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# Evaluation of a connecting teaching format in teacher education

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## Motivation and teaching format

More than 100 years ago Felix Klein described the problem of the double discontinuity and its consequences. With his “three worlds of mathematics” model, David Tall (2008) saw the problem in the existence of different worlds in school and university, namely a conceptual-embodied respectively proceptual-symbolic world at school and an axiomatic-formal world in university. We wanted to counteract the problem of the double discontinuity by connecting the different worlds of mathematics. Therefore we designed a teaching format that should illustrate the interdependencies of mathematics taught at school and mathematics taught at university in the field of geometry. That these connections do not emerge automatically was already stated by Bauer and Partheil (2008). To aid the learning process of the students our teaching format was supported by a blended learning course and mathematical maps. Mathematical maps are a didactical tool to show interrelations between topics and development of a subject matter in time (Brandl, 2008; Schwarz et al., 2017).

## Theoretical framework and empirical study

To measure the effect of our teaching format, we used the theoretical framework of concept image by Tall and Vinner (1981). The term concept image describes “the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes” (Tall & Vinner, 1981, p. 2). It is built up over time by engaging with the concept and changing as new mental connections are made or older connections are exposed as imperfect. So for us it is interesting how the concept image of the students changed after attending the teaching format and if more mental connections between aspects of the concept taught in school and aspects taught in university are observable. We focused on the concepts of line, circle, congruence and sum of interior angles for this evaluation because they are broadly discussed in school and university.

## Data collection and analysis

For our exploratory pre-post-survey we used a guided interview because it allowed both students who attended the full teaching format to answer freely on the question what they connect with a certain concept and then we were able to specifically ask questions on aspects that were not addressed. In all interviews the questions were the same to provide comparability. For each of the four concepts the theoretical notion of concept image was operationalized into different subcategories to which the statements of the students were assigned. We then examined the relations between aspects taught in school and aspects taught in university in these subcategories.

## Results and discussion

We want to show changes in the concept image exemplarily. For both students, we call them S1 and S2, the summed up concept image of a line in the first interview was very close to school matters.

Lines were only seen as a linear function,  $y = m \cdot x + t$  (S1) and as a slanting dash upwards (S2). The line equation, the slope of a line, parallelism of lines and the number of points of intersection were correctly explained by both students. In the second interview lines were seen as an infinite set of points (S1 & S2) that satisfies the line equation,  $y = m \cdot x + b$  (S2) and also from an analytical perspective as reference point and direction vector (S1 & S2). S1 even gave the axiomatic definition of a line as a set that contains at least two different points and that there is exactly one line for any two points. Contentwise you see a change in the concept image of a line. The conception of a line changed to include different aspects, a set theoretical approach of a line as an infinite set of points, which is mainly used in university and an algebraic and analytical approach which is used in school.

In the first interview S1 understood congruence as being the same shape and size, which is the graphic description from school. She also made a connection to the criteria for congruence of triangles which are taught in school. In the second interview S1 added the following excerpt.

Student 1: [Congruence means that] you can map it onto each other with a motion. And a motion is achievable with a maximum of three reflections [...].

This shows that S1 was able to connect the previously very school based knowledge with formal mathematics. The answer to the question which functions are congruence functions reinforces this.

Student 1: Well, so reflections and rotations definitely are congruence functions. Translations too, but dilation and contraction are no congruence functions.

Overall these results suggest that our teaching format positively influenced the concept image of different terms of geometry. Aspects from school were not replaced by but extended with aspects from university. Especially with the abstract concepts of congruence and sum of interior angles more mental connections between aspects of the concepts taught in school and aspects taught in university can be observed. This suggests that the students were able to establish interrelations between what they learned – and will be teaching later – at school and what is taught at university.

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