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External Didactic Transposition in Undergraduate Mathematics

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Undergraduate mathematics programmes in Europe are typically delivered in hierarchical structures of modules whose contents are in general assumed to be quite similar across universities. These programmes and the mechanisms through which they are maintained, revised and developed, have so far not been the subject of systematic research, as most research (including intervention-based research) takes the programme for granted. This paper furnishes a theoretical and methodological framework for undertaking didactic research in this area and provides some results from a first study in four universities in Denmark, France, Germany, and Spain.

Keywords: undergraduate mathematics, external didactic transposition, curricula.

Introduction

As one of the main challenges for future research on university mathematics education (UME), Artigue (2016, p. 22) noted the need to “maintain some connection between the living field of mathematics [...] and undergraduate mathematics education”. She relates this to the fact, already noted by Kahane (cited in Bass, 2005, p. 417), that “in no other discipline is the distance between the taught and the new so large”. It is an illusion that the contents of university mathematics teaching flow directly from the source of research, being continuously updated with the advances of mathematics.

Most research studies consider teaching and learning processes related to a given mathematical piece of knowledge that takes place within one or a few curriculum subjects, domains or modules. These modules seem to function as a context of the study that is not in itself submitted to research—neither at the level of the entire study programmes nor at the level of the modules and their syllabi. In some studies, this context is found to offer constraints or even obstacles to student learning and teaching innovation of the piece of knowledge considered, but the module itself—its structure and place in the curriculum—is rarely put under question. The need for research to go beyond this relationship to curricula was emphasized in the conclusion of the chapter on university mathematics education of the recent volume celebrating the 20th anniversary of ERME:

We should mention also the potential, but currently quite limited, impact [of research on UME] at the level of innovation of curricula and policy. Some of the more global problems identified by [research on UME] [...] clearly call for research and impact at this level (Winsløw, Gueudet, Hochmuth, & Nardi, 2018, p. 71).

Creating a research-based approach to the development of undergraduate mathematics programmes appears as an important challenge. Bosch and Winsløw (to appear) have started to address this

question from the perspective of university didactic transposition, using some research available on the evolution of university mathematics programmes (e.g., Huntington, 2015; Tucker, 2013). In this line of research, this paper presents the first results from a study of the structure and evolution of undergraduate mathematics curricula at selected universities in four European countries. Its main aim is to outline some theoretical and methodological directions for this line of research.

Theoretical framework and research questions

We problematize the elaboration and evolution of university curricula by using the framework of the didactic transposition (Chevallard, 1991; Chevallard & Bosch, 2014). We use this framework as a tool to investigate the precise organisation and structure of the *knowledge to be taught*. Study processes—at all levels—are defined in terms of learning goals to be reached by a given group of people under certain conditions. In the case of European university degrees, these goals are defined in terms of general competencies, materialised in *study programmes*, which are first concretised in a set of *modules* (or subjects). Modules receive a name that can correspond to a sector or domain of a discipline (Algebra, Analysis, Geometry) or to a specific selection of contents (Perspectives in Mathematics, Mathematical language and reasoning, Mathematical modelling). At a second level of concretion, modules are described through a *syllabus* where the learning goals are more precisely defined in terms of subject-based contents and often also skills, learning outcomes, textbooks, prerequisites, etc. Taking the word “knowledge” in a broad sense—to include formal knowledge but also informal practices, skills and competencies, attitudes, etc.—we can say that study programmes, modules and syllabi are the way used by universities to define the knowledge to be taught.

