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Connecting mathematical knowledge with engagement in mathematics teaching practices

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Research suggests that mathematical knowledge is likely to influence how mathematics is taught. In turn, how mathematics is taught impacts students' opportunities to learn mathematics. We report on a study examining the connection between preservice teachers' mathematical knowledge and the nature of their eliciting and interpreting of student thinking. Our findings suggest that preservice teachers elicit and interpret student thinking with more emphasis on student understanding in situations in which they have strong mathematical knowledge of an algorithm used by the student compared to situations in which they have weaker mathematical knowledge about the algorithm used.

Keywords: Teacher education-preservice, instructional activities and practices.

Mathematical knowledge, dispositions, and pedagogical skill

Mathematical knowledge and pedagogical skill are crucial to effective teaching. Studies show teachers with substantial mathematical knowledge (Hill, Rowan, & Ball, 2005) and command of core teaching practices like formative assessment (Black, & Wiliam, 1998) have a greater impact on student learning of content. In the United States (US), mathematical knowledge and pedagogical skill are assets delineated in standards used to appraise both accomplished teachers (NBPTS, 2012) and beginning teachers (CCSSO, 2013). That mathematical knowledge and pedagogical skill are important in mathematics teaching is seldom questioned, but this does not mean that the field has determined the nature of the relationship between them. Mathematics teaching is a complex professional activity involving teachers in interaction with students, content, and context (Cohen, Raudenbush, & Ball, 2003). It is easy to imagine a multitude of ways in which mathematical knowledge and pedagogical skill are mutually influencing, if not mutually defining. For example, mathematical knowledge impacts the content and characteristics of questions that teachers ask during a lesson. Oriented in the other direction, pedagogical skill impacts the extent to which a teacher has access to student thinking that can be a substantial driver of a teachers' own mathematical development. This paper focuses on understanding the interrelations between mathematical knowledge and pedagogical skill empirically with respect to preservice teachers who are enrolled in a teacher preparation program.

Prior studies on teacher effectiveness connect teachers' mathematical knowledge with the achievement of their students. Often these studies examine the number and type of professional development experiences (Blank, & Atlas, 2009) or teachers' performance on mathematics content tests. When mathematics content tests are used, they tend to focus on general mathematical knowledge or mathematical knowledge identified as central to mathematics teaching (Hill, Rowan, & Ball, 2005). Such studies provide a connection between the mathematics that teachers know and the achievement of their students. What these studies do not explain is the way in which teaching is shaped by mathematical knowledge in ways that generate enhanced achievement for students. In a

comparative study of Chinese and US mathematics teachers, Ma (1999) advanced the notion that the way that teachers hold mathematical knowledge (e.g. the depth of understanding of foundational ideas interwoven with knowledge of how those ideas are shown across the breadth of mathematical topics) supports their engagement in teaching. Some examples of engagement in teaching are the questions teachers are able to ask students, the examples they create, and their analyses of student work. Among other mathematical foci, Ma explored differences in teachers' understanding of subtraction algorithms. Her findings illustrate the potential impact of the teachers' understanding and the ways in which different types of understanding might provide the basis for teaching that would support students' learning about subtraction. Needed are studies that can show connections between teachers' knowledge and the ways in which they engage in teaching practices. This would especially support teacher preparation, where resources are most plentiful, to scaffold improvement.

Since mathematical knowledge is likely to impact/influence mathematics teaching, it is important to understand how those resources impact teaching. To enhance the utility of what is learned, it will be important to focus studies on practices of teaching that are routine and crucial to students' learning of mathematics. Two such practices are eliciting student thinking and interpreting student thinking. Teachers use these practices to encourage students to share their thinking about specific academic content in order to evaluate student understanding, guide instructional decisions, and surface ideas that will benefit other students (TeachingWorks, 2016). Research on learning supports the use of teaching practices that enable teachers to know about and build on students' ideas. It also shows that teachers who know about their students' thinking and use that knowledge to influence their instruction have a significantly greater impact on student achievement (Black, & Wiliam, 1998). Hence, understanding the connection between mathematical knowledge and preservice teachers' engagement in the teaching practices of eliciting and interpreting student thinking is likely to be use to the wider field.

