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► **To cite this version:**

Gulseren Karagoz Akar, Esra Bukova Güzel, Serkan Özel. Prospective Secondary Mathematics Teachers' Development of Core-Practices During Methods Courses: Affordances of Quantitative Reasoning. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02422560

**HAL Id: hal-02422560**

**<https://hal.science/hal-02422560>**

Submitted on 22 Dec 2019

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# Prospective Secondary Mathematics Teachers' Development of Core-Practices During Methods Courses: Affordances of Quantitative Reasoning

Gulseren Karagöz Akar<sup>1</sup>

Esra Bukova Güzel<sup>2</sup>

Serkan Özel<sup>3</sup>

<sup>1</sup>Boğaziçi University, Faculty of Education, Turkey; [gulserenkaragoz@yahoo.com](mailto:gulserenkaragoz@yahoo.com)

<sup>2</sup>Dokuz Eylül University, Faculty of Education, Turkey; [esra.bukova@gmail.com](mailto:esra.bukova@gmail.com)

<sup>3</sup>Boğaziçi University, Faculty of Education, Turkey; [ozels@boun.edu.tr](mailto:ozels@boun.edu.tr)

*To investigate the development of the progressive incorporation perspective for effective mathematics teaching, a multi-case study with seven out of twenty-six prospective secondary mathematics teachers was conducted for fourteen weeks during a methods course. The course was built on four-core ideas: quantitative reasoning, task-analysis, conceptual analysis and interviewing. In this paper, we report on the change in prospective teachers' conceptions of mathematics and mathematics learning upon completion of working on quantitative reasoning. Results showed that quantitative reasoning might bring about change on prospective teachers' awareness on practices such as attending to students' thinking and distinguishing their mathematics from their students' mathematics. Results suggest the inclusion of quantitative reasoning in methods courses in mathematics teacher education.*

*Keywords: quantitative reasoning, teacher education, prospective teachers*

## Introduction

Education of prospective teachers is an ongoing challenge (Chapman, 2016; Grossman & McDonald, 2008). Although there has been considering effort on the change of prospective teachers' practices in methods courses, there is still need for focusing on significant change in prospective teachers' thinking about mathematics teaching and learning (Chapman, 2016). In fact, researchers argued that the possible reasons for the difficulty of change might be due to the lack of pedagogical principles prospective teachers need to hold (Jin & Tzur, 2011; Simon, 2006b). Such principles involve developing i) a view of mathematical understanding consistent with the problem-solving view, ii) a view of mathematics learning (Simon, 2006b) and (iii) the teachers' distinction between their mathematics and their students' mathematics (Silverman & Thompson, 2008; Simon, 2006b). Silverman and Thompson (2008) also suggested that prospective teachers need to have an image of (e.g., hypothesize) how their students might come to think of mathematical understandings they envision. In this paper, we embrace these as core practices.

Previous research depicted evidence of these proposed difficulties. Particularly, Yeh and Santagata (2015) study showed that prospective teachers regarded students' procedural fluency as evidence of conceptual understanding. Results also showed that prospective teachers detected student actions as evidence of student learning rather than attending to their thinking. Similarly, Simpson and Haltiwanger (2016) study showed that prospective secondary mathematics teachers had difficulty in interpreting student mathematical thinking in written work. They suggested studying the ways to develop prospective teachers' abilities to make sense of students' thinking. It is in this respect that,

in this study we investigated the possible affordances quantitative reasoning (Thompson, 1994) might provide prospective teachers in terms of the pedagogical principles they need to hold regarding the nature of mathematics, mathematics learning and mathematics teaching. We investigated the following research question: What changes in the prospective teachers' awareness on the practices of effective mathematics teaching might develop focusing on quantitative reasoning during methods courses? Results showed a change in prospective teachers' views on the nature of mathematics. They were also able to characterize procedures and concepts in relation to quantitative reasoning. In addition, they realized the need to distinguish their thinking from students' thinking as well as developing a sense of what mathematics learning means. With this report, we propose that in methods courses prospective mathematics teachers need be given opportunities to engage in tasks focusing their attention on quantitative reasoning. We make suggestions on how to include such tasks pointing to our experience as teacher educators. In the next section we provide our reasons as to why we considered quantitative reasoning as part of the methods course we developed. We explain our reasons with connections to the teacher perspectives framework (Jin & Tzur, 2011) based on which the larger study was conducted.

