Working with example sets: A productive focus in Lesson Study

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The Wits Maths Connect Secondary Project, a research linked professional development project, included Lesson Study with teachers in school clusters. Planning and reflection is carried out using a Mathematics Teaching Framework, a key aspect of which is exemplification. We illustrate that and how working on example sets in a lesson has been productive in our (LS) work. Through this we build a case for a focus on exemplification when studying and working on mathematics teaching, and for support at a more general level for theoretically informed Lesson Study.

Keywords: Lesson study, mathematics, secondary, exemplification, South Africa

Introduction

The Wits Maths Connect Secondary Project (WMCS) is a research linked professional development (PD) project that has included Lesson Study (LS). Based on a view of professional learning as a collaborative practice, and thus with some similarities to the Japanese model, the LS model developed in the WMCS project has two distinct features. It is framed by a theoretically driven and empirically elaborated framework for mathematics teaching that includes an emphasis on exemplification; it has been shaped by conditions of learning and teaching in the WMCS project schools, and thus by constraints typical of low-income schooling contexts (Adler, 2017).

Lesson study has become widely used in PD globally and differently interpreted across contexts (e.g. Quaresma et al., 2018). Adaptations of Japanese LS have taken root in the US, the UK, and Australia (Wood, 2018; Huang et al., 2017), and spread across countries in Asia and Africa. Fujii (2014) reported on a project where LS was promoted in countries such as Malawi and Uganda and argued that cultural practices in Japanese schools were not 'transferred' together with doing LS. In South Africa, while there is recognition of the benefits of LS, Jita et al. (2008) state many challenges, in particular time for running the training and enacting the process and more critically the role of knowledgeable others which was undermined through a cascade model of expanding the reach of LS. The difference with practices in Japan is apparent, particularly in relation to LS in Japan being an in-school practice, where experienced and knowledgeable others are part of the systemic culture.

It is precisely these conditions that inspired the WMCS to (re)design lesson study in ways that both supported the goals of the wider project (see below), and the conditions of teachers' work in South Africa. Hence the key design features of the WMCS model mentioned earlier – it takes constraints on time into account, and it focuses on current teaching practices like exemplification.

Exemplification and variation

There is an extensive literature on the significance of exemplification in mathematics teaching (e.g. Antonini et al. 2011; Watson & Mason 2006). Bills & Watson (2008) and Sinclair et al. (2011) argued the need not only for deliberate and careful selection and sequencing of examples, but that

teaching needs to bring learners' attention to connections between selected examples, and to the underlying patterns of variance amidst invariance that enable generalisation and/or appreciation of structure. This linked work on examples and variation in and for mathematics teaching has inspired and influenced our attention to exemplification in our research and PD work.

Zaslavsky's research has focused on teachers' awareness of their example use in their teaching and students' use of examples in their thinking (Ellis et al., 2017). Teachers' were not necessarily aware of how they were using examples. Teachers' thus can be supported in this work. For Kullberg et al. (2017), the application of variation theory in teaching enables critical features of the object of learning to come into focus. They argued further that multiple examples are not simply cumulative. Their ordering, simultaneous presentation, and the teacher drawing attention to similarities and differences are critical. We agree and have thus worked with teachers on their use of examples. In order to bring the object of learning into focus and enable learners to generalise through the use of examples, we have highlighted two necessary aspects for a sequence of examples, and the accumulating example set in a lesson. There needs to be *similarity* from one example to the next - where a feature is kept *invariant* while other features vary. In addition, there need to be *contrasting* examples that can draw attention to difference between key features. With a focus on variance amidst invariance, and contrast, it is possible to build generality and create possibilities for appreciating the critical features and structure of particular mathematical objects.

Of course, examples are always embedded in a task. While examples are selected as particular instances of the general case, tasks are designed to bring particular capabilities to the fore (Marton & Tsui 2004). In our LS work, constructing an example set across a lesson includes attention to tasks, and as we will show this was the collective work of the LS group – teachers and researchers.

The WMCS, framework of mathematics teaching and model of Lesson Study

WMCS has aimed at improving mathematics teaching in disadvantaged schools in one province in South Africa. With a sociocultural orientation to learning as mediated, and to mathematics as a network of scientific, connected and hierarchic concepts (Vygotsky, 1978), we zoomed in on exemplification and explanatory talk in the lesson, viewing these as key mediational means in instruction. We developed an analytic framework, detailed in Adler & Ronda (2015), for describing and interpreting shifts in mathematics made available in teaching. It focuses on four key elements of a mathematics lesson: First is the *object of learning*, the 'what' of learning or lesson goal, and what students are expected to know and be able to do at the end of a lesson. This leads to *exemplification* - to the sequence of examples, with their associated tasks and representations, that can be used to bring the object of learning into focus with learners. As reflected in our discussion on exemplification, attention is on how to work with variance amidst invariance, to build generality and illuminate structure. We also focus on *Explanatory communication,* and *Learner participation* but these are not in focus in this paper.

