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► **To cite this version:**

Sigal Rotem, Michal Ayalon, Shula Weissman. Pre-service mathematics teachers interpret observed teachers' responses to students' statements. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02422679

**HAL Id: hal-02422679**

**<https://hal.science/hal-02422679>**

Submitted on 22 Dec 2019

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# Pre-service mathematics teachers interpret observed teachers' responses to students' statements

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*In this paper, we will present and exemplify a three-level category scheme used for categorizing the depth of interpretations pre-service mathematics teachers offer for teachers' responses to students' mathematical thinking in observed critical events. The category scheme is a result of top-down literature analysis and bottom-up analysis of 38 critical event reports submitted during one academic year within a clinical preparation context. This category scheme may help teacher educators to gain a better understanding of PTs' interpretations of teachers' responses, and therefore to plan field-based training programs that help PTs to broaden their theory-practice connection.*

*Keywords: pre-service mathematics teachers, critical events, noticing, reflection.*

## Introduction

Reflection has had a growing role in teacher education over the past four decades (e.g., Zeichner, 1981) and is considered to lie at the heart of teachers' professional practice (Clarke, 2000). Teacher educators argued for the integration of reflection with field-based preparation, in order to help pre-service mathematics teachers (PTs) learn how actions connect to particular purposes in context and to support PTs' ability to link theory with practice (e.g., Ball & Forazani, 2009; Zeichner, 1981). In this research we follow Karsenty and Arcavi's (2017) definition of reflection, which is "analytical and careful observation of 'what was done' in order to attempt to understand intentions, plans, actions and utterances" (p. 435). A common use of reflection in teacher education (in-service and pre-service) is asking teachers to analyze classroom situations (e.g., van Es et al., 2017). However, research indicates that when PTs are merely asked to reflect on situations that they observed, their reflection may focus on aspects of teaching that are irrelevant to building further mathematical instruction (e.g., Santagta & Guirno, 2011). This suggests that a structured framework for reflection is needed. Recent research indicates that *professional noticing framework* is an appropriate framework for structured reflection (Jacobs et al., 2010). In this research we built on Jacobs et al. framing of professional noticing framework, which requires PTs to: (1) identify a critical event (2) interpret the mathematical thinking of the student/s in the critical event, and (3) offer an alternative response as if they were the teacher responding to the critical event. This framework is used here to analyze authentic mathematics classroom situations that PTs observed in their clinical preparation program. We refer to these situations as *critical events*. In this study we built on Stockero and Van Zoest (2013) and Goodell (2006) definitions and define critical events as moments in the classroom that if pursued by the teacher may enhance students' mathematical learning, and therefore provide the teacher with an opportunity to reflect and thus to learn about teaching.

In most studies PTs are asked to focus on the student's mathematical thinking as it is expressed in student-teacher interactions (e.g., Jacobs et al., 2010; Simpson & Haltiwanger, 2017). This study takes a slightly different approach, and seeks to focus not only on the student but on the teacher as

well. The premise is that an analysis of teacher responses is an inseparable part of the critical event analysis and that focusing on the relationships between the student's statements and the teacher's responses in the given event will contribute to reflection. In addition, research shows that PTs struggle when it comes to offering an alternative response as if they were the teacher in critical event (e.g., Jacobs et al., 2010). Although not in the scope of this paper, we began looking for ways to support PTs to base their alternatives for teaching on the students' mathematical thinking as expressed in the event. A possible approach for achieving this aim is to direct the PTs to focus their attention to the teacher's response. The current paper takes a first step in building a characterization for the PTs' interpretations for the teachers' statements in the observed events.

This paper presents an analysis of PTs' written interpretations for a teachers' responses in critical events which were submitted during a clinical training program. We analyzed 38 reports submitted by the PTs. The reports contain PTs' analysis of the critical event according to the three elements of the professional noticing framework (Jacob et al., 2010) plus analysis of an observed teacher's response to the student. Our analysis of the reports was conducted bottom-up together with adaptations of some existing frameworks to construct a three-level rubric to assess the depth of PTs' interpretations of a teachers' response in observed critical events. This rubric may help teacher educators to gain a better understanding of their PTs' interpretations of an observed teacher's response, and therefore, may help to plan field-based training programs for teachers.

## **Literature review**

In correspondence with the importance specified above of offering interpretation to the teachers' responses in critical events, we built mainly on three bodies of research: (a) reflection, which we ask PTs to perform in the context of their clinical training; (b) critical events as a construct for reflection; and (c) teachers' professional noticing which is the basis of our structured reflection framework.