## **Theoretical Framework**

In the larger study of which this study was part, our purpose was to examine the prospective secondary mathematics teachers' development of the progressive incorporation perspective to teach mathematics effectively during methods course. Jin and Tzur(2011) postulated the teacher perspectives framework following on the work of (Heinz, Kinzel, Simon, & Tzur, 2000; Simon, Tzur, Heinz & Kinzel, 2000; Tzur, Simon, Heinz, & Kinzel, 2001). They used the term "perspectives" to refer to both the knowledge and beliefs teachers might hold regarding the nature of mathematics, mathematics learning and mathematics teaching and also the practices they might engage in based on such acknowledgment. Tzur and his colleagues proposed that teachers holding CBP (conception based perspective) would act accordingly with the views of radical constructivist epistemology. Therefore, they suggested that PIP would be a more realistic target for the teacher education since " a PIP-rooted teacher's practice can engender students' learning processes envisioned by CBP without requiring the teacher's explicit awareness of such view...".

Therefore, for the first part of the methods course, we used quantitative reasoning framework (Thompson, 1994). Thompson (1994) defined quantitative reasoning as an individual's analysis of a situation into a quantitative structure. So, quantities live in an individual's conception of situations such that in order for an individual to comprehend a quantity, the individual must have a mental image of an object and attributes of this object that can be measured (Thompson, 1994). For instance, a rectangular shape could be thought as an object with the attribute, area, that could be measured. Then, a fraction could be thought as a quantity as the measure of a particular size of the total area of the rectangular shape. For the construction of quantities, Thompson differentiated numerical operations from quantitative operations. Quantitative operations are the non-numerical operations an individual mentally act. For instance, equi-partitioning the total area of a rectangle in purpose of determining a particular size would be an example of quantitative operations. Numerical operations are the result of the evaluation of quantitative operations. So, an individual imagining the fraction such as  $\frac{1}{2}$  as a quantity, can engage in equi-partitioning of the shaded area of a rectangular

shape showing  $\frac{1}{2}$  into more equal pieces (e.g., equi-partitioning into five more equal pieces). This numerically results in the fraction  $\frac{5}{10}$ . Thompson stated that over emphasis on the numerical operations prevent students to learn mathematics meaningfully. Therefore, he proposed that prospective teachers need to pay attention to the mental processes (i.e., quantitative operations) required of an individual for the development of mathematical understandings (Silverman & Thompson, 2008). They stated,

“the prospective teacher must put herself in the place of *a student* and attempt to examine the operations that a student would need and the constraints the student would have to operate under in order to (logically) behave as the prospective teacher wishes a student to do.” (p. 19).

So, we hypothesized that if we engage prospective teachers’ in tasks focusing their attention on the distinctions between quantitative and numerical operations, their awareness on the students’ mathematical reasoning might develop. This also might have allowed them to pay attention to the nature of mathematics. That is, in this framework mathematics is built through an individuals’ mental actions and the results of these actions. This view of mathematics aligns with the problem-solving view in which “...mathematics is built from human activity: counting, folding, comparing, etc.” (Confrey, 1990, p. 109). A teacher with progressive incorporation perspective or conception based perspective is also expected to hold this view of mathematics (Jin & Tzur, 2011)

## Method

### Participants

Seven prospective secondary mathematics teachers participated in the study. They were at their fifth-year of study at one of the universities, in which the medium of language is English, in Turkey. We chose these prospective teachers for the following reasons: First, their GPA’s (Grand Point Average) ranged between 3,44 and 2,76 out of 4,00. This provided a spectrum of GPA’s of all prospective teachers taking the methods course; two in the top, three in middle and two in the lower range. Second, based on the first week of the classes and the written-pre assessment we gathered prior to teaching sessions, we observed them as verbal individuals. That is, they were talkative, and, volunteered to participate in the continuing five-set of interviews till the end of the study.