Between 2013 and 2016 we explored a model of LS in the project. We worked in a school after school hours in small collaborating groups of teachers from clusters of schools. We used the framework above as structuring device to guide lesson planning and reflection, adapting it into a teaching framework referred to in the project as a Mathematical Teaching Framework – MTF, and

illustrated in Figure 1 below. The MTF includes all the components of the analytic framework mentioned above and is intended to assist teachers in planning and then as an observation/reflection tool on the 'quality' of mathematics offered in their teaching in LS. In this paper we focus on exemplification in the MTF framework in our LS work.

Lesson goal		
Exemplification	**Learner Participation**	**Explanatory communication**
Examples, tasks and representations	Doing maths and talking maths	Word use and justifications
Coherence and connections to the lesson goal		

Figure 1 WMCS Mathematics Teaching Framework

Our Lesson Study model

While our cycle follows mainstream practice of LS, differences, particularly in relation to time and place, are a function of resource constraints. Our LS cycle takes place one afternoon a week after school hours for three consecutive weeks, and involves teachers from a cluster of neighbouring schools. In week 1, the LS group plan a one hour lesson on a topic agreed by the teachers. In week 2, one teacher teaches the lesson to a class of learners who agree to remain after school for the lesson. After the learners leave, the LS group spends the following hour reflecting on the enacted lesson and planning the second lesson which is taught in the third week, to a different class, but the same grade. *A Lesson Study cycle thus involves six hours* of face to face collaboration, an evolving lesson plan, and reflection on both taught lessons. In our model, a LS cycle takes place over a relatively short time, outside of normal class teaching.

The study: data and method

While we carried out LS in three clusters of schools, in 2016 we undertook a systematic study of our most participative and sustained LS cluster. A specific question in the study was on the evolution of the example set and in particular:

- What changes occur in the example set across the lesson plans over a cycle?
- How do these changes evolve?

We focus on one LS cycle where four teachers (Thembi, Lerato, Thabi and Sipho) from three different schools in the cluster, and four researchers (Frank, Linda, Jehad and Jill) from the project met in May to plan the first teaching. The teachers chose to work on "simplifying algebraic expressions with brackets in different positions" in Grade 10 because of persistent errors they observed with their learners' manipulation of algebraic expressions when these included brackets in various positions. In order to systematically research our co-learning, we collected all relevant information: written lesson plans; audio and/or video-recordings of all LS sessions. We draw particularly from the lesson plans, post lesson reflective discussions and teacher reflections to engage with our research questions.

Our analysis proceeded with a description of the exemplification in the first lesson plan in terms of the MTF framework, and so our interpretation of variance amidst invariance, and similarity and contrast in the accumulating example set. We identified examples that were removed/added, where this occurred, and how this linked with changing representations and tasks in Plans 2 and 3. Our analysis of the reflective discussions after each lesson included the identification of "example change moments" – moments in the reflective discussion where there was specific attention to exemplification, *and* the discussion led to some change. We described by whom and how the discussion was initiated, and how it evolved.

The lesson plans and evolving example sets

In Table 1 below, we present the example sets for the Introductory and Activity 3 in each lesson plan produced: the initial plan (P1), the revised plan (P2), and then a re-revised plan (P3). The substantial changes are highlighted, and we now use these to illustrate the collective work of the LS group.

P1	P2	P3
Introduction: Calculate the following: a) $4 + 3(4 + 5)=$ b) $(4 + 3)4 + 5 =$ c) $(4 + 3)(4 + 5)$	Introduction: simplify the following expressions: a) $4 + 3(4 + 5)=$ b) $(4 + 3)4 + 5 =$ c) $(4 + 3)(4 + 5)=$	Introduction: simplify the following expressions: a) $4 + 3(4 + 5) =$ b) $(4 + 3)4 + 5 =$ c) $(4 + 3)(4 + 5) =$ d) $(4 + 3) - (4 + 5) =$ Insert brackets in the left side to result in the given sum e) $4 - 3 + 5 = -4$ f) $4 + 3 - 5 = -35$
Activity 3: Simplify a) $(x - 3x) + 5 =$ b) $(x - 3)x + 5 =$ c) $x(-3x + 5) =$ d) $x - (3x + 5)$	Activity 3: insert bracket(s) in the expres on the left side so that the two sides are equal a) $x - 3x + 5 = -3x^2 + 5x$ b) $x - 3x + 5 = -2x - 5$ c) $x - 3x + 5 = x^2 - 3x + 5$	Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal 1. $x - 3x + 5 = -3x^2 + 5x$ 2. $x - 3x + 5 = -2x - 5$ 3. $x - 3x + 5 = x^2 - 3x + 5$

Table 1 Examples in the introductory activity and activity 3 across the three plans