Reflection, as in analytical articulation of instructions and teaching actions in order to understand consideration that led those actions (Karsenty & Arcavi, 2017) is, according to Clarke (2000), "not just an option pursued by good teachers; rather, to teach is to reflect." (p. 201). Karsenty and Arcavi (2017) offered a bottom-up characterization for different aspects that in-service teachers attended while reflecting on observed mathematics lessons. Potari and Jaworski (2002) looked at teachers' reflection using the teaching triad framework, which perceives teaching practice as an integration of management of the lesson, being sensitive – cognitively and affectively – to the students and managing the mathematical challenge of the lesson. Teachers' reflection has been found to be a productive tool to link practice with mathematics teaching theories (e.g., Jacobs et al., 2010). Additionally, recent research suggests that teachers' involvement in deep reflection and analysis of teaching strategies and intentions may hold opportunities to learn from such examination new ways of instruction, insights and horizons (Karsenty & Arcavi, 2017). In this research we ask PTs to reflect on critical events they observed during their clinical training classroom observations. The theoretical base for critical events is cases which are descriptions of events that represent a broad pedagogical phenomenon or a dilemma with theoretical aspects (Shulamn, 1986) and which exhibit unexpected moments that are valuable in regard to students' learning (e.g., Stockero & Van Zoest,

2013). In the research literature critical events also appear as ordinary teaching-learning situations that make teachers question their practice and, through reflection, provide a gateway for the teacher to improve his/her teaching (Goodell, 2006). As our use here of critical events is meant to promote PTs' learning about teaching mathematics, we define critical event as moments in the classroom that if pursued by the teacher may enhance students' mathematical learning, and therefore provide the teacher with an opportunity to reflect and thus to learn about teaching.

But just asking PTs to reflect on observed critical events is not enough; there is a need to teach PTs how to reflect on teaching in disciplined and structured ways (e.g., Santagta & Gurino, 2011). Therefore, the third body of research that informed this study is the teachers' professional noticing framework (Jacob et al., 2010) which is defined as: (1) attending to student thinking within student-teacher interactions; (2) interpreting student understanding based on these interactions; and (3) offering a response based on this analysis. For example, deciding when to ask students for more clarification or to follow up on a student's statements requires the teacher first to attend to the student's statement, then to interpret its mathematical meaning and then to formulate an immediate response. Therefore, these skills are considered core elements of teaching practice that should be practiced in teacher training programs (Ball & Forzani, 2009). Research focusing on cultivating these skills among PTs indicates that PTs struggle when asked to offer alternative responses for the teaching reflected in a critical event (e.g., Jacobs et al., 2010). For example, Rotem and Ayalon (2018) found that even when PTs offered a rich interpretation for the students' statements in the critical event, interpretations for the teacher's response and the suggested teaching alternatives were general in their nature and detached from the interpretation they offered for the students' statements. To date, most researchers have focused on assessing the interpretation teachers ascribe to students' statements in the event (e.g., Simpson & Haltiwanger, 2017). Interpretations for the teachers' statements were seldom in the focus of research. An exception for this is van Es et al.'s (2017) study, which investigated the development of PTs' noticing of ambitious mathematics pedagogy in the context of a video-based course designed to foster PTs' skills of interpreting classroom interactions. They used the term 'making thinking visible' to capture the extent to which the PTs attended to the teacher's role in making student thinking visible. Van Es et al. (2017) identified three levels of skill: (a) not paying much attention to the teacher's strategies making student thinking visible, but rather to management and arrangement of the students and class; (b) identifying the teaching strategies and judging their effectiveness; and (c) inferring the ways the teacher made student thinking visible, and how they influenced on student thinking and learning. For the study presented in this paper, we built on van Es et al.'s three levels of skill to develop a three-level category scheme for categorizing the depth of interpretations PTs offer for teachers' responses to students' mathematical thinking in observed critical events. Consequently, our research question is: *What are the main characteristics for the levels of interpretations PTs offer to a teacher's response in a critical event, which they identified?*

### **Research context: ACLIM-5 clinical training program**

The study took place in the context of ACLIM-5 (a Hebrew acronym meaning "clinical training for unique 5-unit (high track) mathematics teaching"). ACLIM-5 is a large university's three-year

training program designed to support the development of high track mathematics pre-service teachers. Due to the limited space of this paper we will not elaborate regarding the program. We will note, however, that this study focused on the first year of the program, in which PTs study for their teaching certificate. PTs participated in a course on critical events in which they were required to submit a critical event reports based on lesson observations. The reports consisted of four main parts: (a) prompts for describing the critical event that was identified during classroom observation, the mathematical context, and the student-teacher interaction. (b) prompts for interpreting the students' statements. (c) prompts for interpreting the teacher's response, E.g. "offer interpretations for the teacher's actions. What was going through his mind? What was it based on?" and (d) a request for alternative ways of responding, other than the teacher's response. Sections (a), (b) and (d) of the report resonance Jacobs et al. (2010) framing of professional noticing. Here, PTs' answers for (c), served as the data source for this study.

