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Two situations for working key properties of R

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Introduction

Teaching and learning real numbers is not an easy topic, and it is probably for this reason that most secondary school curricula do not develop it, even if they are used. Several studies have been conducted on this subject, but often focusing on the double decimal representation of decimal numbers or on irrationality (Oktaç & Vivier, 2016) made a recension of researches on these theme). More recently, the emphasis has been placed on the completeness of **R**, for the topology of the order, as one of the key characteristic properties to be achieved (e.g. Bergé, 2010; Durand-Guerrier, 2016). It has been hypothesized that these properties of **R** are important for understanding real analysis. We are in line with this hypothesis with a positioning in favour of an algebraic approach, but how to develop the simultaneous study of algebraic and topological properties of **R**? In particular, we have built situations that may introduce the completeness of **R**, on the basis of the algebraic properties of **R**. In the two preliminary studies we present here, we analysed the students' mathematical work using the Mathematical Working Space theory – MWS- (Kuzniak, Tanguay, & Elia, 2016).

Calculations with real numbers

This situation presents the basic algebraic operations in relation to the completeness of **R**. In fact, the definitions of the sum and the product in **R** are based on the completeness of **R**. They are rarely worked at the secondary level, as if these definitions were trivial.

We asked the students to find the roots x_1, x_2, x_3 of the polynomial $P(x) = x^3-60x^2+980x-4700$ using adjacent sequences of decimal numbers. At this educational level, it is only possible to calculate approximate values of these roots since exact values need more advanced mathematics (unless using an algebra calculator). We, then, asked them to calculate the results of the basic operations on the three real roots, such as $x_1+x_2+x_3$, that is actually 980 with the root properties of a polynomial. The visualization process is based on increasingly precise approximations (of the adjacent sequences) defining *small* nested intervals, and the signs (such as the tables of values), are used to support these processes. The main goal was to encourage work of visualization to conjecture the limits of each adjacent sequence, to recognize the polynomial coefficients.

This situation has been proposed to 6 grade 12 students by pairs, in France in 2018. The data are made on the student's dialogues and their written productions. Finally, the visualization of the limits did not take place, but this did not prevent elements of completeness from appearing spontaneously, as for the convergence and the intersection of the nested intervals, or from students confronting the epistemological obstacle of the potential infinite and the actual infinite.

Functional equation of the real exponential

The fact that it is hard to distinguish the density and completeness of **R** by graphical representation (especially in technological tools). In this sense, we propose to construct algebraically a function that allows a visualization of the discrepancy between density and completeness. This situation is attractive because it allows using and connecting of a wide range of academic knowledge.

The mathematical task consists in searching for the functions f defined on $\boldsymbol{E}$, where $\boldsymbol{E}$ is the usual numbers sets **N**, **Z**, **D** and/or **Q** and **R**, with values in **R**, which satisfy the conditions: (a) $f(1) = 2$ and (b) $\forall x\ \forall y$ on **E** $f(x+y)=f(x)\cdot f(y)$. The first question proved that $f > 0$ and that $f(0)=1$. Then, there is a unique function (the exponential) in **N**, **Z**, **D** and **Q** but this is not the case in **R**. More specifically, we ask for certain values and graphical representations. The functional equation is used as an artifact by calculating the values of f algebraically, and a graph is drawn for each set.

A first study in Chile with 6 students by pairs was conducted in 2017. The data are made of written students' answers and direct observation. For sets **N** and **Z**, there were some technical problems in using (b) to compute values and some students drew a continuous line but quickly realize that the context is discreet. For **D** and **Q**, the discussion is based on the line drawn and the density: do we represent a continuous line? Finally, a significant obstacle arose for **R** because of the need of topological arguments. We think that asking more explicitly to build a function f on **Q**.1+**Q**.√3 (additive subgroup of **R** - 1.a+√3.b; a\in **Q**, b\in **Q**) with $f(\sqrt{3})=1$ could help, and so do the use of a technological software like Geogebra.

Conclusion and discussion

We presented two preliminary studies with these situations that have a great potential. Nevertheless, it seems that the introduction of an algebraic work allows to clearly show the topological jump made when "passing" from **Q (**or **D)** to **R**. Moreover, **R** cannot be reduced to its sole property of completeness: the uniqueness of **R** is based on several other properties, like its algebraic structure.

MWS theory has allowed us to design these situations and interpret the results obtained. Thus, the three dimensions of MWS has been activated: the work is done on the visualized signs (semiotic dimension), on the mathematical knowledge itself (discursive dimension) and on the technological tools to produce signs (instrumental dimension).

REFERENCES

Bergé, A. (2010). Students' perceptions of the completeness property of the set of real numbers, *International Journal of Mathematical Education in Science and Technology*, *41*(2), 217–227.

Durand-Guerrier, V. (2016). Conceptualization of the Continuum, an Educational Challenge for Undergraduate Students. *International Journal on Undergraduate Mathematics Education*, *2*, 338-361.

Kuzniak, A., Tanguay, D., & Elia, I. (2016). Mathematical working spaces in schooling: An introduction. *ZDM*, *48*(6), 721-737.

Oktaç, A. & Vivier, L. (2016). Conversion, change, transition… In B.R. Hodgson, A. Kuzniak, & J.-B. Lagrange (Eds.), *The Didactics of Mathematics: Approaches and Issues. A Hommage to Michèle Artigue*, (pp. 87-122). Switzerland: Springer.