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Identifying sense-making in algebra instruction at U.S. post-secondary colleges

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As part of a large-scale study of algebra instruction at two-year post-secondary institutions in the United States, we have developed a qualitative video analysis framework, Evaluating the Quality of Instruction in Post-secondary Mathematics (EQIPM). In this paper, we present two of twelve characteristics used to describe instruction (Instructors Making Sense of Procedures, and Student Mathematical Reasoning and Sense Making), which describe students and instructors engaging with mathematics so their ideas or ways of thinking are evident. We use examples taken from our corpus of videos to exemplify these two characteristics of instruction and how they contribute to its quality. We also offer suggestions for professional development activities.

Keywords: Instruction, College mathematics, Postsecondary education, Community colleges, Instructional quality.

In the United States, 43% of all undergraduate students enroll in public post-secondary institutions that offer courses for the first two years of their undergraduate degree. These institutions, hereafter called community colleges, provide students with many options to further their educational goals, some of which include remediation, transfer to university undergraduate programs, vocational training, general education, continuing education, and workforce development (Blair, Kirkman, & Maxwell, 2018). Community colleges target diverse students who are mostly non-traditional (e.g., older, working, or with family responsibilities), offer flexible schedules, and charge very low tuition compared to universities. Because of the wider range of students who enroll, these colleges offer a broad range of mathematics courses, from developmental mathematics (designed to prepare students for collegiate level study of mathematics) to mathematics courses taught in the first two years of an undergraduate major. The failure rates in algebra courses at community colleges range from 30% to 70% (Bahr, 2010). Because algebra skills are of paramount importance for advancing into higher level mathematics, investigating algebra instruction at community colleges becomes important; however, community colleges are largely under-researched (Mesa, 2017).

In *Algebra Instruction at Community Colleges: An Exploration of its Relationship with Student Success (AI@CC, Watkins et al., 2016)*, we seek to investigate the relationship between the quality of characteristics of algebra instruction and student learning as evidenced by student grade as a percent. In this paper, we exemplify two characteristics of algebra instruction at community

colleges, Instructors Making Sense of Procedures, and Student Mathematical Reasoning and Sense Making. We focus on observed instances that evidence mathematical sense-making¹, to start addressing the wider question: What is the nature of sense-making evidenced in algebra instruction at the community college level of post-secondary mathematics education?

Background and supporting literature

We used the instructional triangle to define algebra instruction as the interactions between instructors, students, and the content studied in community college environments (Figure 1, Cohen, Raudenbush, & Ball, 2003). To characterise algebra instruction, we drew on two frameworks that analyse the quality of instruction created for school-level mathematics education (Mathematical Quality of Instruction (MQI), Hill, 2014; Quality of Instructional Practices in Algebra (QIPA), Litke, 2015). These frameworks describe and code (with ratings from 1 to 5)

instructional practices from video-recorded class sessions by rating consecutive 7.5-minute segments. Although the two frameworks provided useful language to describe the characteristics of instruction, the actual descriptions were tied to elementary or high-school content and interactions that bore little resemblance to instructional practices at community colleges. We retained some of their characteristics, but modified the descriptions for our purposes, through multiple iterations that involved the use of video recordings of 15 class sessions in Fall 2016, ranging between 45 and 120 minutes (see Cawley et al., 2018). From this work we developed the video-coding framework called Evaluating the Quality of Instruction in Post-secondary Mathematics (EQIPM). This paper includes two EQIPM characteristics, Instructors Making Sense of Procedures and Student Mathematical Reasoning and Sense Making, which emphasise ways in which instructors or students engage in algebraic sense-making in community colleges. Version 3.0 of EQIPM is presented in Figure 2².

The definition of ambitious mathematics instruction (NGA, 2010) supports 11 of 12 characteristics of instruction in EQIPM. In ambitious instruction, the instructor engages students in challenging tasks, observing and listening while they work on those tasks providing an appropriate level of support to diverse learners. The instructional goal is to ensure that all students do high quality work instead of only expecting fast and correct execution of mathematical procedures. Classroom Environment emerged from our observations of community college lessons. This characteristic is informed by Danielson (2015) who advocated that in a respectful and open environment, student opinions are encouraged, and student contributions are validated in ways that make them feel safe.

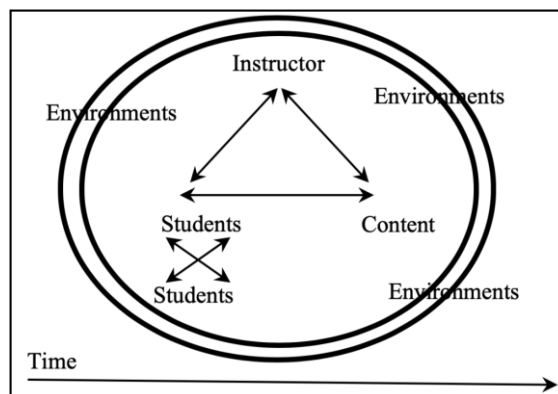


Figure 1: Instructional Triangle (Cohen, Raudenbush, & Ball, 2003)

¹ We use the NCTM (2009) definition of sense-making which accounts for students discovering coherence across mathematical domains and seeing connections between new concepts and their existing knowledge.