In this context, this paper reports on research driven by a desire to better understand the ways in which mathematical knowledge influences preservice teachers' engagement in eliciting and interpreting student thinking. We studied these teaching practices of preservice teachers within a mathematical content strand that dominates the primary grades – number and operation – zooming in on the operation of subtraction. Within subtraction, we focus on the use and understanding of different algorithms to solve problems, an area in which preservice teachers experience difficulty connecting their knowledge of traditional algorithms to alternative approaches (Son, 2016). Our research was guided by the question: What is the relationship between preservice teachers' mathematical knowledge of particular mathematical algorithms and the ways in which they approach eliciting and interpreting student thinking about those algorithms?

Using simulations to learn about preservice teachers' engagement in eliciting and interpreting student thinking

A practical problem that arises is how to study the ways in which preservice teachers' mathematical knowledge of particular mathematical algorithms impact their eliciting and interpreting of student thinking. We chose to design and use simulations in which preservice teachers engage in eliciting and interpreting a student's thinking about the use of a particular subtraction algorithm. Simulations are approximations of teaching that place authentic, practice-based demands on a participant while

purposefully suspending or standardizing some elements of a teaching situation (Shaughnessy & Boerst, 2018). For each simulation, we developed a piece of student work and then designed an elaborate protocol detailing how a student would: complete the process shown in the work; reason about the process used; talk and act when questioned about the process. Those taking on the role of the student (teacher educators) were trained so that they could use information from the protocol as the basis for engaging in the situation and so that they could respond in ways that aligned with the profile of student thinking and acting.

Our decision to use simulations was driven by four factors. First, simulations of practice have proven to be usable in many professions to gauge clinical knowledge and skills (e.g. Boulet et al., 2009). Second, simulations provide reliable access to teaching situations that other methods, such as interviewing real children or interviewing in school contexts, could not. Third, simulations allow for the standardization of the assessment context (Shaughnessy, Boerst, & Farmer, 2018). Fourth, our goal required a design that would purposefully put preservice teachers in situations where their mathematical knowledge was low. The use of simulations allowed us to accomplish this while preserving our own ethical commitments to educating children and consistently positioning preservice teachers to work productively with children.

Methods

We collected information about the mathematical knowledge of preservice teachers and used that information to assign them to three teaching simulations that each strategically positioned preservice teachers to elicit a student's thinking about a particular algorithm that had been used to compute an answer to a subtraction problem. Specifically, the simulations were designed to present opportunities to elicit a student's thinking about algorithms about which the preservice teachers had different degrees (known to the researchers) of mathematical knowledge. Twenty-four elementary preservice teachers from one teacher preparation program participated. Because our goal was not to bound our analysis to any point in time during a teacher education program, we purposefully recruited preservice teachers who were at different points in the program. Specifically, two were at the beginning of the teacher education program, eight were at the midpoint of the teacher education program, and 14 were at the end of the program. Next, we describe the pre-assessment, the assignment of preservice teachers to specific simulations, and the administration of the teaching simulations.

Pre-assessment of mathematical knowledge

We designed an assessment of mathematical knowledge inclusive of four subtraction algorithms that could be encountered in a subsequent teaching simulation. These algorithms, shown in Figure 1, include: A) the US traditional algorithm for subtraction; B) adding up subtraction; C) subtracting from the base; and D) expand and trade subtraction. Largely, these algorithms represent processes that are either directly taught through common US mathematics curriculum materials or are likely to surface in some form during mathematics work in US classrooms. One was deliberately picked to represent an algorithm that would likely be unknown to preservice teachers.

Figure 1. Pre-assessment subtraction algorithm examples

Preservice teachers were shown two examples of students' use of each of the subtraction algorithms, resulting in eight total examples to analyze. Half of the examples showed how the algorithms could result in correct answers and the other half of the examples showed how the algorithms could result in incorrect answers. This was done because of the potential for inaccuracy to influence preservice teachers' understanding of the different algorithms. For each example, preservice teachers were asked to determine accuracy of the answer, describe the steps that the student likely used to arrive at the answer, provide an explanation about the mathematical validity of the student's method, and apply the student's approach to another problem. The intent was to capture information that would allow us to categorize each preservice teacher's mathematical knowledge of each algorithm as "strong", "moderate", or "weak" relative to the mathematical knowledge shown across the algorithms. For each preservice teacher, the subtraction algorithm about which they demonstrated the most understanding was designated as "strong" and the subtraction algorithm about which they demonstrated the least understanding was designated as "weak. Those algorithms that were neither strong nor weak were designated as "moderate."