The notion of didactic transposition points to the fact that teaching and learning processes do not begin when teachers and students meet in a classroom. The starting point is the selection and elaboration of the *knowledge to be taught* from what is called the *scholarly knowledge*, which gives legitimacy to the knowledge to be taught. Three institutions intervene in this process: the scholarly institution of knowledge producers and users (the “experts” or scholars); the school institution where the scholarly knowledge has to be transposed (which in our case corresponds to the university); and the *noosphere* which is defined in the neighbourhood of both and includes all those who make decisions about the teaching processes at play. In the case of UME, these three institutions have a large intersection. However, even if they share many of their subjects, we should differentiate the *positions* occupied and the roles assumed by their subjects. Therefore, a mathematics researcher can act as a subject of the scholarly institution—a scholar—when she produces mathematics and acts as an expert of the discipline. She can also act as a subject of the noosphere when she makes decisions about the study programme of a given degree. And she is also the subject of the school institution—this or that university or faculty—when she teaches a module.

The process of selecting, adapting, organizing and declaring the “knowledge to be taught”—beginning from scholarly knowledge and ending with all the teaching materials that can be proposed in a given course—is called *external didactic transposition* (EDT), while the subsequent step towards the “actually taught knowledge” is called *internal didactic transposition* (Chevallard, 1991, p. 35). One can interpret almost all existing research on UME as primarily concerned with internal didactic transpositions, since (as noticed above) the programmes and syllabi are merely

considered a context of teaching and learning processes. When putting the EDT at the core of the study, we consider the following research questions: What are the main processes and rationales in the EDT processes leading to UME study programmes and modules? What are the institutional forces that influence them (from the scholarly institution, the noosphere, and the school)? What common tendencies can be found in European universities, regarding these questions?

Methodology

This paper only presents a first attempt to provide some pieces of answer to these research questions throughout an exploratory study mainly based on four one-hour interviews with experienced undergraduate mathematics teachers from Denmark (DK), France (FR), Germany (GE) and Spain (ES). Teachers were selected for their experience with recent curriculum drafting. They are all professors in mathematics departments and researchers in different areas of pure mathematics: topology, number theory, algebra, combinatorics, and functional analysis. The main selection criterion was our facility to approach them and their willingness to participate in the study. The bachelor programmes of mathematics of the four universities were analysed before preparing the interviews and used as the empirical basis of our interaction with the interviewees. They are not necessarily representative of other programmes in the same country.

The interview guide contains four sections. The first section asks a rough description of one of the latest undergraduate modules the interviewee had taught. The second section is about the history of this module, the genesis of its syllabus and possible conditions for changing it. The third section includes questions about the whole study program, compared to past programs and to programs of other universities. Finally, the fourth section refers to processes involved in the elaboration of the program, obstacles for changing it, and factors of influence within and outside the department. To support the discussion, the informants were given a copy of the course structure diagram of their faculty, showing modules with titles, ECTS credits and time organisation. In particular, the informants drew arrows on this diagram to show the dependency between modules and referred to it while commenting on the design of the study plan as a whole. Both the design of the interview guide and the choice of informants reflect our assumption that EDT in this context is closely related to the internal didactic transposition, including the daily teaching experience.

The interviews were audio-recorded and afterwards transcribed, and translated into English if not conducted in this language (one of the four was not). After a first reading of the interviews, we identified four themes, which were also to some extent built into the interview guide. Given our research questions, the themes focus on the product of EDT (themes 1 and 2) and its functioning as a dynamic process (themes 3 and 4). We scanned the interviews for utterances about these themes and then identified emerging, shared viewpoints on the EDT. These are presented in the next sections. When quoting informants, we refer to their country (DK, ES, FR, GE).

Theme 1: Form and contents of study programmes

We first note some initial observations mainly based on the documentation of the study programmes. We focused on bachelor programmes in (pure) mathematics. All four universities also offer master programmes in this area. Therefore, the bachelor programmes may not qualify students for a job in the “real world” but could mainly provide a basis for master studies, which in all four

