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Towards an interplay between TDS and ATD in a Design-Based Research project at the entrance to the university

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It is often said that understanding the complex teaching and learning issues at university level requires the networking of several theories. In this paper, we study how the Theory of Didactic Situations and the Anthropological Theory of the Didactic can be combined to address a developmental research issue at university level. We refer to criteria provided by Design-Based Research to compare and contrast the two theories. This preliminary study should allow us to design a teaching intervention related to Calculus at university.

Keywords: Calculus, hypothetical learning trajectory, Theory of Didactic Situations, Anthropological Theory of the Didactic, Design-Based Research.

Delimitation of the project's context

The final aim of our research is to design a teaching intervention in relation to real numbers and numerical sequences at university. At the entrance to university, real numbers and numerical sequences represent an important part of the real analysis curriculum in the mathematical sciences and pre-engineering in France and Tunisia where our experiment will take place. The goal of this chapter is to establish rigorously the properties of real numbers and their interpretation by means of numerical sequences, and its structure is generally founded (as requested by the curriculum) on the continuity principle of the set of real numbers: *Any non-empty subset bounded from above has a least upper bound*. This principle in conjunction with the ordered field properties achieves the implicit axiomatic construction of the theory of real numbers and other properties follow: Archimedean property, density of rational numbers and sequence's interpretation, sequence's interpretation of the continuity principle (monotone convergence theorem), adjacent sequences theorem, Bolzano-Weierstrass theorem and Cauchy completeness theorem. In line with Winsløw, Gueudet, Hochmut and Nardi's claim (2018) about the stability of the mathematical contents (including Calculus) at the first year of university, this chapter requires typical practices involving logical reasoning about the formal existence of real numbers or the aforementioned properties. Relying on different paradigms (e.g., cognitive, socio-cultural, semiotics), research on the secondary-tertiary Calculus transition has highlighted over the years students' difficulties to engage in these practices at the entrance to university and beyond (e.g., Bressoud, Ghedamsi, Martinez-Luaces & Törner, 2016; Winsløw et al., 2018). We decided to use a Design-Based Research approach (Anderson & Shattuck 2012) with the aim of designing a teaching intervention for learning real numbers and sequences that provides details about what is likely to happen and why. The DBR approach leaves an open choice concerning the theories involved in the design. Moreover, comparing and contrasting theories can contribute to improve these theories (Prediger, Bikner-Ahsbals & Arzarello 2008). Here we have chosen to network the Theory of Didactic Situations (TDS, Brousseau 2008) and the Anthropological Theory of the Didactics (ATD, Chevallard 2015).

TDS and ATD are indeed two neighboring theories with strong epistemological concerns and with the same research program, so that: "[...] *each one reinterprets and reformulates the problems raised by the other.*" (Artigue, Bosch, Gascón & Lenfant 2010, p. 1535). In this paper, we firstly focus on basic variables for comparing and contrasting TDS and ATD by means of the DBR methodology. We secondly present our preliminary thoughts on the elaboration of the Hypothetical Learning Trajectory (HLT) of Calculus at the entrance to university (related to the interconnected properties of real numbers and sequences). The HLT is a fundamental component of DBR. It mainly contains mathematical learning goals, mathematical problems and specific learning and teaching assumptions in the context of these problems; it has a dynamic character and can change during the different phases of the research. The interaction between the HLT and the empirical results is the cornerstone element for developing the teaching intervention.

Design-Based Research as a methodological research

Reports on what is DBR are numerous. In this paper we retain Anderson and Shattuck claim (2012): "Design-based research (DBR) evolved near the beginning of the 21st century and was heralded as a practical research methodology that could effectively bridge the chasm between research and practice in formal education." (p. 16). We focus our description of DBR on its main methodological components: 1) iterative cyclic process; 2) deciding on orienting theory and design guidelines framework; 3) HLT; and 4) selection of research instruments and techniques. The DBR methodology classically consists of intertwined cycles of three phases each: elaboration of the intervention (design); teaching experiments and analysis on the fly; and retrospective analysis. The collaboration between researchers and practitioners (teachers) is the key feature of the quality of the iterative adjustment and refinement of the intervention. This collaboration is carried out through prospective and reflective processes before, during and after each teaching experiment. The results of the retrospective analysis phase mostly supply a new cycle of three phases. Concretely, the research's aim (i.e. the development of an empirically and theoretically grounded teaching intervention for specific mathematics education aims) has to be firstly transformed into research questions and subquestions with the help of the orienting theory and of the design guidelines framework. The design guidelines framework is a theoretical construct describing how to design the intended learning and teaching environment. For an illustration of these choices, see for example the DBR project for statistics education of Bakker and Van Eerde (2015) where a semiotic approach is combined to the Realistic Mathematics Education framework. These theoretical frames guide the development of the HLT through the different phases of the research. In the first cycle, the design phase involves the reflection on what students should learn about the targeted mathematical topics and how this should be done. This first identification leads to the formulation of temporary mathematical learning goals which initiates the design of mathematical problems, identification of learning processes and teacher management of these processes. Informed by concrete mathematical tasks, the HLT is progressively shaped during this phase. An elaborated HLT contains: "[...] *mathematical learning goals, students' starting points with information on relevant pre-knowledge, mathematical problems and assumptions about students' potential learning processes and about how the teacher could support these processes.*" (Bakker & Van Eerde, 2015, p. 446). In the teaching experiment phase, the HLT is used by both teachers and researchers and could be adjusted

