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The connectivity in resources for student-engineers: the case of resources for teaching sequences

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This paper concerns the teaching of mathematics for student-engineers. We focus on the theme of sequences in the first year of engineering education. We claim that teaching sequences in the first year of engineering education requires different kinds of connections: connecting different topic areas, different concepts, and different registers. We use here the concept of connectivity, to analyze resources in the case of an engineering school in France. We provide elements from interviews showing that the teachers can strengthen some forms of connections during their use of resources.

Keywords: connectivity, resources in engineering education, mathematics for engineers, sequences.

Context of the research

The present paper concerns the teaching of mathematics in the context of engineering. We present an analysis of resources collectively designed by teachers in a French Engineering School (undergraduate education). We address the issue of the connections made in mathematics for student-engineers. We focus on the theme “sequences”.

The textbook is a crucial artifact for university teachers (González-Martín, Nardi, & Biza, 2018). However, the appropriateness of the content offered by textbooks in some academic paths remains a controversial issue; Randahl (2016) shows that student-engineers do not perceive the mathematics offered by the textbook as relevant for the engineering context. She considers that the issue of how the textbook is combined with the context of lectures and task solving sessions might “provide learning opportunities with meaningful use of the textbook” (Randahl, 2016, p.67).

Research literature highlights the gap between the mathematics taught in mathematics courses and the use of them in engineering courses (Biehler, Kortemeyer, & Schaper, 2015). Minding this gap supposes to understand the reasons behind it. Research evidences also the necessity of making explicit connections between theory and practice to make the mathematical content in engineering education relevant to students (Flegg, Mallet, & Lupton, 2012). According to Gueudet and Quéré (2018), engineers in the workplace identify missing links in the mathematics taught during their studies: links between mathematics and the real world, and between different mathematical contents.

All these sources suggest that the teaching of mathematics in engineering education should propose specific “connections”. The main question we address here is: what type of connections is present in the resources used and in teaching practices regarding mathematics for engineering students?

Connectivity in resources: topic areas, concepts and semiotic representations

“Connections” can be related with (at least) three issues that could influence the teaching of mathematics; some of them are specific to the context of engineering education.

- (1) Within mathematics, the networking between mathematical objects and their representations (Hiebert & Carpenter, 1992), identified as essential for the understanding of mathematical facts.
- (2) The need to build links between mathematics courses and engineering courses.
- (3) The modifications of learning processes in the digital era prompted Siemens (2005) to introduce the concept of *connectivism*. He considers learning as an “actionable knowledge” that can reside outside ourselves. The learning in this case depends on the ability to build a network of connections between, and across, the available resources (or databases) and users of these resources.

Based on these works, we have proposed the concept of *connectivity* (Gueudet, Pepin, Restrepo, Sabra, & Trouche, 2018) developed in the frame of e-textbooks analysis. This connectivity has two components: “macro-level connectivity” that refers to the potential of linking to and between users and resources outside the textbook (practical aspect); and “micro-level connectivity”, where the focus is on a particular mathematical topic within the e-textbook to look for connections in, between, and across individuals’ cognitive/learning tasks and activities. We define therefore *connectivity* in a set of resources for teaching a mathematical theme as a linking potential for a given user (student or teacher) both practically as well as cognitively. We only use here micro-level connectivity. We indeed consider, in the case of sequences, the connections present in resources between different topic areas or mathematics fields, between different concepts, and between different semiotic representations.

For Duval (2006), semiotic representations are productions constituted by the use of signs belonging to the same “register” (natural language, formalism, algebraic formula, graph, figure, etc.). A register of semiotic representation allows the following activities: to represent a concept; to treat representations within the same register; to convert representations from a given register into another. According to Duval (2006), the conceptual acquisition of a mathematical object necessarily passes through the acquisition of one or several semiotic representations. However, the connection between different representations, seems absent in engineering programs. Quéré (2017) interviewed French engineers about their mathematical needs in the workplace. They declared that the teaching of mathematics they received during their studies “did not make enough connections” particularly connections between different mathematical contents and between different representations.

From the institutional point of view, the mathematics taught are shaped by the institution where they are taught (Chevallard, 2003; González-Martín et al., 2018). We consider, as Quéré (2017), that in engineering education, mathematics courses constitute an institution different from engineering courses. Different institutions could shape the teaching of mathematical topics in different ways. Moreover, teachers’ *personal relationship* with those topics, which results from their experiences in different institutions (Chevallard, 2003; González-Martín et al., 2018), could shape the mathematics taught. Indeed, the teaching actually implemented by a teacher using a given resource can offer more connections than this resource. Teachers might develop an “unexpected” connectivity in practice, by their discourse, examples produced “on-the-spot”, and how they show proofs, etc. Here, there is the issue of the combination between *potential connectivity* and *effective connectivity* developed in use, which could be also related to the teacher’s perception of students’ needs and difficulties.