cases may lead to a job as a secondary school teacher, among other options. All programmes offer a common core of courses on analysis, linear algebra, algebra, numerical methods, and stochastics. The programmes also include some auxiliary subjects such as programming or informatics (all), physics (France and Spain), or a secondary “minor” subject (Germany), but the interviews show that the core mathematics subjects do not rely on such auxiliary modules. Even the use of digital tools such as CAS for analysis courses is mostly optional. A quite substantial difference among the programmes is that in the Spanish and the French ones, almost all modules in the three years are fixed and mandatory, while the Danish and German programmes reserve at least 40 ECTS for elective courses. The elective courses cover many fields, some of which are mandatory in the other two programmes: geometry, topology, functional analysis, advanced algebra, stochastics or statistics, etc. The size of individual modules is also quite different: In the Spanish case all modules are 6 ECTS, the French programme contains courses from 2.5 up to 7.5 ECTS and the Danish and German programmes mostly have 10 ECTS-modules. All larger modules include lectures, and usually also tutorials, labs or problem sessions.

There seems to be a tendency to keep a core of courses that cover the same mathematical domains, with a strong consensus on these core domains:

This program gives a solid foundation for most of the mathematical directions that you would, we want to cover here. I mean you do a solid background in analysis and in algebra... (DK)

On the other hand, details, and emphases of the study programme also depend on local conditions like staff composition, established research units and so on:

If people from the staff are more, say, predominantly from applied mathematics, they tend to shift pure mathematics from the syllabus. And the reverse is true too. (ES)

In spite of differences between the programmes, the interviewees claim that the structures of modules are hard to change (DK) and that there is not very much to play with (ES).

Another common facet relates to the basic idea underlying the overall structure of the programmes and in particular the sequencing of the modules: they are organized according to the logical dependency of notions, definitions, and theorems (mathematics as a product). In the interviews, there was only the case of a discrete mathematics course where the content seems, at least partly, to be organized around problems:

[We] rather start from a concrete problem, a problem that they know, the Fibonacci numbers, and look how the method is developed in that problem you see, how to do, how to get a functional equation for the generating function and then use partial fractions. (GE)

Theme 2: Form and usage of module syllabi

The first part of the interviews centres on a recent module taught by the informant. This provides further information about one particular syllabus, and the ways it functions to structure and direct the actual teaching. Considering all four interviews, we can identify some common points, as hypotheses about how undergraduate mathematics syllabi are elaborated and used more generally. The first common point is that the syllabus consists of a short list of mathematical concepts or results that form the core of the module, while still leaving some initiative to the teacher:

There is still some flexibility, particularly how far one would go with certain topics. (GE)

Not surprisingly, more choice is left to the lecturer in the case of modules that are more “free-standing”, such as a course on order structure and social choice theory. In courses within a longer sequence, such as the first analysis course, the constraints are stronger:

The curriculum was sort of written beforehand and I basically had to stick to it [...] You do have some influence on the weight you would put on certain topics, and I think I spent more time and focus on continuous functions than was perhaps originally intended. (DK)

When the informants describe one of the undergraduate courses they have taught recently, they almost exclusively refer to their lectures, including the particular choices they make in terms of emphases, what to cover, etc. When asked, some of the informants also mention the exercises, often taught by assistants in different sessions, but without giving them crucial importance in the structure of the course. Tutorials are strongly related to the lectures, as the relevant theory should always be treated before being applied in exercises. The exercises are also related to the final exam in the module:

[Tutorials teachers] teach on the blackboard, have 100 students in front of them and they solve the exercises on the blackboard. [...] the list of exercises is a list that comes from father to son. It's the same list that has been there for the past 10 years. [...] they are trained in the classroom with the same kind of exercises that will be at the exam. [...] the key for 60 or 70% of the students to pass is to do an exam that is not essentially different from previous ones. (ES)

All informants agree that their own activity as researchers does not directly influence their activity and choices as lecturers, because the material of the undergraduate course is very elementary:

It's so basic, all of it. [...] I mean, I think most people would agree these are the kind of things that you would include in a course like that, so I am not sure that my particular line of research has played a significant role. (DK)