Assignment of preservice teachers to particular simulations

We used information from these pre-assessments to assign preservice teachers to three simulations. One simulation involved a "student" using an algorithm for which the preservice teacher was characterized as having strong mathematical knowledge (relative to the other algorithms). Another simulation involved a "student" using an algorithm for which the preservice teacher was characterized as having weak mathematical knowledge (relative to the other algorithms). An additional simulation involved a "student" using an algorithm for which the preservice teacher was characterized as having moderate mathematical knowledge (relative to the other algorithms).

Administration of the teaching simulations

All simulations were facilitated through the same sequence of stages. First, the preservice teacher prepared for an interaction with one simulated student (i.e., a teacher educator taking on the role of the student) by reviewing a specific piece of student work and considering what would be relevant to ask the student. For each simulation, they had 10 minutes for this preparation work. Second, the preservice teacher elicited and probed the simulated student's thinking to understand the steps she took, why she performed particular steps, and her understanding of the key mathematical ideas involved. Preservice teachers had five minutes to interact with the student. All preservice teachers finished their interactions within this time frame. All simulations were video recorded to support

scoring and research purposes. Third, following the simulation, the preservice teacher responded to questions about their interpretation of the student's process and understanding and their prediction about the student's performance on a similar problem. Interviewers followed up on the preservice teachers' responses as needed.

Methods of analysis

We analyzed the simulation performances in which the preservice teachers elicited student thinking and the follow up interview in which the preservice teachers interpreted student thinking. Because the simulations made use of highly specified protocols for the student's processes, understandings, and ways of being, we were able to use checklists to track on the absence or presence of particular moves (Shaughnessy, & Boerst, 2018). Our checklists for the eliciting portion of the teaching simulations are based on an articulation of "high-quality" eliciting of student thinking. For example, high-quality eliciting student of thinking includes: eliciting the process/steps involved in using the algorithm and probing understanding of the algorithm and ideas underlying the algorithm. The checklist includes specific things that the preservice teacher might do in the context of each teaching simulation (e.g., probes why borrowing works) and specific responses that the student provides based on their preparation and training (e.g., "When you borrow, you're not changing the problem. You're just moving the same amount to a different place) when prompted by the preservice teacher. The practice of interpreting student thinking includes describing the components of the algorithm used by the student, making claims about the student's understanding of the algorithm, and substantiating claims by drawing on evidence from a larger body of data gathered from the eliciting. The checklist includes specific things that the preservice teacher might do in the context of each teaching simulation (e.g., describes the student's understanding of why borrowing works). For interpreting, we also cross-check any evidence that a preservice teacher cited with the information gathered during the interaction with the student to ensure that they had the evidence that they cited. For each situation, we further specified crucial "core" components of the process and understandings that were at the heart of the student's algorithm, ones that would be particularly important to attend to in the interpretation.

Each performance was appraised by two members of the research team. Disagreements were resolved through review of the data and remediating differences of interpretation by developing and refining a codebook.

Findings: Connecting the preservice teachers' eliciting with their mathematical knowledge and disposition

As noted previously, we analyzed the quality of each preservice teacher's eliciting performance in a context in which the preservice had "*strong mathematical knowledge*" of the subtraction algorithm versus their performance in which they had "*weak mathematical knowledge*" of the subtraction algorithm. We next turn to findings related to their eliciting and interpreting of student thinking.

Eliciting practice

For each teacher, we compared their eliciting performances in the strong mathematical knowledge simulation context (SM) and the weak mathematical knowledge simulation contexts (WM). There were no differences across the two conditions with respect to the preservice teachers' eliciting of the student's procedural steps to solve the problem. However, preservice teachers probed for the

students' understanding of the process more frequently in the SM simulation ($M = .792$, $SD = .2518$) than in the WM simulation ($M = .313$, $SD = .3848$), $t(23) = 5.468$, $p < .001$. In other words, when preservice teachers had stronger understanding of mathematics of the student's algorithm (relative to the second algorithm), they more frequently elicited the student's understanding about core ideas.