Our aim here is to consider the connectivity in the case of teaching sequences in the first-year in a French Engineering School. Therefore, we reformulate our general question as follow:

Q1: Which connections concerning sequences appear in resources for a mathematical engineering course?

Q2: Which is the effective connectivity of the curriculum implemented by different teachers using these resources?

Sequences in the French curriculum and in the engineering education

Our choice of to focus on sequences is related to its importance in some engineering paths. Within mathematics courses, sequences constitute an entry for other concepts (single and double integrals, series, etc.). They are also essential for studying Fourier series. In engineering courses, sequences are needed in a different way: in the study of some processes of discretization (transferring of continuous models into discrete models), in numerical analysis (numerical resolution of differential equations or heat equations, etc.), and in the “signal processing” course.

In France, the first encounter of students with sequences happens in grade 11. The curriculum recommends that the teaching of sequences in grade 11 goes through the modelling of real-life situations. Connections in terms of registers are recommended. For instance, the curriculum requests using the spreadsheet (numerical representation) or the calculator (algorithms) to compare the evolution of arithmetic and geometric sequences, as well as to represent the term of a sequence graphically. According to the curriculum, in grade 12 the concept of “limit of a sequence” should be studied in order to prepare the introduction of “limit of a function”.

In different engineering education paths, the introduction of sequences usually appears in the calculus courses. Randahl (2016) notices that calculus is part of the basic mathematics courses for engineering students, and it has many formal aspects. Compared to the secondary level, more time is dedicated for mathematical reasoning and the use of formal representations. Research literature on the teaching and learning of sequences acknowledges the existence of strong links between both concepts “limit” and “sequences” (Alcock & Simpson, 2004; Roh, 2008). Regarding the concept of “convergence”, studies highlight the importance of the graphical representation to make sense of it, as well as to facilitate the appropriation of its formal representation (Alcock & Simpson, 2004).

Context of the study and methodology

The context of our study is a French Engineering School. It is one of the three technological universities in France. Mathematics teachers in this Engineering School have different statuses (teacher-researcher, full-time teacher, etc.) and various specialties (mathematicians, engineers, physicists, etc.). Four semesters structure the first and second academic years, where there are “basic science courses”, including mathematics, physics and chemistry. There are four mathematics courses. Each of them followed in consecutive semesters. Sequences are introduced in the first chapter of the mathematics course of the second semester. In this context, our methodology consists in two parts: 1) collection and analysis of resources on sequences; 2) interviews with 3 teachers and their analysis.

The resources collected are collectively designed and used by teachers. They contain the textbook's lesson distributed to students; the slideshow projected during the lecture sessions; the tutorial that contains tasks to solve and problems; and the Practical Work tutorial (PW) composed of online exercises on a moodle platform.

To analyze the resources, in terms of micro-level connectivity, we used the grid developed by Gueudet et al. (2018). We adapted it to our case, according to the definition of connectivity we discussed above. We then analyzed the resources following four steps. In the first step, we identified concepts used in the resources, we mentioned the connected concepts and the way they are connected by definitions, propositions, theorems, examples, and/or exercises. Within this step, we studied the connections that concern "progression" chosen by the designers (connections with previous knowledge and further knowledge). In the second step, for each concept (or connected concepts), we characterized the different moments of their appropriation (lesson, examples, exercises, problem solving, or practical tasks); we considered also the context of this appropriation (different topic areas within mathematics, different disciplines, real-life situations, etc.). In the third step, for each concept (or connected concepts), we identified the registers of semiotic representation used or that can be used in solving tasks; we characterized them in terms of the three cognitive activities "to represent", "to treat", or "to convert" (Duval, 2006). In this step, we determine, where appropriate, the connections made with particular tools (software, calculator, etc.). The fourth step concerns particularly the exercises; we determined the different strategies to solve them, and the variations of the same exercises. The fourth step allowed us to understand the connections that the designers emphasize between concepts, registers and mathematics topic areas.

We also conducted interviews with three teachers. We chose teachers with contrasting profiles in terms of their specialties and academic education, and in terms of their status. T1 is a physicist, he has a PhD in theoretical physics and modeling, he teaches mathematics, modeling and physics; T2 is an engineer, he has a PhD in Engineering, he teaches mathematics and engineering courses; T3 is a mathematics teacher, he has an "agrégation" in mathematics (French competitive examination that must be passed to teach at university). We hypothesize that the contrasted profiles can inform us on different forms of connectivity developed in implementation of mathematics resources.