The interviewees all agreed that syllabi can be revised in reaction to experiences while taking into account the function of the module as a prerequisite for other ones. However, most changes described in the interviews seem to respond to more global changes of an administrative character, such as the passage from quarters to semesters. In two of the universities, changes were also due to the passage from a structure where some basic courses, including calculus, were shared with other study programmes, to a programme where all courses are tailored exclusively to mathematics students. The contents are mainly justified by such a collegial consensus, the needs in other courses and the preparation for the master level, and sometimes also students' prerequisites. Two informants (GE and DK) also made passing references to the perceived needs of future teachers, but otherwise, the contents are not related to needs or factors outside of the programme. All the informants express, in various ways, that the contents of the core modules must be kept:

Changes on the content are really dangerous because either you omit things which are important for other modules or [*you add something new in relation to previous years, with the result that*] the exam, it's going to be a full disaster. (ES)

Theme 3: Process of design of study programmes and modules

We seek to identify the main constraints of the EDT as a process conducted inside the mathematics department, seen as an institution within the university. External constraints may originate from other disciplines within the university (interrelations of programmes), the university as a whole (coherence and economy of study programmes), the ministry of education (accreditation), the European Union (the Bachelor-Master-Doctorate (BMD) structure or other aspects of the Bologna process), or the socio-economic sphere (needs for qualified labour force in specific domains).

The collected interview data gave an overall picture of EDT as a rather informal process:

There may be two or three people who go teaching this course, and I think we will (for the new accreditation) just sit together again and see what everyone agrees to. (GE)

The module descriptions are thus done by teachers who volunteer, motivated by their involvement in the teaching. The formal constraints imposed by national authorities seem weak and concern only the formal structure of the study programme:

Before the BMD reform, the size of the modules and their names were given by law. The legislation regulated the number of hours of integration theory, differential calculus, and so on, and checked on the syllabus that the study programme fulfilled the expectations of the degree. With the BMD, universities are totally autonomous. (FR)

The autonomy of institutions is indeed furthered by the Bologna Accord. In some cases, this is used as an opportunity to create more independent and pure mathematics programmes. External constraints due to other fields are limited to the few cases when modules are shared with them:

In this version of the programmes, great autonomy was given to the departments, with little constraints on the potential navigation of students between different study programmes. This choice was made by the ruling committee of the faculty of sciences and the consequence that I can see is that it leads to a disciplinary retreat. (FR)

Constraints due to the socio-economic environment were also mentioned: “Here, you have a lot of insurance companies and banks, so we lose some students in this direction” (GE). However, its impact on the EDT seems weak: “In the curriculum, you don’t see it” (GE). Again, students’ mobility was also pointed out, to justify that mathematics programmes need to be fairly similar.

Because of the overall weakness of perceived external constraints, the interviewees came to identify students as the main constraint that EDT meets as a process:

The real constraints are the students. How to build a study programme that is suitable to open to advanced studies in mathematics but also matches the level of students that we find in front of us? This is very complicated. (FR)

Theme 4: Evolution of programmes and modules

When asked about major evolutions of the study programmes over the last decades, two main changes were mentioned: the tendency to structure programmes in a higher number of (smaller) modules, and the incorporation of new modules related to computer science, programming and

discrete mathematics. All interviewees agreed that the strong compartmentalization of knowledge leads students to specific study strategies, resulting in a loss of connections among subjects:

So, it's not so much the curriculum, but what matters is how people learn. I take my exam after the end of this course right away and then I can forget about it and come to the next thing. So this was different in the diploma system. You had the oral exams after two years and you had a big exam at the end after four or five years. This makes a big difference. (GE)

Another change that can be observed is the incorporation of what we can call “propaedeutic” courses in the first semester of some universities: “Mathematical language and reasoning” (ES), “Perspectives in Mathematics” (DK). In the French case, a more unified start has been achieved by merging the modules of analysis and algebra in the first semester:

In the first years, you need to give substance to a set of mathematics, to have more possibilities of interaction between the different elements of the subject to facilitate the links. (FR)

As for the modular structure, the fragmentation seems to be reinforced by the repartition of subjects among groups of researchers:

[What introduces changes in the curriculum is] the sociological evolution of the laboratory, that is, the number of colleagues that are in this or that team, their thematic orientations. (FR)

The tendency towards disciplinary “purism” also means that subjects which are not linked to research groups in the department tend to be reduced or even disappear.