or changed on the fly (i.e. during the lessons linked to one teaching experiment phase) sometimes by using additional theoretical considerations. In the retrospective analysis phase, the evolving HLTs are the researcher's guideline to generate conjectures about students' learning and thus to create the teaching intervention. Reciprocally, the teaching intervention is progressively enriched by the evolving HLTs. The DBR methodology involves several instruments and techniques that connect theoretical concerns and concrete experiences in the form of testable hypothesis (potential HLT). For instance, task oriented analysis and longitudinal cyclic approach are useful to test and refine the HLT in the retrospective phase. Researchers who are familiar with TDS can quickly see similarities between Didactical Engineering (DE) as the research methodology of TDS (Artigue, 2000; Brousseau, 2008) and DBR methodology. The specialists of ATD may possibly connect DBR methodology with the theoretical construct of Study and Research Path (SRP) (Chevallard, 2015; Winsløw Barquero, De Vleeschouwer & Hardy 2014). Nevertheless, designing teaching experiments drawing on DE or SRP, specifically in the case of university mathematics education (UME), remains a challenging issue that requires further investigation. We claim that DBR can contribute to this investigation.

TDS and ATD from the DBR perspective

Variables of the networking

We draw here on the methodological components of DBR to generate appropriate variables for comparing and contrasting TDS and ATD. The selection of these variables is conducted with respect to the strategies that have been discussed and adopted by the researchers in the field of networking theories (e.g., Kidron et al., 2018). For the sake of clarity, we find helpful to firstly reorganize the whole DBR methodology through three stages according to the developmental process of HLTs: 1) the upstream stage; 2) the ongoing stage; 3) the downstream stage. In the following description of each stage, the role of both teachers and researchers and their potential collaboration, if any, are carefully stated. The instruments and techniques (questionnaires, interview, methods of analysis, etc.) used are a main issue that we cannot discuss here. The upstream stage is substantially linked to the chosen theoretical approach and leads to the formulation of the research question or problem by means of the epistemological and cognitive assumptions of this approach. At this stage, researchers need to carefully reflect on the (educational) aim of their project/intervention, by using two questions: V1: how to model the mathematics to be learnt and to be taught? V2: how to model mathematics learning and teaching processes? These questions represent the two first variables V1 and V2. The ongoing stage involves the development of series of HLTs and should be carried out by both the researchers and the teachers mainly at the experimental step. The potential variables are then: V3: the design guidelines; V4: the process of implementation. The downstream stage is related to the evolving HLTs and the improvement of the intervention, and it is supposed to be conducted by the researchers. The last two variables are: V5: the potency of the elaborated HLT and V6: transferability of the teaching intervention in other contexts.

Primary interpretations of the variables for TDS and ATD

Starting from a concrete phenomenon or a set of data, the problem of the questions formulated by theories on them, was raised from the beginning by the researchers working on networking theories (Kidron et al., 2018). In the case of our project, the research question refers to the design and implementation of a replicable teaching intervention that focuses on transition issues and enhances students' learning of Calculus at the beginning of the university. A more general question forms the basic goal of the *research program* of both TDS and ATD: 1) TDS's formulation: "*how to design, regulate and make controlled observations of experimental situations where mathematical content appear as the optimal way to address a mathematical problem?*" (González-Martin, Bloch, Durand-Guerrier & Maschietto, 2014, p. 120- slightly adapted); 2) ATD's formulation: "*how to design a didactic organization that places central questions at the starting point of mathematical activity, making mathematical content appear as models constructed to provide answers to these questions? And what is the ecology of these didactic organizations?*" (Winsløw et al., 2014, p. 106). In the case of our study, the refinement of our research question depends primarily and mostly on the values of each of the following variables.