### Data Collection

Data were collected for a total of five weeks during the methods course the first author taught. The methods course was four hours per week. For this multi-case study (Yin, 1984), we conducted classroom teaching experiments for the teaching sessions(Cobb, 2000) (see Figure 1).

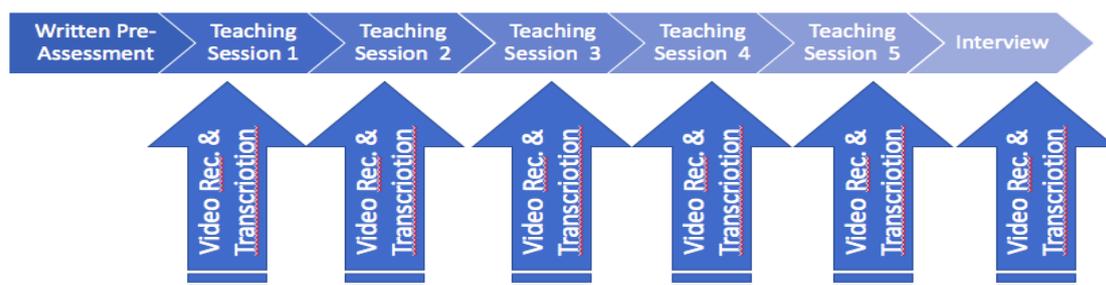


Figure 1: The data collection period during the methods course

In this method, the teaching sessions are planned in advance. However, for each teaching session, we revised the (sub) learning goals depending on our hypotheses about prospective teachers' development. Prior to the teaching sessions, we gave a written pre-assessment to prospective teachers to collect their thoughts on mathematics learning and teaching. Then, each of the teaching sessions was videotaped and transcribed afterwards. Prospective teachers also kept weekly journals concerning both in-class and out-of-class discussions, due online of the night after each class. The classes were Tuesdays and Thursdays, so that we could watch the videotapes and read the journals in between the teaching sessions. In the journals, we purposefully asked prospective teachers to reflect back and forth on the relationships among the constructs discussed. In this way, each teaching session was revised and re-planned based on the ongoing assessment. Then, we conducted an interview upon completion of working on quantitative reasoning. Data sources included prospective teachers' written responses to pre-assessment and the journals; transcripts of the videotapes of i) whole-class discussions and iii) the interviews. These data seemed central to understanding the shifts in the practices contributing to advances on the part of prospective teachers.

### **The Procedure**

The first set of classes in the methods course included mostly concrete examples to focus on the nature of mathematics. First, we included tasks in terms of the distinction between prospective teachers' thinking on quotitive and partitive division types based on measuring and equal sharing (quantitative operations). For this, we asked prospective teachers to come up with a word-problem for which the solution was given as  $10/5=2$ . Then, we asked them to sort different division problems they came up with into two categories by paying attention to the similarities and differences. They worked in groups to sort the word-problems. Following they shared their ideas during the class discussion. With this task, we wanted to direct their attention to both their classmates' and their own mental actions (i.e., measuring and equal sharing) and the numerical operations, such as division, they used. Then, we asked them to think about the two examples in chapter 1 from Van de Walle (2007). In these examples, Marlana and Darrell, two students' work on division problems are shared. We wanted to observe and assess if they could focus on these students' mental actions—measuring and equal sharing. After that, we had a classroom discussion on chapter 1 from Van de Walle (2007). The next week, we asked prospective teachers to work in groups to come up with a way to teach improper fractions to 5<sup>th</sup> grade students. For this, we also asked them to explain their thinking for instance what  $11/6$  means to their group members. We wanted to take their attention to again to their classmates' mental operations as well as theirs. Following, we provided them with the first author's articulation of the understanding of improper fractions (Tzur, 1999) from the point of view of quantitative operations. Also, for the out-of-class activity, prospective teachers considered the first author's articulation of between-state ratio (Karagoz Akar, 2007) based on a task from a reform curriculum (Everyday Mathematics) and had a discussion on it during the next class hours. Then, in the third week, we taught the geometric meaning of differentiation using a task we modified from the national reform-based curriculum. Following week, we used a task sequence to teach the algebraic and geometric meaning of the

