical knowledge was expanded, through integrating MNSs for example I learned from the MNS about Efron cubes that the transitive rule does not always holds*"; "*Through integrating MNSs, I learned with my students a lot about contemporary*

mathematics"; "There are many mathematical contents I was not familiar with. For example, the frequency of the digit 1 as a leading digit". The findings from the interviews with the teachers reinforced the findings from the questionnaires. They indicated that all the interviewed teachers (Seven out of seven) perceived an expansion and deepening of their mathematical knowledge. "It was new to me that nowadays mathematicians employ technology not only for proving theorems but also for validating proofs". In addition, the teachers were exposed to the protracted work of proving mathematical theorem: "I did not know that to prove the Fermat Last Theorem it took 300 years"; "I am happy to learn new things in mathematics, as the MNSs power point presentation there are many references, to mathematical background materials and applets".

Teachers' perceptions about integrating MNSs as an attribute for their recognition of the importance of exposing students to contemporary mathematical subjects beyond the curriculum

Table 2 presents the distribution of teachers' degree of agreement with the statements: To what extent integrating the MNSs contribute to your recognition of the importance of exposing students to contemporary mathematical subjects beyond curriculum?

| Teachers | Degree of Agreement | | | | |
|----------------|---------------------|-------|--------|--------|------------|
| | Very Large | Large | Medium | Little | Not at all |
| Group A (N=14) | 21.43 | 50 | 21.43 | 7.14 | |
| Group B (N=11) | 18.18 | 63.64 | 18.18 | | |

Table 2: Teachers' grading of their recognition about the importance of exposing students to contemporary mathematical subjects beyond curriculum (in percentage)

The teachers in Group A reported that their recognition about the importance of exposing the students to the content of the MNSs motivated them to join the project to start with, and to carry on their participation in it. In addition, teachers in Group A, reported about their desire to change the students' image about mathematics as a dynamic, and constantly evolving field: "The exposure of the students is very significant to their image of mathematics"; "It's important for me that my students really recognized that mathematics is an ever-growing field". The teachers in Group B added: "It is important that students recognize the world of mathematics beyond matriculation exam". The teachers' awareness of the importance of exposing students to MNSs also related to an opportunity to deal with the fears of students from math: "One of my goals in integrating MNSs in my class is to try to remove fear of mathematics and try to develop a love for mathematics". In addition, integrating MNSs helped teachers emphasize the use and the connection of mathematics to daily life: "It was very important to me at every MNSs presentation to note the fact that mathematics connects to science, which has many uses and is constantly evolving". The teachers' interviews strengthened the findings from the questionnaire. Five out of seven interviewed teachers reported that they chose to participate in the project to expose themselves and their students to mathematics as a field of constant development: "Mathematics is perceived by students, as frozen and unchanging". Integrating MNSs enabled teachers to present mathematics in an orderly,

interesting and exciting way: “the integration of MNSs gave me an opportunity as a teacher to present a story after story of things that can ignite students’ imagination”. With integrating the MNSs “I can open to my students horizons for what is happening in mathematics. I did not have the ability to connect things together without participating in this project”

Teachers' perceptions about integrating MNSs as a way to deepen teachers' knowledge about the relationships between mathematics, and its applications to science and society.

Table 3 shows the distribution of teachers' degree of agreement with the statement: To what extent integrating the MNSs contribute to your knowledge about the relationships between mathematics and its applications to science and society?

| Teachers | Degree of Agreement (in percentage) | | | | |
|----------------|-------------------------------------|-------|--------|--------|------------|
| | Very Large | Large | Medium | Little | Not at all |
| Group A (N=14) | 14.29 | 35.71 | 42.86 | 7.14 | 0 |
| Group B (N=11) | 18.18 | 36.37 | 27.27 | 18.18 | 0 |

Table 3: Teachers' grading of their recognition of the relationships between mathematics and its applications to science and society

The teachers in Group A, reported on important knowledge from integrating MNSs in their lessons, which links mathematics to its uses: “It is Important as a teacher to have the knowledge beyond the curriculum that connect mathematics to other fields such as: agriculture, the connection to taxes, music, world maps”. Group B teachers also reported a rich expanse of examples illustrating the relationship between mathematics and science through integrating MNSs: “I was amazed to discover the connection between origami folds and the development of science and medicine”. All the interviewed teachers reported about the contribution of the MNSs to their knowledge of the relationship between mathematics, sciences and society. The integration of the MNSs has given teachers the exposure, and understanding, the relationship between mathematics and scientific phenomena around them: “for example, soap bubbles were very interesting. Why the bubble is round, and it has to do with both science and mathematics, I knew that the Hungarian cube knew has math around it, but now I did deepen my knowledge about it. The MNSs enriched my knowledge, opened my horizons, and the integration of the MNSs in students instruction forced me to understand it more deeply”. In addition, all the interviewed teachers reported the contribution of integrating MNSs, to their knowledge about the relationship between mathematics and computer science. An example of teachers report: “The fact that the computer becomes a significant player in the stages of proof is very impressive, such as the color of the map”. The teachers reported a deepening of the relationship between mathematics and phenomena occurring in nature: “For example, in the Fibonacci series, I was very excited by the physical explanation of the series' shows in the flowering mechanism. MNSs help me to understand why, and the explanation took me a few steps forward”.

Discussion

The present study focused on high-school mathematics teacher`s perceptions of the contribution of integrating MNSs to various component of their MKT during the first two years. The findings as appeared above in Tables 1, 2, 3 indicate that teachers perceive the interweaving of MNSs in their mathematics lessons, as a catalyzer for the development of at least three components of their MKT. This goes hand in hand with the claim by Shulman (1986) and Ball et al (2008) that this is the unique mathematical knowledge required for teaching mathematics.

Teachers also expressed their perceptions that by integrating MNSs in their mathematics lessons, they developed their SCK as well as their KCS. By intelligent integration of MNSs during instruction teachers perceived that they deepened their knowledge about the connection between mathematics and other field, about different ways for proof theorems, hence they have expanded their SCK. By introducing various ways in which students' curiosity can be stimulated, and identified topics in which students were interested in the curriculum and beyond, teachers have expanded their KCS.

Another component that the findings illuminate is the HCK as defined by Ball et al. (2008). The findings indicate that teacher`s perceptions the significant contribution of integrating MNSs on their MKT component, related to HCK (Ball et al. 2008). By exposed to the MNSs, they strength and deepen their awareness of the dynamic, lively and ongoing mathematics development. The teachers found out about the connections between the development of mathematics and the development of science and society. According to HCK, defined by Ball & Bass (2009), by integrating MNSs in their mathematics lessons, the teachers reflected in the various HCK components: They developed a sense of the mathematical environment surrounding the current “location” in instruction, by identified links between topics taught in the mathematical curriculum to topics outside the curriculum. They were exposed to major disciplinary ideas and structures such as various possibilities of mathematical theorem, to the logical structure of mathematical content. Finally, the teachers stated they appreciated being exposed to core mathematical values such as, collaborative work and persistence, as expressed in the MNSs, with the description of mathematicians work with their colleagues in other fields.

In conclusion, this study will continue in examining more components as the third year of this study unfolds.

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