Relations between academic knowledge and knowledge taught in secondary education: Klein's second discontinuity in the case of the integral

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Felix Klein (1872) put forward a double *discontinuity*: in addition to the secondary-tertiary transition, a second discontinuity occurs at the end of university studies when a student who obtains a teaching position is appointed to teach mathematics in high-school. This discontinuity is observed through the difficulties faced by postgraduate students in France to perceive the links between the knowledge of mathematics learned at university and the knowledge acquired in high school (for example between the integral as a measurement of an area and the Lebesgue integral, even with the Riemann Integral). These postgraduate students are therefore often reluctant to work on advanced mathematics (e.g. upper division undergraduate courses) as part of their teacher training.

Winsløw and Grønbæk (2014) formalized this double discontinuity in the language of the Anthropological Theory of The Didactics (ATD), using the notion of personal relationship R to an object of knowledge within an institution, introduced by Chevallard (2007). Let us consider: the institution, which here will be either high school (L) or university (U); an individual x who occupies different kinds of positions in his path through the institutions: first as a student in high school (s), then as a student at university (σ) and then back again in high school as school teacher (t); an object of knowledge, that lives in the institutions and noted o in high school and ω when considered at university level. Klein's double discontinuity may be expressed as the following diagram:

$$R_L(s,o) \longrightarrow R_U(\sigma,\omega) \longrightarrow R_L(t,o)$$

The purpose of my research project (PhD thesis) is to study the second transition between $R_U(\sigma,\omega)$ and $R_L(t,o)$ by choosing an object of knowledge from the field of analysis: the integral. Students meet integrals in different institutions and in different theories (Newton Integral in high school, Riemann and Lebesgue integrals, and Measure Theory at university). So a first research question is: *how does a math teacher in high school mobilize his knowledge of university mathematics to teach integrals in high school*? To study this question, we formulate sub-questions: *What links are put forward by the institutions between the different integration theories? Conversely, which gaps can be observed in the curricula*? *How are the continuities and discontinuities perceived by teacher students*? *What training courses may be proposed to help students draw connections*? The poster presents the overall methodology of my study and some preliminary results obtained so far.

The methods used are of a qualitative nature. *Epistemological Models of Reference* (EMR; Florensa, Bosch, & Gascón, 2016) are built in order to describe the relationships to knowledge, beginning with $R_U(\sigma,\omega)$ and $R_L(s,o)$. The EMR is a tool for the researcher who operates a reconstruction of knowledge. The elaboration of the EMR is based on epistemological analyses related to historical epistemology (supplemented if necessary by a study of contemporary epistemology by means of interviews with mathematicians) and the analysis of official syllabi and curricula, teaching materials and textbooks. This model is constructed both to analyze continuities, discontinuities and gaps in the path of the learner through institutions and to identify possible entry points for the design of a didactic engineering.

A first study (conducted in the context of my Master's degree thesis) consisted of comparing EMRs between the logos block of praxeologies related to the Riemann integral at university and the integral in high school. The main results and their consequences for learning (in the form of hypotheses for further research) are as follows: on the one hand, the praxeologies related to the measurement of areas at high school contain an implicit logos that a teacher student will probably have difficulties to relate to academic knowledge on the integral (a substantial mathematical work would be necessary to link the notion of area from high school with Measure Theory, taught as a highly abstract subject at university); on the other hand, the academic praxeologies involving Riemann's integral focus on computational techniques for antiderivatives rather than working on the corresponding logos, apart from abstract questions on integrability (probably out of reach for a majority of students). As a result, it is likely that students will draw little connections between the block of the logos of Riemann praxeologies and the methods for approximate computation of integrals at high school since they will not grasp the conceptual scope of Riemann sums.

In order to study Klein's second transition and to propose training courses for future teachers, didactic praxeologies of the teacher need to be investigated: indeed, the tasks of the teacher are not limited to solving problems, but also to elaborate problems that promote adequate conceptualization and learning, make pertinent choices of didactic variables, and provide adequate feedback to students. The development of a model to describe these praxeologies will be based on observations of classroom practices and teacher interviews, in order to determine the role and then explore the possibility of strengthening the use of academic knowledge in didactic organizations of the teacher.

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