² The development of EQIPM is ongoing. See Mesa et al. (2019) for the most recent changes in the code structure.

EQIPM: Evaluating Quality of Instruction in Postsecondary Mathematics			
Features of Segment	Quality of Student-Content Interaction	Quality of Instructor-Content Interaction	Quality of Instructor-Student Interaction
Mathematics is Focus of Segment	Student Mathematical Reasoning and Sense-Making	Instructors Making Sense of Procedures	Instructor-Student Continuum of Instruction
Procedure Taught	Connecting across Representations	Supporting Procedural Flexibility	Classroom Environment
Modes of Instruction	Situating the Mathematics	Organization in the Presentation of Procedures	Inquiry/Exploration
Technology Use	Mathematical Errors and Imprecisions in Content and Language		Remediation of Student Errors and Difficulties
	Mathematical Explanations		

Figure 2: Dimensions and characteristics of instruction in EQIPM

The American Mathematical Association of Two-Year Colleges' IMPACT³ guide (2018) presents a vision to improve mathematics education in the first two years of college. The IMPACT guide concludes that for students to achieve proficiency in mathematics

it is necessary for them to, among other things, (1) know mathematics procedures and execute core computations fluently, (2) make sense of and solve problems, and (3) demonstrate evidence of mathematical understanding (AMATYC, 2018, p. 25). Research by Star (2005) provides additional evidence of the importance of linking conceptual understanding with procedural skills.

Methods

The data for this paper were collected in the Fall of 2017. Trained observers video-recorded all class sessions on two of three topics from 40 instructors: linear, rational, or exponential equations and functions. The instructors, who volunteered to be observed, were at six diverse community colleges, which represent a range of institution size, degree of urbanicity, region (Southwest, Midwest, and Central in the U.S.), and student background. This round of data collection resulted in 131 class sessions, which ranged in duration between 45 and 150 minutes. Fifty-four percent of these sessions were solely in a lecture context and 85% included some elements of a lecture.

Starting in the Summer 2018, 17 trained coders used EQIPM to rate the characteristics of instruction that appeared in each segment⁴ (n = 1,576 coded segments of which 1,236 are full 7.5-minute segments) of every class session. For each segment, the quality of each characteristic was coded along the 12 characteristics using a rating from 1 to 5. The coders provided a justification for their rating, including the specific time stamp in which the coder noticed evidence that justified that rating, and a brief explanation of the quality of the characteristic based on specific guidelines of EQIPM. For the most part, a rating of 1 was reserved for "no evidence" except for the Mathematical Errors and Imprecisions in Content and Language in which the ratings are reversed. A rating of 3 was considered an acceptable quality, usually indicating a modal rating. A rating of 5 was reserved for exemplary cases; cases we interpreted as best examples of that characteristic against EQIPM. Ratings of 2 and 4 were reserved for 'in between' cases. When some features were present but they

³ Improving Mathematical Prowess and College Teaching (IMPACT)

⁴ Hill et al. (2012) used a segment length of 7.5-minutes after several comparisons of coding using various lengths. All lessons were parsed into 7.5-minute segments; the last segments of the lessons were usually shorter than 7.5 minutes.

were of less quality than what a rating of 3 would have been, a 2 was assigned. When the quality was better than a rating of 3, but not truly illustrative of an exemplary quality, a 4 was assigned.

Instructors Making Sense of Procedures refers to instructor work that fosters students' sense-making about how a procedure moves from one step to the other, rather than merely reproducing the steps of the procedure (Star, 2005). For this code, the coder is trained to assess the extent to which instructor practices of using mathematical definitions, symbols, properties, structures, examples, and interpretations of solutions motivate understanding of a mathematical procedure. Examples of such instructor practices include, but are not limited to attending to the type of solution generated by a procedure and its interpretation, using a series of examples of an abstract formula to extract key features, and clarifying symbols of definitions. For a 7.5-minute segment, an exemplary 5 rating of the characteristic is assigned in the event that the coder interprets that the practices identified motivate mathematical understanding and belong to either a range of instances or a sustained, prolonged instance with instructor practices for fostering student sense-making. A rating of a 5 only indicates that sense-making happened in an exemplary way, attending to elements of a procedure or the algebraic concepts in a procedure. Other codes in the framework will pick up additional elements of instruction that together may allow one to determine high-quality instruction.