### **Data collection**

The data for this paper was taken from the critical event reports submitted in the first year of the first ACLIM- 5 course. In the course participated one male and 12 female. Five studied toward a dual degree in Mathematics and Education, and eight graduated with a B.Sc. from the university's department of Mathematics. The data consist of 38 critical events reports describing real-life classroom events from PTs' observations, submitted during the academic year.

### **Data analysis**

Data analysis was conducted in three phases. First, we used bottom-up characterization for PTs' answers within (c) section. Then we searched for compatibilities between the bottom-up characterization and research literature that was mentioned above (e.g., Potari & Jaworski, 2002; van Es et al., 2017). Finally, we built the coding scheme, using both the bottom-up and literature-based characterization.

### **Findings**

Our analysis of 38 critical reports resulted in a three-level scheme for categorizing the depth of interpretations that were assigned by PTs to the teacher's response in critical events. Van Es et al. (2017) three-level framework for analyzing PTs' attention to the teacher's making student thinking visible, served us as a base ground for developing the scheme, while using a bottom-up approach to allow other characteristics for each level of interpretation to emerge. Eventually, we have received a refined coding scheme for level of interpretation, as presented in Table 1.

**Table 1: Coding scheme for level of interpretation**

<b>Levels of interpretation of teacher's response</b>	<b>Characteristics of level of interpretation</b>
Descriptive/affective	Participant describes the teacher's response in general terms while attending to affective aspects of the students or to the management and arrangement of the students and class;
Semi interpretation	Participant points out the teaching strategies that the teacher used when responding to the students' statements; and/or judges the effectiveness of the strategies.
Full interpretation	Participant details the teaching strategies that the teacher used when responding to the students' statements and proposes considerations that may have led to the teacher's response and judges the effectiveness of the strategies.

### **Low level of interpretation to teacher's response- descriptive/affective interpretation**

The results of the bottom-up analysis indicated that when asked to interpret teachers' statements, PTs tended to describe teachers' statements while attending to the management and arrangement of the students and class without offering an explanation for the considerations behind the teacher's statements. This echoes van Es et al. (2017) low level of interpretation - attending mainly to management and arrangement of students and class. Some PTs interpreted the teacher's actions or statements as aiming to motivate the students by addressing their affective needs. For example, in a critical event which was described by Faith and in which the teacher asked a student to share with the class an original way of drawing the graph of  $f(x) = \sin(2x - \frac{\pi}{3})$  within the segment  $[0, \pi]$  without analyzing the function, but through transforming the basic *sin* function, Faith interpreted the teacher's response as:

- 1 He [the teacher] gave the student a positive feeling
- 2 and she [the student] felt that she had deep and beautiful thinking [...]
- 3 and this provided an opportunity for other students to think differently.

Faith started by attending to the teacher's intentions (line 1) and then she offered her interpretation of the student's feeling as result of the teacher's response (line 2). Together, lines 1-2 could imply that Faith sees the teacher's response as an expression of his attention to the student's affective aspects. Line 3 could imply either that Faith considered the teacher's response to be a way to motivate other students in the class or a way of promoting students' awareness to the different ways to solve the problem. However, Faith's interpretation is very general in the sense that it can be assigned to various lessons (not just mathematics) and it is disconnected from the specific critical event that was observed. Her general interpretation can be characterized by attending to teacher's affective sensitivity to students (Potari and Jaworski, 2002)

### **Medium level of interpretation to teacher's response- Semi interpretation**

The bottom-up analysis indicated that at this level PTs tended to point out at the teacher's teaching strategy and sometime judged its effectiveness but still without regarding considerations behind the teacher's statements. This echoes van Es et al. (2017) medium level of interpretation - "identifies teaching strategies and choices the teacher makes in the lesson to make thinking visible; and/or judges the effectiveness of strategies."

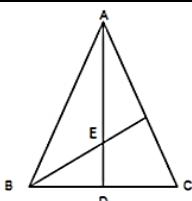
Vanessa observed and reported on the same critical event as Faith. Vanessa’s interpretation was as follows:

- 1 The teacher enjoyed the student’s response,
- 2 and shared it with the rest of the students
- 3 to see whether she was correct or not, and asked them to apply it.
- 4 When he shares the response with the class, and discusses it
- 5 it means she is going in the right direction [...]
- 6 It encourages the student to think more deeply while answering other questions.
- 7 The teacher wants to encourage students to think analogously while analyzing functions
- 8 and not just to rely on rules, so he shares the student's response with the class to discuss.