The interview grid covers four axes, according to which we analyzed the teachers' declarations. The first concerns the connections to make across the different mathematics topic areas, across different disciplines, or with real-life problems. The second axis concerns the teachers' point of view on students' difficulties, which influences the connectivity they could emphasize in their use of resources. The third axis concerns the place of the examples they add, which allows to make connections between concepts or to use the registers of semiotic representation. The fourth axis concerns the place of mathematics proofs in engineering education, and their role in enhancing connectivity. We aim by the analysis of the interviews to identify elements on the effective connectivity of the curriculum implemented by different teachers using the same resources. Nevertheless, we are limited here to what teachers declare about the use of resources.

Potential connectivity in resources for teaching sequences

We identified in the resources for teaching sequences: 7 definitions, 10 theorems, 29 exercises, and 9 examples. The PW contains three exercises, where the use of Wiris CAS calculator is requested.

In terms of previous knowledge, some basic concepts from secondary school are reintroduced but in a formal register (convergence and limit of sequences). We note that continuity and limits of functions are taught in the first semester. We also note that the sequential definition of limits of functions is not taught. The order used to teach the concepts of “limit of sequences” and “limit of functions” is different from the choice made in the curriculum of secondary school (see above).

The connections between both concepts “limit of sequences” and “convergence” are emphasized. The resources also emphasize, in the properties and exercises, the connections between “convergence”, “variation of sequences” and “bounded sequences”. For instance, there is in the tutorial an exercise with nine different sequences; students are asked to study the convergence of sequences and calculate their limits when it is possible. The sequences given need different strategies to solve the proposed tasks (a sequence bounded by another one that tends to 0; a decreasing sequence and bounded from below by 0; a task that needs the use of properties about the quotient of convergent sequences; etc.). Recursive sequences are particularly emphasized in the different components of the resources. The exercises related to recursive sequences allow students to establish connections with “upper bound”, “lower bound” and functions.

There is no connection between different topic areas within mathematics. In terms of different semiotic representations, the resources contain particularly one semiotic representation, the formal representation. In general, the treatment of tasks remains within the same representation. The resources contain rarely tasks where students are asked to represent mathematical objects in a given register. The PW constitutes an opportunity to make new connections between mathematics topic areas as well as between semiotic representations. Nevertheless, it is restricted to three exercises. The connections between semiotic representations are strengthened; there are, for example, tasks that need the graphical study of the given sequences. Besides, the algorithmic topic area changes to study the behavior of recursive sequences by dynamic representations (Figure 1). The use of Wiris CAS software offers the potential to make connections between algorithms and the graphical register.

In the analyzed resources, sequences constitute a theme where the mathematical materials are mainly limited to the calculus topic area. This is very similar to the content presented to

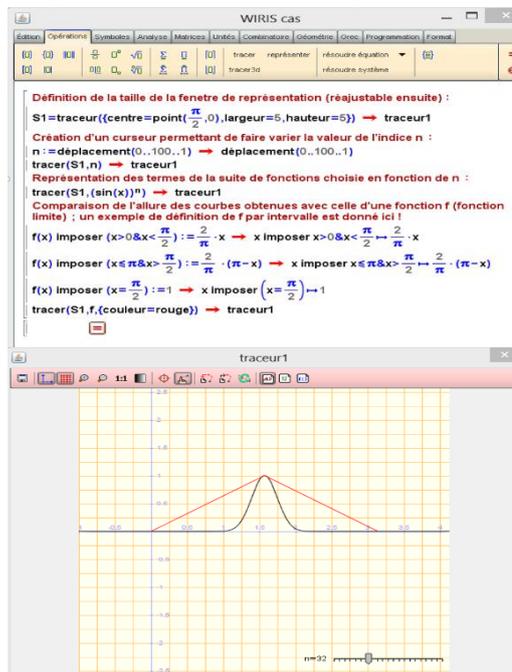


Figure 1. It is required to act on the cursor “n” to relate the two dynamic representations (Wiris CAS).

mathematics majors. There are strong connections between concepts (convergence/divergence, limit of a sequence, function, upper/lower bound, recursive sequences). The tasks and activities propose mainly treatments within the formal register. In addition, there are no authentic situations, real life problems, or problems referring to other disciplines. All the examples, exercises and applications are intra-mathematical, which could strengthen a possible gap with the institution of engineering courses.