Interpreting practice

For each preservice teacher, we compared their interpreting performances in the SM simulation context and the LM simulation context. Preservice teachers did not differ in their skills in interpreting the procedural steps of the algorithm used by the student in the two contexts. Almost all of the preservice teachers named all of the student's steps in both contexts. However, the quality of their interpretation of the student's understanding of the process that was used was significantly higher in the SM simulation ($M = .500$, $SD = .3612$) than in the WM simulation ($M = .188$, $SD = .3234$), $t(23) = 3.498$, $p < .002$. In other words, preservice teachers were better at interpreting the students' understanding in situations in which they had strong mathematical knowledge of the algorithm compared to a situation in which their mathematical knowledge was comparatively weaker.

Implications and next steps

Preservice teachers demonstrated different eliciting and interpreting skills across two situations – one in which they had relatively high mathematical knowledge of a particular subtraction algorithm being used by a “student” and one in which they had relatively weak mathematical knowledge of a different subtraction algorithm. Preservice teachers focused their eliciting and interpreting practices more on student understanding when they had relatively strong mathematical knowledge. In some ways this finding is not surprising. Strong mathematical knowledge of an algorithm may enable preservice teachers to know what to ask about in terms of understanding. Further, in interpreting student thinking, mathematical knowledge of an algorithm may impact preservice teachers' abilities to identify the mathematical understanding that are worth naming. To be clear, in our work with preservice teachers we have seen cases where preservice teachers with strong mathematical knowledge of a situation did not elicit much thinking from the student, but instead made assumptions about the process and the students' understanding based on a combination of the student's written work and their own understanding of the algorithm. A contribution of the study was gain a better sense of the degree to which such engagement was common (or not) in our context. While this study does not provide insight into the reason(s) behind the differences in the practices, this outcome is useful because it shows concretely that differences in practice predicted by researchers like Ma (1999) who had theorized that greater mathematical knowledge would enable engagement in teaching practices that support student understanding of mathematics.

Some elements of the preservice teachers' eliciting and interpreting were not significantly different across the two situations. Notably, there were not marked differences in the extent to which preservice teachers elicited procedural aspects of the student's algorithm. Likewise, there were not marked differences in preservice teachers' interpretations of procedural aspects across the different algorithms. When coupled with the previous analysis, this indicates a nuanced relationship between knowledge and teaching practice. The degree to which mathematical knowledge impacts the focus of teaching practices with respect to some facets of mathematics (understanding) more than others

(procedural) is interesting. It is plausible that differences were not seen due to the fact that eliciting information about steps in a mathematical process, like an algorithm, may be more straight forward than eliciting information about understanding and thereby are not as heavily impacted by the degree of mathematical knowledge one has about an algorithm. The similarity of eliciting and interpreting with respect to the steps in an algorithm is notable for the questions it suggests. What enables preservice teachers to elicit and interpret student thinking about the procedural aspects of an algorithm irrespective of the depth of their mathematical knowledge about the algorithm? What compels preservice teachers to elicit student thinking about the procedural aspect of an algorithm and to attend enough to the procedural aspects to be able later to produce accurate evidence-based interpretations?

A clear limitation of the study findings presented here is consideration of other factors that might be contributing to the differences in eliciting and interpreting practices. For instance, a teacher's mathematical dispositions may shape teaching practices in ways that impact students' opportunities to learn mathematics (Thompson, 1992; Philipp, 2007). As an example, in our own work with preservice teachers, we frequently hear differing, and often strongly expressed, dispositions toward particular computational algorithms. These dispositions influence and give structure to how teachers perceive students' engagement in mathematics. Studies have illustrated ways in which dispositions can impact the classroom practices of teachers (e.g., Fennema et al., 1996). Some of the connections between teaching practice and content knowledge that we have shared in this paper could be explained by the influence of mathematical disposition. For instance, it is just as plausible that a positive mathematical disposition may result in preservice teachers believing that it is worth asking questions about understanding because they personally know that the algorithm can be understood as it is that strong mathematical knowledge is the resource driving that approach to the practice. Given the potential for mathematical knowledge and mathematical dispositions to be mutually influencing (e.g. the more robust the mathematical knowledge of the teacher about an algorithm the more favorable the disposition to engage in and with the algorithm), in current work, we are teasing apart their influences with respect to engagement in teaching practices.

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