Here, in lines 2-4 Vanessa point out the teacher’s strategy in the critical event - the teacher made the student’s solution a focus for a whole classroom discussion. From line 3 it may follow that Vanessa sees the teacher’s strategy as aiming to evaluate the student statement “to see whether she was correct or not”. The effectiveness that Vanessa sees in this strategy may be seen as: (I) a way to acknowledge the student’s solution (line 5); (II) a way to encourage the student to “think deeper” when approaching other questions (line 6); and (III) a way to motivate the class to adopt a different kind of approach to function analysis, one that is based on function transformation (line 7-8).

### High level of interpretation to teacher’s response- Full interpretation

The characteristics of this category - *Participant details the teaching strategies that the teacher used when responding to the students’ statements and proposes considerations that may have led to the teacher’s response; and judges the effectiveness of the strategies* - are similar to those of the medium level of interpretation, with two modifications. First – the participant details the teaching strategies - and second - they propose considerations that may have led to the teacher’s response. This characterization is meant to keep the overall language of the levels of interpretation while also echoing our purpose in interpreting teachers’ responses, which is to attempt to understand the intentions, actions, statements and considerations that led to the response. This level differs from van Es et al. (2017) high level of interpretation as in this category the emphasis is on the consideration that PTs ascribe the teacher’s response. An example of high-level interpretation of a teacher’s response can be found in Adel’s critical event interpretation. In the critical event two students, one after the other, asked the teacher whether E is the orthocenter of the triangle (the mathematical task that was at the center of the critical event is presented in figure 1).

<p>In the isosceles triangle ABC (<math>AB=AC</math>), E is the bisectors intersection point. When the line AE is extended, it intersects the base of the triangle at point D (see the figure).</p> <p>Given that: <math>\sphericalangle ABC = \alpha</math>, <math>AE=m</math>.</p> <p>(a) using <math>m</math> and <math>\alpha</math> express the length segment ED.</p> <p>(b) using <math>m</math> and <math>\alpha</math> express the radius of the circumscribed circle to triangle ABC.</p>	
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**Figure 1: The mathematical task that was presented in Adel’s critical event.**

The teacher answered the first student privately and then, when the second student asked the same question, he facilitated a discussion with the whole class. Adel’s interpretation of the teacher’s response was as follows:

- 1 When the first student gave a wrong answer,
- 2 the teacher explained to him privately why what he said was wrong.
- 3 The teacher explained to the student that E is not the orthocenter of the triangle,
- 4 but rather the bisectors meeting point.
- 5 After a second student gave the same answer,
- 6 the teacher thought that there was something wrong with his [the teacher's] explanation,
- 7 and perhaps the rest of the students had also misunderstood.
- 8 So, the [teacher] asked the class: was what the student just said correct? [...]
- 9 The discussion was an effective way to understand the mistake
- 10 and for the teacher to understand why they [the students] think their answer is correct.

In the beginning of her interpretation Adel repeats the course of events (line 1-5). Then Adel offers her idea regarding what led to the teacher's response: because more than one student had this confusion the teacher might have thought that his instruction was not clear enough (line 6-7). In line 8 Adel goes back to the critical event and articulates the teacher's strategy: making the students' statements public within a whole class discussion. Adel reflects about the effectiveness of the response. She sees this strategy as a way that might help the teacher to understand his students' thinking (line 10).

Applying this three-level analytic framework to the 38 critical event reports, we found that 58% of PTs' critical event interpretations for teachers' responses were low-level, 26% of PTs' critical events interpretations were of medium level and 16% were high-level. These findings raised several thoughts regarding ACLIM-5 clinical training programs as well as theoretical insights which are elaborated in the next section.

## **Discussion**

Our bottom-up analysis of critical event reports together with inspiration drawn from van Es et al. (2017) rubric allowed us to categorize coding scheme for depth of levels of interpretation to an observed teacher in a critical event, which does not appear in the literature so far. The first level has a descriptive/affective nature, the second level indicates that the PT tries to interpret the teacher by pointing out the teaching strategy the teacher used and its effectiveness. In the third level, the PT offer considerations that may have led to the teacher's response in the critical event.

From a practical standpoint this scheme will guide our further research regarding the ACLIM-5 clinical preparation program, to investigate whether there is a change in the level of interpretation as PTs become in-service teachers. Furthermore, to investigate whether there is a connection between in-depth interpretation of a teacher's response in a critical event and offering a teaching alternative to that is based on the students' mathematical thinking as it is interpreted by the PT in the critical event. From a theoretical standpoint, a connection between in- depth interpretation for a teacher response in a critical event and suggested teaching alternatives, could serve as a basis for further conceptualization of the professional noticing framework by adding a subprocess - interpreting the teacher response in the student-teacher interaction - in the gap between student interpretation and suggested alternative (e.g., Jacobs et al., 2010).



## Acknowledgment

The project is generously supported by Trump (Israel) Foundation (Grant # 213) and the University of Haifa. We thank Prof. Roza Leikin for her help in monitoring the project.

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