V1. The ATD models mathematics through the construct of Epistemological Reference Model (ERM) which is formulated by taking into account all the institutions involved in the process of didactic transposition (i.e. the adaptation of mathematical scholarly objects to mathematical objects to be taught and learnt). The construct of Fundamental Situation (FS) models mathematics in the TDS approach. Its formulation takes into account the sources of the meanings of mathematical objects and defines the conditions for saving these meanings when they are concerned by didactic transposition.

V2. In TDS, the process of learning and teaching mathematics is modeled under the central construct of Situation: the system of relationships between the students, the teacher and a mathematical milieu (Ghedamsi, & Lecorre, 2018). Situation is defined by means of two levels: didactic (situation of institutionalization) and a-didactic (situation of action, situation of formulation, and situation of validation). In the didactic level, the use by the teacher of knowledge developed by the students in a-didactic level improves learning and its link to the teaching goal. In the case of ATD, this process is modeled into didactical organization (a set of didactical praxeologies modeling teaching and learning activities) which is strongly connected to the mathematical organization (a set of mathematical praxeologies) that this didactical organization aims to implement. The different didactical organizations are defined by means of the notion of moment of study: exploratory moment, technical moment and technological moment (Bosch & Gascón, 2001).

V3. In the case of TDS, the design guidelines are divided into two steps. In the preliminary step, the researchers conduct an epistemological, cognitive and didactical analysis in order to identify specific characteristics of the targeted mathematical objects, the complexity of the potential cognitive process for students and the actual teaching environment. These analyses aimed to identify the didactical variables (namely the parameters that influence students' learning of the targeted mathematical object(s)) that structure the whole design. In the second step, the construct of the milieu which structures the interactions between students, the teacher and the mathematical

milieu in the a-didactic level is used to test several values of these variables. The researchers deploying ATD analyze all the steps in the didactic transposition process linked to the targeted mathematical objects or domain of objects: their origin, their relation with other objects, their integration in mathematical praxeologies, etc. in order to elaborate the related ERM and then to establish the generating question. This question should go over the school level (see the notion of scale of levels of determination (Bosch & Gascón 2006)) and be linked to specific subjects raised by the society. The study of this question will lead to new questions that make the study open and the learning goal not determined in advance. However, in the rare case where the analysis must guarantee that students meet specific praxeologies, the learning goal should be clearly defined. Otherwise, it is the responsibility of students and teachers, during the study, to choose the trajectory which will determine the praxeologies they encounter.

V4. The implementation with ATD follows at least three criteria: a) the distribution of responsibility between teacher and students is continuously renegotiated, b) the access to intermediate answers is mediated by media (books, journals, TV, internet, etc.) and validated with regards to the didactical milieu, c) the final answer, if any, must include a learning goal. In the case of TDS, the improvement of the interactions among students is ensured by the teacher's enrichment of the mathematical milieu. Depending on the complexity of the mathematical objects, especially at the entrance to university, the teacher can manage the values of the didactical variables without minimizing students' responsibility in generating knowledge. The experimental situation evolves according to the rules of the didactic contract: "*the implicit set of expectations that teacher and students have of each other regarding mathematical knowledge and regarding the distribution of responsibilities during the teaching and learning processes.*" (González-Martin et al., 2014). The teacher should support the change in the didactic contract by strengthening the link between the outcomes of the a-didactic level and the institutionalization level. Due to space constraints, we will not say more about the remaining variables. Figure 1 synthesizes the values of these variables for TDS and ATD.

	TDS	ATD
Global Vision	Socio-constructivist vision of learning and teaching mathematics.	Socio-cultural vision of learning and teaching mathematics.
V1	Fundamental situation of a mathematical object: Mathematical game (or problem). The problem should satisfy the same criteria as those developed in mathematical games theory. The object targeted by the problem should not be explicitly referenced and must provide an optimal method for solving it.	Epistemological Reference Model (ERM) related to mathematical object or domain of objects: Organization of local and regional mathematical praxeologies through sequences of connected praxeologies. Praxeologies (composed of two interrelated blocks: practical and theoretical) model human activity (including mathematical).
V2	Situation at two levels didactic and a-didactic	Didactical organization and moments of study
	Preliminary design	