Analysis of interviews: towards effective connectivity in the enacted curriculum

Concerning the characteristics of a mathematics teaching well suited to engineering education, we identify three different trends in the answers of the interviewed teachers: T1 emphasizes the need for “contextualization”; T2 perceives his profile as interesting for teaching mathematics in engineering education; T3 perceives himself as a teacher who applies the mathematics as are “recognized in the university institution”. T1 notices that teachers who are mathematicians have a reluctance to integrate problems referring to other disciplines. He supposes that they need some support in the design and the implementation of this kind of problems. T2 states he has “ability” to make connections between contents to ensure a continuum of learning between mathematics courses and engineering courses. He sees himself as a “specialist in mathematics for engineers”. He says:

T2: Specialist of mathematics no, I am not a mathematician. Otherwise, am I a specialist in mathematics for engineers? I think I have a lot of experience to know how math tools are used in the workplace. [Our translation]

He presents mathematics content as tools that have utility in the workplace of engineers:

T2: We try often to remind students that later on they will need mathematical tools and they have to build them. They have to be able to manipulate them properly. [Our translation]

T3 only considers the mathematics courses in terms of utility within mathematics.

In their answers about students’ difficulties, the teachers referred to: 1) reasoning (T1, T2, T3); 2) treatment of tasks within the formal representation (T2, T3); 3) the capacity of abstraction (T1, T3). It seems that the emphasis on formal representations in resources accentuates these difficulties.

According to their declarations in the interviews, the examples they add during the classroom sessions appear as revealing their will to support connectivity in different ways. Each teacher, in his preparation of the course, designs his own examples. T1 says that the examples he designs should enhance the meaning of the concepts at stake. He presents examples referring to other disciplines. T2 says that he gives examples particularly during the introduction of concepts, “intra and extra-mathematical examples”. According to T1 and T2, examples referring to other disciplines of real life situations are opportunities to make sense of the contents. T3 says that he gives examples after having stated a theorem. He uses examples to illustrate the “operability of the theorem”.

Concerning proofs, all interviewed teachers stated that they do not do, in the classroom sessions, all the proofs given in the resources. The reasons they gave could be interpreted in terms of personal relationship with calculus [our translation]:

T1: the proofs I like are usually the useful proofs. I do not like for instance the “trapping” proofs for which if you do not have the right intuition, you cannot start.

T2: When you look at the theorem and the proof does not come to your mind right away, and you finally find out that the proof is based on tricks, I think this proof should be replaced by examples.

T3: If the proof requires little effort and seems important to me, I do it. I do not do it if I think we can understand the theorem without the proof, sometimes the proofs help to understand.

T1 and T2 show in class the proofs that can support the learning of a form of reasoning, which is, according to them, essential for future engineers. T3 shows the proofs that can help to understand theorems and properties, even if they sometimes require fastidious techniques. However, the interviews allowed us to identify criteria the teachers consider for choosing the proofs to do in the mathematics courses in engineering education: they can help improve the understanding of theorems; they allow highlighting the importance of a given theorem; they can help – during the proof – implement a mathematical reasoning. Each of these criteria could promote a specific form of connection between concepts.

The additional resources that the teachers design individually can strengthen specific connectivity in their teaching. T2, for instance, prepares examples to motivate the introduction of a given concept or to replace “tricky” proofs. T1 designs resources to show situations from other disciplines. T3 designs additional resources (exercises and examples) to strengthen links between mathematics concepts.

Discussion and conclusion

We present in this paper the issue of connectivity concerning a mathematics course in engineering education. We analyze the potential connectivity of resources, collectively designed, for teaching sequences theme. We highlight elements on effective connectivity that teachers, as users of these resources, could develop in their implementation.

The potential connectivity in resources for teaching sequences appears as restricted to the calculus topic area. The register of formal representation occupies an important space. According to two of the interviewed teachers (T1 and T2), making connections with engineering contexts, real-life problems or other disciplines “remains a dimension to enhance”.

It appears by the interviews that the way the teachers use resources is likely to lead to an effective connectivity different from the potential connectivity, and this could be related to their personal relationship with the topic at stake. The teachers stated that students struggle with making sense of formal properties. Hence they attempt to support students by designing additional resources. They add examples and select separately the proofs to show in lectures sessions. These individual teachers’ choices could strengthen some forms of connections. The physicist (T1) tries to enhance the connections in terms of other disciplines; the engineer (T2) constructs mathematical tools and tries to make connections with the engineering courses; the mathematics teacher (T3) tries to maintain connections within mathematics.

The analysis of the interviews draws our attention to the need to take into consideration the macro-level connectivity (Gueudet et al. 2018), especially connectivity across disciplines, in terms of teachers' own resources, as well as in terms of teachers' and students' joint work. Besides, based on the analysis of the interviews, we highlight the necessity to observe how the teachers select and implement proofs in their sessions, which could tell us more about the effective connectivity built.

The issue of connectivity in resources and the connectivity developed by their use seems to be relevant in the context of engineering education, where the connectivity between theory and practice is crucial. Each of these elements requires further study with a larger sample of teachers, considering different mathematical topics and articulating the analysis of resources and classroom observations.

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