There appears to be strong stability in the core-curriculum shared by the four programmes, and we see no signs of substantial disagreement about the overall profile of the programmes.

Conclusion and outlook

Our exploratory study considers the EDT, its processes and rationales, institutional forces and common tendencies, mainly through the lens of experiences reported by lecturers and their reflections on internal didactic transposition processes—how it is framed, determined by obstacles and goals. Our main results (in fact, hypotheses of general trends) are the following:

- Although the programme structures of the universities look quite different, they have a common core of courses (analysis, algebra, probability, and statistics, etc.), reflecting a broad consensus among mathematicians at large. The interviewees describe and justify relations between the modules in terms of inner-mathematical (logical or theoretical) dependencies.
- A module is a lecture-based course and it is described essentially through a short list of topics, especially notions and theorems. This list outlines what the lectures should cover, leaving room for variations, but with very little reference to what the students are asked to do (types of problems, exams, etc.).
- The process of curriculum design is very informal, with no systematic and explicit organisational principles, thus relying on a set of shared assumptions and on the dependence between faculty and teaching needs. Moreover, the interviewees experience weak external constraints (influence of the socio-economic environment, students' mobility) together with a high autonomy to make changes, which seem to reinforce disciplinary purism.

- An increased modularisation is seen in the last 20 years with a trend towards early specialisation in mathematics. When changes are operated in a study programme, the core of the programme is preserved but some auxiliary modules may be added or replaced, to support students' learning of generic skills (e.g. mathematical reasoning), to introduce new domains (graphs, modelling, etc.) or provide new tools (computer programming).

At this stage of the research, we are far from being able to propose or support initiatives towards a more systematic and knowledge-based EDT—that is, to produce curricula and syllabi that optimize mathematics programmes in relation to institutional contexts and specific sets of constraints. One major motivation is that, if we want to pursue specific agendas—such as broader and more efficient preparation of secondary school teachers, new paradigms of “inquiry-based” teaching, and so on—, we need to understand the processes of EDT that determine the set of conditions and constraints under which such agendas are pursued.

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References

- Artigue, M. (2016). Mathematics Education Research at University Level: Achievements and Challenges. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), *Proceedings of the First INDRUM Conference* (pp. 11–27). Montpellier, France: University of Montpellier and INDRUM.
- Bass, H. (2005). Mathematics, mathematicians and mathematics education. *Bulletin of the American Mathematical Society*, 42(4), 417-430.
- Bosch, M., & Winsløw, C. (to appear). The external didactic transposition of mathematics at university level: dilemmas and challenges for research. *Educação Matemática Pesquisa*.
- Chevallard, Y. (1991). *La Transposition didactique: du savoir savant au savoir enseigné* (2nd edition). Grenoble, France: La Pensée sauvage.
- Chevallard Y. (2015) Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm. In J. S. Cho (Ed.). *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 173-187). Switzerland: Springer.
- Chevallard, Y., & Bosch, M. (2014). Didactic Transposition in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 170-174). Netherlands: Springer.
- Huntington, H. (2015). *A historical analysis of the mathematics major requirements at six colleges in the United States from 1905 to 2005*. Ph.D.-thesis, University of Columbia.
- Tucker, A. (2013). The History of the Undergraduate Program in Mathematics in the United States. *The American Mathematical Monthly*, 120(8), 689-705.
- Winsløw, C., Gueudet, G., Hochmut, R., & Nardi, E. (2018) Research on University Mathematics Education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds.), *Developing research in mathematics education - Twenty years of communication, cooperation and collaboration in Europe* (pp. 60–74). Oxon, UK: Routledge.