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Between natural language and mathematical symbols (<, >, =): the comprehension of pre-service and preschool teachers' perspective of “Numbers” and “Quantity”

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This paper deals with a study that relates to the understanding of the concepts that constitute part of symbolic thinking. The goal of the study was to understand how pre-service teachers (PST) and preschool teachers (PT) understand and use the mathematical symbols <, >, and = when comparing numbers, figures and shapes of different sizes and thicknesses. Using both quantitative and qualitative methods, we examined a study population of 71 PST attending a course for teaching mathematics to pre-schoolers and 149 PT. Our results show that the majority of participants did not answer the questions correctly, with a significant difference between how the two groups validated their answers, indicating that the participants do not correctly understand that mathematical symbols should only be used in the mathematical context.

Keywords: Preschool teachers, pre-service teachers, mathematics education, early childhood, mathematical symbol.

Introduction and theoretical framework

Mathematical language in Early Childhood is a language of symbols, concepts, definitions, and theorems. It does not develop naturally like a child's natural language, but needs to be taught (Ilany, & Margolin, 2010). In essence, children are engaged in mathematics in daily life from birth, and today's global trend is to introduce “formal” mathematics at a young age. Preschool math practice aims to develop mathematical awareness and cultivate mathematical thinking from an early age, thus shaping the child's future mathematical thinking, general thinking, and cognitive abilities. Studies have shown that the volume and quality of preschool math practice predict a child's success in math in elementary school (Clements, & Sarama, 2006, 2015).

The first skills include being able to use the concepts of “bigger”, “smaller,” and “equal to” to recognize differences between objects. Some PT introduce the mathematical symbols =, <, and > already in preschool and, unfortunately, ask the children to use these mathematical relational symbols to compare non-mathematical objects. This leads children to believe that these symbols are not restricted to mathematical values and, moreover, even when comparing numbers, to use them incorrectly. For example, a child in grade one may write “6 < 4” because the four looks bigger and thicker than the six, indicating that he is looking at the numbers as graphical entities and not mathematical ones. Such instances have led to the study of how PST and PT themselves use these mathematical symbols (Hassidov, & Ilany, 2017). Different quantities are compared through relations of order using various strategies based on the properties of these relations. According to Cantor's (1971) sorting principles, the set of real numbers has an intrinsic linear order. In other words, between any two quantities, one and only one of three following options holds true: (i) the two values are equal to each other; (ii) the first is greater than the second; (iii) the first is smaller

than the second. If we plot real numbers on a number line, two numbers, a and b , are equal only if the points that represent them coincide. If b is greater than a , the point representing b will be to the right of a . Here, we can also say that a is less than b . It is useful to present the ways that a relationship between two quantities can be described by using three pairs of relations, where each proposition of the pair is the negation of the other: $a = b$ $a \neq b$; $a > b$ $a \leq b$; $a < b$ $a \geq b$. Recall that the strategies used to compare two quantities are based on the general properties of comparison relations. The relation of equality is an equivalence and maintains the three properties of any equivalence relation: reflexivity, $a = a$ (each value is equal to itself); symmetry, $a = b \Leftrightarrow b = a$; and transitivity, $a = b \& b = c \Rightarrow a = c$ (two values equal to a third are also equal to each other).