binomial form of complex numbers (Saraç & Karagoz Akar, 2017). We also had a discussion on Thompson(1994). For the out-of-class activity, for both of the teaching sessions on differentiation and complex numbers, prospective teachers were asked to articulate on reasoning processes they have engaged in.

### **Analysis**

We analyzed the data using the constant comparative method (Strauss ve Corbin, 1990). In this method, the researcher constantly reads and analyzes data both during and at the end of the data collection. Also, s/he reads the data and compares and contrast his/her hypotheses (conjectures) both within all data from one participant and among participants. Therefore, we read the transcripts of the teaching sessions, the journal entries and the transcripts of the interviews line by line. Our goal was to determine what core-practices these seven prospective teachers' recognized and developed during these five weeks. Reading the transcripts of the teaching sessions and the journal entries from the first week together with the written-pre assessment, and the transcripts of the data from the interviews at the end of the five weeks, we all coded the data individually. Then, comparing the list of codes on the data from each participant, we came up with a set of codes representing the whole data. Following, we came together to discuss and have a consensus on the codes. We then finalized the list of the codes depicting prospective teachers' awareness of the core-practices. In the following section, for illustrative purposes, with reference to the data from one prospective teacher, Mina, in the middle range of seven prospective teachers, we share the results. Focusing on data just from one prospective teacher, we hope to maximise the possibility of the reader's achieving some familiarity with the context for the analysis.

### **Results**

#### **Beginning of the study**

As stated earlier, in the second week of the study, we had asked prospective teachers to discuss in groups how to teach improper fractions to 5th grade students. Then, all the groups choosing a representative shared their ideas in the class discussion. Mina spoke on behalf of his group members and other prospective teachers agreed upon the idea: They first started with proper fractions and used it to work towards improper fractions.

Mina: First, we thought that they (referring to students) need to know proper fractions. Like, they need to know that they are between 0 and 1. Like this could be shown by shading some part of shape, like rectangle. Like for instance dividing the shape into 6 pieces and shading 5 pieces and counting 5 pieces. But then this should be shown on the number line dividing the interval between 0 and 1. Then, in the same way we can divide the interval between 1 and 2 into six pieces. Now, we divided the interval between 0 and 1 into six pieces and divided the interval between 1 and 2, then when we count like 1,2,3,4,5 until 11, the number  $11/6$  will be in between 1 and 2. Then what  $11/6$  means because because we divided each piece into 6, they will know that the denominator is 6 and since we counted until 11, they will know that that point will correspond to  $11/6$ . Like they will see that  $11/6$  is there.

S1: Super idea

S2: I think it is very meaningful showing it with the number line.

As the excerpt showed, prospective teachers viewed mathematics as obviously apparent to their students as it seemed to them. Such knowledge existed in the drawings they shared with their students. That is, their view of mathematics corresponded to a Platonist view of knowledge. They also agreed on using the number line to depict the connections to their students supposedly acquired previous knowledge such as proper fractions. This suggested that they considered that their students would think the same way as they think.