V3	Researchers: Identification of didactical variables to determine global and local organizations of the design.	Researchers: Identification of a question with generating power for praxeologies and with significance in the context of students' life.
	A priori design	
	Researchers/Teachers: set the variables at certain values to achieve the learning goal.	Researchers/Teachers: the question is rarely accompanied by a learning goal.
V4	Teachers: a) more active role of teachers at the university (manage the values of didactical variables that are already decided), b) gradual change of didactic contract.	Teachers: a) distribution of responsibilities between teacher and students, b) dialectic media/milieu (Chevallard, 2008), c) final answer, if any, with a learning goal.
V5	Researchers: Testing the learning goal and the validity of the theoretical assumptions of TDS through the actual teaching and learning process	Researchers: Testing the actual teaching and learning process.
V6	Reproducibility and paradigmatic didactic contracts	Ecological issues (institutional conditions and constraints; scale of levels of determination)

Figure 1: TDS and ATD from the DBR perspective

This comparison enables the discussion on the potential of each theory by means of DBR methodology. In our case, this discussion will be developed through the project of elaborating a teaching intervention for real numbers and sequences at the entrance to the university. Among the DE studies on the transition towards university Calculus, Ghedamsi (2008) (see also González-Martin et al., 2014) discussed ways of designing a milieu that helps students deepening their understandings of the interrelated properties of real numbers and sequences at the entrance to the university. The epistemological hypothesis of the design is formulated as follows: numerical approximation methods connect the practical and theoretical existence of the mathematical objects stated in the interrelated properties of real numbers and sequences (see first section). These properties become the arguments to validate the used methods. The elaboration and the experimentation of the design have been undertaken through TDS constructs where the epistemological hypothesis has been cognitively formulated so that it can be verified or falsified. In the last section, we use the choices made in this DE as a filter to start a brief discussion on how to elaborate the HLT of the present research.

First steps towards the elaboration of the HLT

This paper presents a theory networking study which is a preliminary step required to design the HLT. The succinct comparison above highlights the necessity to keep the coherence of the whole theoretical foundations when engaging in an intervention project. We retain three fundamental results from this study: 1) the two theories complement each other to address transition issues; 2) the implementation of the design in the context of university mathematics needs the planning of an ad-hoc didactical milieu; 3) the study of the replicability of the intervention necessitates the identification of paradigmatic university expectations about learning and teaching mathematics. We

will not develop more the conclusion of this study; we have chosen to end this paper by presenting the first step towards the elaboration of the HLT. The process of modeling the mathematical contents related to the interconnected properties of real numbers and sequences is a fundamental step towards the elaboration of the HLT. The epistemological aspect constitutes the starting point for engaging the elaboration of the design for both theories i.e.: 1) the design of a collection of mathematical problem(s) where the interconnected properties of real numbers and sequences appear as providing the optimal solution, according to TDS; 2) the design of central questions where the interrelated properties of real numbers and sequences appear as models (mathematical praxeologies) constructed to provide answers to these questions, from the ATD point of view. As claimed by TDS, the construction of mathematical problems, that create the need for these properties, requires firstly investigating the mathematical meaning of their relationship and its fundamental significance. This investigation should be done by using fundamental mathematical sources (i.e. the several theorizations of the set of real numbers) and by deepening insights on their historical growth. As researchers involved in TDS approach know, designing a collection of problems that connects rationally these properties remains a real challenge. The main reason seems to be that the didactical variables that may emerge must preserve the meaning of the relationship between these properties at the moment of their transformation into mathematics to be learned. For instance, the DE experimented by Ghedamsi (2008) shows that the mathematical organization related to the interconnected proprieties built upon the expectations of the curriculum (i.e. start with the self-evidence of the continuity principle) does not fit cognitive requirements. The fundamental reason was that during the teaching experiments, the students employed spontaneously the nested intervals theorem which is closely linked to the adjacent sequences theorem. However, the nested intervals theorem, in conjunction with the Archimedean property, generates all the targeted properties and leads to another foundation for real numbers theory. So, this theorem deserves more attention in the elaboration of the HLT. ATD materializes a mathematical organization by a set of connected mathematical praxeologies. In the case of the properties of real numbers and sequences, each set of praxeologies, including the one proposed by the official curriculum, is grounded on a specific theory of real numbers. Thus, the theoretical components of each set achieve a fundamental significance (in the terms of TDS) of the relationship between these properties. In the case of our project, the mathematical problems of the HLT must include in particular the theoretical aspects of the mathematics involved. At least two questions may guide the elaboration of these problems: what are the mathematical meanings produced by each set of praxeologies? How should several sets of praxeologies be combined to support learning? These comments provide some information to start thinking about the development of the HLT but to go further supplementary analysis is needed, in particular for formulating the learning goals.

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