Symbolic reasoning means the ability to grasp the meaning of a symbol representing an object or idea, without having an expression in the symbol itself (Bialystok, 1992). It is an evolving ability and one of the developing expressions of thought (Thomas, Jolley, Robinson, & Champion, 1999). Its development is characterized by changes that occur in the form of the mental representation of an object. Young children believe that the symbolic representation reflects the nature of the object it represents (Bialystok, 1992). For example, children may write the names of large objects using large letters (Thomas et al., 1999). Nemirovsky and Monk (2000) noted that young children do not distinguish between the symbol and the object that the symbol represents. The early development of symbolic reasoning in children should allow them to properly use mathematical symbols later in formal math. Teaching mathematics to pre-schoolers today requires professional knowledge on the part of the PT (Charalambous, Panaoura, & Philippou, 2009). Unfortunately, studies conducted in recent years indicate that PT assigned with teaching preschool mathematics do not have adequate knowledge. This may stem from negative personal experiences or a lack of appropriate training in college (Hassidov, & Ilany, 2014, 2015). They often use the knowledge and experience they bring from daily life, meaning that they might not always give the correct mathematical importance to the symbol. If PT incorrectly understand the use of mathematical symbols, it is reasonable to assume that they will subsequently pass this misinformation on to the children, leading to incorrect use in the future. It is thus crucial to teach the proper mathematical use of symbols from the preschool level (Hassidov, & Ilany, 2017). PT often use the knowledge and experience they bring from daily life, meaning that they might not always give the correct mathematical importance to the symbol. If the teachers incorrectly understand the use of the symbol, they will subsequently pass this on to the children, leading to their incorrect use in the future. Although young children can identify symbols and write them, this does not necessarily reflect an understanding of the symbol's mathematical meaning or their relationship to numbers. The concept of equality is an especially difficult concept to comprehend for children, since this term can be used both relationally and mathematically. Using the “=” symbol incorrectly with children makes it even harder for them to properly understand its concept.

Many studies have examined how children of various ages comprehend the “equal” sign. They show that children aged 5–12 tend to perceive the equal sign as an operational symbol and not as a sign of comparison. PST translate the symbols as a command to perform a mathematical operation. It is important to grasp that the meaning of a symbol cannot be changed by non-mathematical factors (such as a change in size or other physical factor). In a study dealing with the knowledge of PST and PT regarding their understanding of the significance and use of mathematical symbols

between numbers, Hassidov and Ilany (2017) found that PST and PT do not fully understand that mathematical symbols should relate only to the mathematical nature of the object. If one number was written in a larger, smaller, or thicker format than another, they often regarded the physical qualities and not the mathematical (i.e., the values of the numbers). Furthermore, even when they used the symbol correctly, the reasoning behind its use was often flawed.

Research Questions

This study examines how PST and veteran PT understand the concepts of $>$, $<$, and $=$. Its objectives were twofold:

- 1) How do PST and PT comprehend and use the relational symbols ($>$, $<$, and $=$) in perspective of "Numbers" and "Quantity"?
- 2) Is there any difference between how the two groups comprehend and use these symbols?

Method

The study population comprised 71 second- or third-year PST participating in a year-long course dedicated to the teaching and learning of mathematics in early childhood and 149 veteran PT. Data were collected via semi-structured interviews and a 25-item questionnaire designed by the authors. Of the 25 questions in the questionnaire, eight (questions 1, 2, 3, 17 and 7, 9, 10, 16) addressed the use of mathematical symbols between shapes and numbers that had some graphical difference (size, thickness, placement) (Table 1). Respondents were asked to either place a relational symbol between two figures or indicate "X" if they believed there was no appropriate answer, and then justify their answers. Analysis was both qualitative and quantitative.

Questionnaires were filled out by the PST before any formal study of the subject. The researchers interviewed a random sampling of 30 PST. This was followed by a class discussion on the use and meaning of mathematical symbols, and the subject's place in the preschool curriculum. Questionnaires were filled out by the PT and then individual interviews were conducted to ascertain the PT reasoning for their answers. Relevant background information was collected (e.g., professional experience).

Results

Overall, not one of the participants gave the correct answer and justification for questions 1, 2, 3, and 17. Even the very few who gave the correct answer ("X") gave flawed justifications, the correct one being that these symbols cannot be used for graphical objects and only for numerical entities. A significant difference was found between the two groups: a large number of PT did not supply any justification for their reasoning (58.4% for question 1, and 57.7%, 60.4%, and 66.4% for questions 2, 3, and 17, respectively) compared to the number of PST who did not (19.8%, 14.1%, 16.9%, 28.2%, respectively).