Student Mathematical Reasoning and Sense Making entails the components of reasoning and sense-making for students. Mathematical reasoning includes student practices such as drawing logical conclusions from patterns identified, providing conjectures, offering counter-claims in response to a proposed mathematical idea, and mentioning the mathematical properties that underlie a procedure. Evidence for sense-making includes student practices such as asking mathematically motivated questions while requesting explanations, leveraging previous mathematical ideas while responding to a question, and interpreting the solution generated by a procedure. A 5 rating includes either a combination of instances that meet these criteria or a sustained instance that saturates the 7.5-minute segment with student reasoning or sense-making so that student ideas are evident and contribute to the development of the mathematics.

Coders adhered to three additional rules when coding: (a) assigning a higher rating rather than a lower rating when in doubt, (b) rating each segment in isolation from other segments within the same class session, and (c) focusing on evidence that was seen and said in the video, rather than what coders thought should be in the video. Following Hill (personal communication, September 2017), 10% of randomly selected class sessions were double-coded; the coders held calibration meetings to discuss discrepancies in ratings and agree upon final ratings. The data used for the analysis reported in this paper came from the coding of 903 segments.

Findings

EQIPM offers a close and granular examination of the quality of algebra instruction at community colleges, but no element of EQIPM can stand alone to suggest high-quality instruction. Breaking down instruction into parts is a first important step towards efforts to understand the current status of instruction at community colleges, and a valuable activity that may help instructors develop their teaching skills. Qualities we interpreted as exemplary included cases of algebra instruction identified with a 5 rating. Only 89 out of 920 segments (10%) were identified as exemplary in at

least one of the 12 codes. There were 398 segments (43%) with a rating of 4 and 5. We identified 29 exemplary segments (5,3%) where instructors supported students in making sense of procedures and seven segments (1%) where students demonstrated exemplary mathematical reasoning and sense-making in class. Examining instances with a rating of 4 or 5, the cases increase to 95 (10%) and 37 (4%) respectively. In light of these percentages in the community college setting, we think that there is much room for improving instruction, and existing practices of exceptional quality can act as resources for improving instruction and student outcomes.

Instructors making sense of procedures

In a lesson on rational expressions, Instructor 110 discussed with the students the procedure of finding domain restrictions. She started her lecture by asking what is $\frac{0}{5}$ and why $\frac{7}{0}$ is undefined, gradually moving student attention to rational expressions and the denominator of $\frac{x+5}{x-7}$. She asked students to find x -values for which the aforementioned rational expression is undefined, and students suggested $x = 7$. After the instructor made the calculations for the suggested value and confirmed that the outcome is $\frac{12}{0}$, which is undefined, she asked whether there were suggestions for other values for which the rational expression is undefined. She waited for 11 seconds for a response, and suggested $x = -5$ for which the expression becomes $\frac{0}{-12}$. The students recalled this fraction equals zero so the rational expression is defined for $x = -5$. The instructor then gave the students time to find domain restrictions for a series of more complicated examples of rational expressions with denominators that are binomials, trinomials, and other polynomials (e.g., $\frac{x+4}{x^2-3x-10}$ and $\frac{x^2-x-2}{x^2-25}$). The use of instructor practices was rated as an exemplary case for this characteristic, because the instructor attended to the type of solution (i.e., denominator equals zero) generated by the procedure of finding domain restrictions, first using integers, which the students are familiar with in this level of education, then interpreting increasingly complicated examples of rational expressions. The students successfully suggested -7 for $\frac{12}{0}$ and related $\frac{0}{-12}$ back to the information in the beginning of the lecture.

In a lesson on exponential functions, Instructor 102 discussed the reason behind the growth pattern of exponential functions. He created tables of values of a linear function, a quadratic function, and an exponential function in which the x -values started at 0 and increased by 1. He calculated the difference of consecutive y -values of each of the three functions, and found that for the linear function the difference was always 2, but for the quadratic function the difference each time increased by 2. He generalised the finding about the differences by saying for both functions that “It doesn’t matter it is a 2, it could be any constant.” He then found that for the exponential function the difference of consecutive y -values, even the difference of the difference of consecutive y -values, would never become constant or increase by a constant value; rather, the difference of consecutive y -values will repeat the growth of the exponential function. The instructor’s use of the structure of three kinds of functions—a linear, quadratic, and exponential—to clarify the meaning of exponential growth was rated as an exemplary case for this characteristic. In particular, he used various numerical y -values of the functions and their differences to find patterns and exemplify the exponential growth. Later in the lesson, all students went to the board to determine the type of

growth of various types of functions and they were asked to classify the growth of functions as either exponential or linear.

