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Students' conceptions of the definite integral in the first year of studying science at university

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Theoretical considerations and method

To explore students' interpretation of the definite integral and to deepen their understanding of its application, I draw on the works of Tall and Vinner (1981) and Sfard (1991). Tall and Vinner (1981) consider an individual's concept image for a given concept is "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p.152). They used the expression "evoked concept image" to indicate the elements of the concept image that were uncovered by their responses. According to Sfard (1991) a mathematical concept possesses two aspects: a process aspect and an object aspect. The first one identifies the operational nature when students focus their thinking on the procedure, action and algorithms contained in the concept. The second one identifies the structural nature when students conceive the mathematical concept as if it is an abstract object. Sfard notes that an individual is said to have a pseudo-structural conception when the object conception manifested by that individual does not refer to the objects of lower levels and to the processes that led to it. For the definite integral concept, the image of the area and antiderivative conceptions are usually due to insufficient abstraction of the integral image. However, the limit of approximation is due to the reification of the summation process into the symbol $\int_a^b f(x)dx$. So I consider that students of this category possess both the process and the object aspects of the evoked concept image. It is important to emphasize that students concerned by this study may possess elements that are never uncovered by their responses. Therefore, I am exploring their involved conceptions in response to given questions. Students participating in this study attended a first-semester calculus course at Bizerte University of Science. The year before this study, they had all succeeded at the baccalaureate examination with mathematics as an option. They were 18/19 years old. Eighteen students were interviewed in January 2017 and fifteen others in January 2018. The test consisted of five questions that focused on students' conception of the definite integral. The students written responses were analyzed according to Jones' categories. In this paper, I focus on the first question of the test:

Q1: Suppose $\int_a^b f(x)dx = k$, k is a real number. Explain what k means and how it was obtained.

This question was designed to explore how students interpret definite integral when asked to describe its meaning.

Preliminary results

Based on Jones' categorization (2015), my preliminary analysis of students' responses found three categories. The first category describes the definite integral in terms of anti-derivative. This category focuses on symbolic representation of integrand function. K is evaluated by manipulating the FTC: one takes an anti-derivative F of f , and substitutes b for x and a for x and finally evaluate $\int_a^b f(x)dx$ by subtracting the values of F at the endpoints of the interval $[a, b]$. The second category describes

the definite integral $\int_a^b f(x)dx$ as an area under the curve, where f is a positive function and that $a < b$, the boundary being the x-axis and the vertical lines $x = a$ and $x = b$ as illustrated in Figure 1. Finally, the third category describes integral as an approximation process. They write this type of response “ $\int_a^b f(x)dx = k$ is the area of the region that lies under the graph of the function f which represents the limit of the sum of the areas of approximating rectangles as shown in the Figure 2 below”.

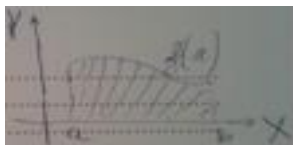


Figure 1: area conception

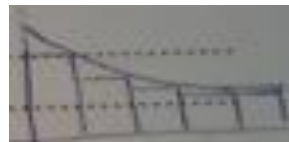


Figure 2: limit of approximation conception

Discussion

Preliminary results from the analysis show that most of the students (42%) interpret the definite integral as an area underneath the graph of the function and roughly, half of the remaining students (27%) describe it in terms of an antiderivative. The area conception involves the idea that an integral represents an area even though f is a negative function on $[a, b]$. It is based on generalization of a special case ($f \geq 0$). This consideration is not always evident, it produces unsatisfactory results in some situations where $f < 0$. Students of this category confuses between definite integral and area. They should be reminded that integral could be negative, zero or positive value. It is interesting that, although area conception leads to the procedural aspect of definite integral, this conception is it is necessary to develop a conceptual learning. The second category bases its reasoning on FTC. However, the response of students in this study is quite unsatisfactory since it conflicts with the formal definition of the definite integral. One of the students of this group thinks about a symbolic expression of the FTC. Even though it is a satisfactory response, it is not sufficient for structural understanding. It joins definite integral to antiderivative and limits it to algebraic representation. It is noteworthy that 15% students describe $\int_a^b f(x)dx$ as an approximation process. This conception enables the students to understand the meaning of definite integral, not only as a process but also as an object. Moreover, it brings out the epistemological aspects of integration. All students show that the definite integral represents an infinite process, however they focus only on positive functions. These preliminary results show that the large number of the students in this study evoked conceptualizations of integration that fail to emphasize the underlying structure of the definite integral. It seems to be a pseudo-structural thinking as described by Sfard (1991).

References

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