Questions for example: 2; 17 (Quantitative) asked which mathematical symbol, if any, should be placed between the shapes of different sizes and thickness.

Question 2: contained three smileys. The results were similar to question 1: most did not answer "X," and those who did, justified it incorrectly. Similarly, there was a significant difference ($p < 0.001$) between the groups (see Table 2). The vast majority of both PST and PT answered "=",

indicating that they focused on the number of smileys (numerical properties). However, one preschool teacher said: “There are the same number of smileys, but the area is different.” That is, her answer was based on quantity, but her justification also considered the shape. Another wrote “I counted the smileys.” One wrote: “Based on my experience, I would teach that the second is larger. But there can be different levels,” indicating that she feels that different criteria can be used under different circumstances. One PST teacher wrote: “I looked at the number of smileys. There is no importance to the length of the rectangle, only the number.” One PST teacher indicated “=” but wrote “The same quantity in each rectangle, although the left rectangle has a greater area.” Those who indicated “<” justified their answer by indicating either the size or thickness of the rectangles. One preschool teacher answered, “They look to me to be the same, except that one rectangle is longer.” A PST teacher who marked “<” wrote “the rectangle on the right is thicker and coloured.” Again, although 4% of the PT gave the correct answer (“X”) their justifications were incorrect. For example: “They cannot be compared because the shapes are not the same.”

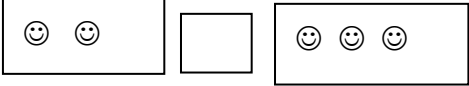
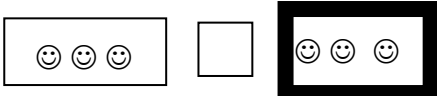
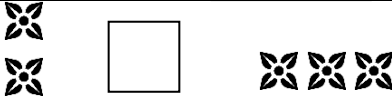
Question	Possible Answers							
	<		>		=		X	
	PT	PST	PT	PST	PT	PST	PT	PST
1 	98	100	0	0	0	0	2	0
2 	5	0	16	6	74	94	4	0
3 	95.4	96	1.3	1	1.3	3	3	0
17 	1.3	0	2.7	4	92	90	4	6
7 $\frac{1}{2} \square \frac{1}{4}$	14.1	0	*80.5	*97.2	0.7	0	4.7	2.8
9 $5 \square 5$	0	0	26.2	14.1	*70.5	*77.5	3.3	8.4
10 $6 \square 4$	17.4	2.8	*77.9	*91.6	0	0	4.7	5.6
16 $2 \times 3 \square 6$	1.3	0	6	0	*86	*98.6	6.7	1.4

Table 1: Quantitative (questions 1, 2, 3, and 17) and Numerical (questions 7, 9, 10, and 16) - Analysis of the responses of PST and PT (all values represent percentages, *correct answer)

Differences between correct answers: Question 7 $p^*=0.001$, distribution $p=0.006$;

Justification for question 1	Graphic properties*		Numerical properties*	Both size and quantity*	No answer*	Number who gave correct justification	
PST	N	1	56	0	14	0	
N=71	%	1.4	78.8	0	19.8	0	
PT	N	6	53	3	87	0	
N=149	%	4	35.6	2	58.4	0	
Justification for question 2							
PST	N	2	52	7	10	0	
N=71	%	2.8	73.2	9.9	14.1	0	
PT	N	18	41	4	86	0	
N=149	%	12.1	27.5	2.7	57.7	0	
Justification for question 3							
PST	N	1	57	1	12	0	
N=71	%	1.4	80.3	1.4	16.9	0	
PT	N	3	54	2	90	0	
N=149	%	2.1	36.2	1.3	60.4	0	
Justification for question 17							Another answer X
PST	N	10	36	4	20	1	0
N=71	%	14.1	50.7	5.6	28.2	1.4	0
PT	N	4	44	1	99	1	0
N=149	%	2.7	29.5	0.7	66.4	0.7	0

Question 10 $p^*=0.01$, distribution $p=0.01$; Question 16 $p^*=0.003$, distribution $p=0.032$.