**End of the five weeks period**

Data from the interviews at the end of the five weeks period showed a change in prospective teachers' awareness on the nature of mathematics and their students' thinking.

| Questions asked during the interview                                                                                                                                                                       | Mina's answers                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | The codes depicting data from the interviews on seven prospective teachers                                                                           |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Why do you think that we have focused on the quantitative reasoning framework?</p>                                                                                                                      | <p>Like I thought while working on them like mathematics includes two processes like it has procedures and the meanings behind those procedures... For instance <math>\frac{1}{2}</math> like was thinking numerically earlier. For instance when I say <math>\frac{1}{2}</math> is equivalent to <math>\frac{5}{10}</math>, like I say now that multiplying with 5 has a meaning when I divide the shape into two equal pieces and then shade one part. That multiplying with 5 means for me now dividing the shape, <math>\frac{1}{2}</math>, into 5 more equal pieces. Like those numbers have meaning....So like we work on quantitative and numerical operations so that like we will know the reasons behind like formulas...</p> | <p>Mathematics is made up of both numerical and quantitative operations—a human activity</p> <p>Awareness of the need for key mathematical ideas</p> |
| <p>Remember we asked you to articulate on your engagement in the tasks focusing on differentiation and complex numbers. How do you think that this experience might benefit you a prospective teacher?</p> | <p>Like yes as I said earlier like we should know the mental processes of our students like how they think... I saw my process now I can focus on theirs...</p> <p>Like to imagine how they might think so that they could learn like construct them connecting them...</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <p>Awareness of the need to focus on students' thinking</p> <p>Awareness of their students thinking differently from theirs</p>                      |

|                                                       |                                                                                                                                                                |                                                                    |
|-------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| How do connections in conceptual understanding occur? | Eeee like linking them ...like it is like to me mathematical understanding, what is the reason behind this. Like I said earlier not numerical, quantitative... | Awareness of the nature of mathematics learning with understanding |
|-------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|

Table 1: Codes depicting prospective teachers' awareness of the core practices.

As the codes depicted, there was a change in prospective teachers' awareness of the views on the nature of mathematics. Data showed that, for instance, Mina regarded mathematics knowledge as human activity, aligned with the problem solving view. Also, he was aware of the need to understand mathematics meaningfully so that he could anticipate his students' thinking to help them construct those ideas on their own. In addition, he was aware that his thinking would be by nature different from his students' thinking.

## Discussion

In this study, we investigated the change in prospective teachers' awareness on the practices involved in effective mathematics teaching once they engaged in tasks focusing on quantitative reasoning during methods courses. Results showed that their views on the nature of mathematics has changed. Contrary to the data from the beginning, after engaging in tasks requiring them to analyze both their and their classmates' thinking in terms of quantitative and numerical operations for almost one and a half-moth period, they started viewing mathematics as a human activity. Regarding the teacher perspectives framework (Jin and Tzur, 2011), this change is the first stage for the development of progressive incorporation perspective. Though, research on beliefs suggest that establishing change is not easy (e.g., Wilson and Cooney, 2002). Still, we argue that asking prospective teachers to reflect on and write about specifically their own thinking might have brought about the change in their views on the nature of mathematics. So, we propose to the field such task as a core practice for teacher educators to use in methods courses. Yet, with caution, we argue that there is need to examine the effect of engagement in tasks focusing on quantitative reasoning on both prospective and inservice teachers' beliefs on mathematics. Results also showed that prospective teachers' awareness on the need for focusing on their students' thinking developed. In addition, they recognized that their thinking processes would by nature differ from their students' thinking processes. Previous research emphasized the need to study the ways to develop prospective teachers' abilities to make sense of their students' thinking since they had difficulty in interpreting thinking (Simpson & Haltiwanger, 2016). In addition, research has shown that prospective teachers confused students' procedural fluency with conceptual understanding (Yeh & Santagata, 2015). To the contrary, results of this study indicated prospective teachers' awareness of the nature of concepts and procedures. Therefore, we suggest focusing on quantitative reasoning during methods courses as a possible way to provide prospective teachers for paying attention to their students' thinking differentiating their procedural fluency from their concepts. We also recommend the use of tasks focusing on quantitative reasoning in professional development workshops for inservice mathematics teachers. As a final note, considering the core-practices as conglomerate components of effective mathematics teaching, we suggest that the effect of quantitative reasoning on both prospective and inservice teachers' mathematical knowledge for teaching might be investigated.

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- Acknowledgement: This paper is supported by Boğaziçi University Research Project, Grant Number: 11220.