Student mathematical reasoning and sense making

In a lesson on rational expressions, Instructor 610 worked with the students on simplifying rational expressions. After the first half hour of the 150-minute lecture, the instructor asked students to tell her what simplifications she should carry out for the expression $\frac{3y}{y^2+3y-10} - \frac{6}{y^2+3y-10}$. A student correctly suggested the instructor to write $\frac{3y-6}{y^2+3y-10}$, to factor the polynomial in the numerator and denominator, and to simplify the rational expression. Another student correctly suggested for the expression $\frac{5}{7} + \frac{4}{21}$ that the instructor could multiply both the numerator and the denominator of $\frac{5}{7}$ by 3 and perform all calculations needed. A third student correctly suggested for the expression $\frac{1}{5} - \frac{2}{3}$ that the instructor multiply the numerator and the denominator of $\frac{1}{5}$ by 3, the numerator and denominator of $\frac{2}{3}$ by 5, and perform all calculations needed. This instance where different students contributed to the lecture by leveraging the mathematical idea of Least Common Denominator (LCD) in various cases (e.g., the LCD is the same as or different from both of the denominators) while simplifying expressions lasts for about two of the 7.5 minutes of the segment. A follow-up instance in the same segment lasted for about four and a half minutes. It included a fourth student who orally provided the calculations he performed to find the LCD for the expression $\frac{19x+5}{4x-12} + \frac{3}{2x^2-12x+18}$. By factoring the latter expression, it becomes $\frac{19x+5}{4(x-3)} + \frac{3}{2(x-3)^2}$. The student reasoned the LCD was $4(x-3)^2$ because the LCD included the highest exponent from all multiples of the factors of the denominators. Two other students found the LCD for each of two different rational expressions, using their fellow student's reasoning. The combination of two prolonged instances that saturate the segment with student reasoning and sense-making so that student inputs contributed to the development of the meaning of the LCD was rated as an exemplary case for this code.

A rule for deciding on the rating of Student Mathematical Reasoning and Sense Making was that there must be clear evidence of students engaging in mathematical reasoning or sense-making through verbal utterance(s) and/or through written work. A limitation for finding such evidence was that despite the premium quality of our audio-visual equipment, the recordings sometimes did not capture clearly student voices or written work. Although we found instances that saturated the 7.5-minute segments with student reasoning or sense-making, the audible or visible evidence found within a segment could not qualify for an exemplary case. That resulted into lowering the quantity of exemplary cases found for this characteristic of instruction in the community college classes.

Discussion

This paper described two characteristics of instruction captured in EQIPM, Instructors Making Sense of Procedures and Student Mathematical Reasoning and Sense Making. The common theme in the three examples given can be summarized as 1) a sustained effort to provide sense-making behind procedures by use of a variety of examples in varying complexity while 2) prompting students to provide solutions to the examples with an explanation of their reasoning. The

instructors' approach paved the way for students to develop proficiency in mathematical procedures; this is a new insight into community college instruction because educational research in this context is still rather limited. Rules were not given to simply be followed but ideas were presented with concrete examples to apply sense-making. We currently work on improving the codes and ratings of EQIPM. In recent iterations for the development of EQIPM, we widened the mathematical content of the codes of the dimension Instructor-Content interaction. Instructor Making Sense of Procedures, for example, includes sense-making whether it occurred while supporting a procedure, a mathematical idea or ways of thinking.

In *Instructors Making Sense of Procedures*, instructor practices that motivated sense-making included the search for patterns, the use of integers or numerical y -values of functions which were familiar to students, and the careful selection of examples with increasing rigor. The latter can help students build their own understanding of rate of change, and later connect this idea to the concept of derivative in calculus. In *Student Mathematical Reasoning and Sense Making*, the instructor offered different students the opportunity to suggest how they would simplify rational expressions and to justify their choices, sometimes using their peers' language and reasoning. This practice also provides the instructor with the opportunity to overlay student discussion with more precise language and connect student ideas. Such video-recorded instances of exceptional qualities of algebra instruction at community colleges can be leverage points to discuss with instructors, compare, and build up the current instructional practices that are in place. EQIPM nevertheless is not intended to become a score system to evaluate teachers. It is created to understand what the characteristics are that constitute instruction at community colleges and what features make that instruction of high quality.

The AMATYC IMPACT guide provides a vision for mathematics instruction in the first two years of college and identifies key attributes that point to the importance of students making sense of mathematics and demonstrating evidence of mathematical understanding (AMATYC, 2018). The opportunities for students to develop these attributes, as evidenced by our video analysis, is not common practice at community colleges. Our findings indicate that exemplary implementation of *Instructors Making Sense of Procedures*, and *Student Mathematical Reasoning and Sense Making* occurred in only a small number of cases. Professional development using instances of exemplary practices that support high quality algebra instruction can help change the landscape of mathematics instruction at community colleges by helping instructors provide opportunities for their students to develop key attributes tied to mathematical proficiency. In the next phase, we will work to identify ways for instructors to support students to develop mathematical proficiency and to subsequently improve student performance and preparation for higher mathematics.

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