Table 2: Quantitative analysis (value and percent) of the justifications given by PST and PT to questions: 1; 2; 3: 17 * $p<0.001$

Question 17: Each side had two triangles, one being “upside down.” On the left, they were in a single row with a plus sign (“+”) between them. On the right, they were one on top of the other. Once again, the vast majority (94% of PST and 96% of PT) answered incorrectly and there was a significant difference ($p<0.001$) between the justifications they gave (Table 2). One preschool teacher who indicated “>” said: “There are two triangles and the addition operation, so that side is larger than the right side.” One who indicated “=” wrote, “We haven’t learned this yet.” Another gave an answer that seemed confused, “They are equal from two standpoints. One is that on each side one triangle goes up, and one goes down. So, they make the shape of an equilateral diamond.” A teacher who indicated “=” said, “The placement of the triangles is not important. What is important is their quantity.” One PST who answered “X” justified it with “There is no answer

because I didn't know which symbol to use. There are two triangles on each side, but they are not arranged the same." Some PT answered "X" because they did not know which of the others to use.

Questions for example: 9; 10; 16 (Numerical) asked which mathematical symbol, if any, should be placed between numbers of different sizes and thickness.

	PST	N=71	PT	N=149
Justification for question 7: $\frac{1}{2} \square \frac{1}{4}$	%	N	%	N
Correctly answered	PST	68	PT	120
None given.	19.7	14	49.7	74
The sequence of numbers.	18.3	13	2.7	4
A quarter plus a quarter equals half.	11.3	8	2.0	3
Because half is greater than a quarter.	32.4	23	12.8	19
Based on number of items.	14.1	10	13.4	20
Incorrectly answered	PST	3	PT	29
None given.	0	0	14.1	21
The $\frac{1}{4}$ is larger according to the picture but $\frac{1}{2}$ is larger according to quantity.	4.2	3	0	0
Based on graphic property.	0	0	0.7	1
We never learned fractions.	0	0	4.7	7
Justification for question 9: $5 \square 5$	%	N	%	N
Correctly answered	PST	55	PT	105
None given.	16.9	12	36.2	54
The sequence of numbers.	45.1	32	16.1	24
Incorrect reason (based on graphic property).	2.8	2	13.4	20
Based on number of items.	12.7	9	4.7	7
Incorrectly answered	PST	16	PT	44
None given	2.8	2	16.8	25
Both have the same value but differ in size and thickness.	8.5	6	1.3	2
The left number is larger than that the right one.	11.3	8	11.4	17
Justification for question 10: $6 \square 4$	%	N	%	N
Correctly answered	PST	65	PT	116
None given.	19.7	14	47	70
The sequence of numbers.	63.4	45	24.8	37
Incorrect justification.	0	0	0.7	1
Based on number of items.	8.5	6	5.4	8
Incorrectly answered	PST	6	PT	33
None given	0	0	14.1	21
There is no answer because 4 is graphically larger but 6 is numerically larger.	5.6	4	0.7	1
The 4 is larger because of the size.	2.8	2	7.4	11
Justification for question 16: $2 \times 3 \square 6$	%	N	%	N
Correctly answered	PST	70	PT	128
None given.	22.5	16	55.7	83
The sequence of numbers.	53.5	38	28.9	43
Based on quantity.	22.5	16	1.3	2
Incorrectly answered	PST	1	PT	21
None given	0	0	8.1	12
We didn't learn this subject	1.4	1	2	3
Because of the size of the numeral.	0	0	4	6

Table 3: Numerical - analysis (value and percent) of the justifications given by PST and PT to questions: 7; 9; 10 and 16

Question 9: Table 1 shows that 77.5% of the PST and 70.5% of the teachers answered correctly, but as can be seen in Table 3, only 45.1% of the PST and 16.1% of the teachers who answered correctly gave the correct explanation. Of those who gave an incorrect explanation, 2.8% of the PST and 13.4% of the teachers gave the reason to be the graphic form of the numbers, and 12.7% of PST and 4.7% of the teachers referred to the quantity of items (one numeral) on each side. One reason given by a teacher indicated her deliberation between the graphic or numerical quality of the numbers: “It depends on how one looks at the question: according to shape, one is larger than the other; according to numerical value, they are equal.” Of those who answered incorrectly, 8.5% of the PST and 1.3% of the teachers argued that no symbol could be put between the digits because there can be multiple answers based on how one looked at the question (“Both numbers have the same value but not the same size and thickness”). 11.3% of the PST and 11.4% of the PT argued the number on the left is larger. One PST wrote: “Looking at the numbers, they are equal in terms of quantity or value, but the type is bigger and it’s confusing.”

Question 10: Table 1 shows that 91.6% of the PT answered correctly compared with 77.9% of PT (Table 3). This question deals with getting to know the first ten numbers. From table 1 it could be seen that there is a significant difference in the scattering distribution between kindergarten PST to PT ($\chi^2_{(2)} = [9.271, p=0.01]$). It can also be seen that 91.6% of the PT answered correctly, compared to 77.9% of the kindergarten PT ($\chi^2_{(1)} = [6.19, p=0.01]$). In Table 3 we see that 63.4% of the PST and 24.8% of the PT correctly explained that it was due to the sequence of numbers. Some participants (8.5% of PST, 5.4% of PT) incorrectly based their answer on the number of items on each side and not their numerical value. Of the incorrect answers, 17.4% of the PT, but only 2.8% of the PST answered that “four” was larger than “six” based on the numbers’ graphic properties.

Question 16: Table 1 shows that 98.6% of the PST answered correctly compared with 86% of PT ($p<0.01$). Of the 21 (14.1%) of PT who answered incorrectly, 10 answered “X,” claiming that a number of answers were possible, and 9 (6%) claimed that 3×2 was greater than 6 due to the graphic properties of the numerals (Tables 1 and 3).

Additional findings. Interviews and discussions with the PST and PT revealed that most of them thought it was possible to use more than one mathematical symbol, between numbers, as an answer.

Discussion and conclusions

This study found that most of the participants failed to answer the questions correctly. The justifications given to the questions show a significant difference between the PST and PT with respect to how many justified their answers, yet it is clear that all participants did not appreciate the significance of the mathematical symbols and how to use them, specifically, that mathematical symbols should be used only for mathematical symbols. This was clear since even when the answer given was correct (“X”), the justification was generally incorrect (Hassidov & Ilany, 2017; Ilany & Hassidov, 2018). The results of this study show that PT feel that mathematical symbols may be used in different ways, depending on context: sometimes with respect to the quantity and sometimes to the shape or size of graphical images and they did not restrict them only to their mathematical significance. The conclusion is that the participants do not properly understand the significance of the symbols =, <, and > nor how to use them. This will, in all probability, mean that they will not teach the concepts properly to preschoolers. Indeed, studies have shown that PT believe the signs

can be used in many ways. Using the same words in everyday life and in mathematics leads to misconceptions regarding the meaning of the mathematical signs. PT thus do not see any problem if a child writes “**5** > 5,” and have stated that they teach the child to use the symbol “>” between two objects, as “in this case the size is important; in another case the length may be important. It depends on the context.” PT may even believe it is correct to use two different signs at the same time; however, they must understand the cognitive conflict that this gives children and must understand that it is never possible to use two different signs between two numbers at the same time. PT must be made aware that the signs “<, >, and =” must be used only in the mathematical sense. PT who incorrectly see quantity as a graphical concept and do not see the mathematical significance will, most likely, pass on this misconception to the children. This might lead the children to think that the size of the number or graphical object is what determines the relationship and which symbol to use.

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