With respect to the examples, the LS group agreed to introduce the lesson with numerical expressions. This representation of expressions with brackets would have meaning for learners and provide a semantic basis for moving on to algebraic expressions, and so more abstract symbolic forms. The numbers 4, 3, 4 and 5 and the + operation in each expression were invariant. Only the position of the brackets changed from one example to the next. The task was to calculate the value of these expressions. Communication with learners was to focus on 'what was the same and what was different' across the examples, thus bringing into focus the impact of the changing position of the brackets. The examples of algebraic expressions that followed in Activity 1 and 2 were similarly structured with x, 3, x and 5 and the + operation invariant while the position of the brackets changed with each subsequent example. Activity 3 included the operation of subtraction/negative terms and if time there was a 4^{th} activity prepared. The accumulating example set reflects the group's interpretation of variation for this lesson where similarity first within each activity example

set and then across these could build generality in relation to the application of the distributive law and the more visible form of the changing position of brackets the expression. Contrast was introduced in Activity 2 where two binomials were now added, and thus potentially bringing attention to the operation between brackets.

The changes made to the examples in P2 and P3

Changes from P1 to P2 can be seen in Activity 3. There are now equations, with the expressions on the left retaining the same numbers and variables as in P1. Brackets now need to be inserted to produce an identity. The task for learners thus changed. Further changes appear from P2 to P3. There are additional examples of numerical expressions in the Introduction. It appears that the change in Activity 3 in P2 led to a change in the Introduction in P3. Across the three plans the accumulating example sets expanded. This was accompanied by a movement between the numerical and algebraic representations of expressions and changes in task demand. How then, did these changes in exemplification come about? What reflective processes provoked and produced these?

How did the change in plans evolve?

We identified key 'example change moments' in the reflective discussion where there was specific attention to exemplification, *and* the ensuing discussion led to a change in the lesson plan. The vignette below, from the reflection after Lesson 1, takes us into the discussion, allowing us to see when the selected change moment occurred, who initiated it, how it was taken up by others and then evolved into a change in the example set.

Vignette - when the exemplification is "not enough"

Thembi taught Lesson 1, and closely followed the joint plan (P1). The hour long reflective discussion began, as was the practice in the WMCS LS, with the teacher herself reflecting on the lesson. After twenty minutes of general reflection, Frank asked the group to reflect on the example sets. It was at this point that Thembi voiced a key concern with her learners' responses to Activity 3.

> Thembi: Remember … activity three … they all answered "yes it will make a difference" … already they picked up from activity one and two that … if the bracket is put in a different place it changes the solution. … But for me … that was not enough …

It was not immediately apparent to others what Thembi meant by "not enough". Some thought she felt that Activity 3 was "too easy". Following further discussion on how to 'change' activity 3, Linda made a suggestion that caught Thembi's attention ("that's brilliant"), and resulted in a change to the example set:

> Linda: I had another suggestion … what if you … gave them the answer and then said: where must I put the brackets to get this answer?
> Thembi: …where you give them x minus three x plus five but then give them the solution and ask them where should we put the brackets? That will be, yes I think that's brilliant. ….

Attention moved to collectively generating the example set that appears in Activity 3, P2 above. Staying with the expressions in b, c, and d in P1, the task now was to "insert brackets on the left expression "so that the two sides are equal". Thabi expressed concern that this would be too much of a "jump" for learners. All agreed that as he was to teach the next lesson, he could give more time to the activity, and then perhaps not complete all of the other activities in the lesson.

This vignette evidences that the moment of explicit attention to examples was initiated by Frank (WMCS), and thus by the project interest supporting teachers to use the MTF to frame and steer reflection. Significantly, it was Thembi (teacher) who responded to this initiative with a teaching/learning concern in her enactment that unsettled her. In this way, Activity 3 came into focus. It took some time for the group to understand Thembi's concern and to offer productive suggestions. Finally, the suggestion from Linda (WMCS) satisfied Thembi, and despite Thabi's concerns, it was accepted, and the example set changed. Interestingly, there were difficulties in Lesson 2, and reflection that again co-produced the changes to the introductory activity highlighted in Table 1. We do not include more detail here due to space limitations. Briefly, discussion focused on how to scaffold the more demanding task, resulting in the suggestion that numerical examples be done first[1].

Discussion

The vignette shows how changes were initiated through "example change moments". Here, as in other instances, the initiation was by a project member/researcher, reflecting the structuring of the reflection by the elements of the MTF framework, in particular by attention to exemplification. However, it was the teachers' engagement with these initiations that directed the discussion in relation to their learners and their teaching/learning concerns. It was possible to trace the substance of these discussions, as we elaborate below. Together this leads us to posit two main themes that emerge from vignette and that illustrate exemplification as an explicit focus in our LS and as powerful for teachers.

Theme 1: Changes are a collective accomplishment

The vignette above illustrates how the initiation and evolution of change to the example set across the lesson, and particularly in the Introduction and Activity 3, was a *collective accomplishment* of both the teachers and the WMCS teacher educators/researchers. While a researcher initiated an explicit focus on the example set, the change was evidently a function of the driving concern of the teacher and the suggestions and additional instructional resources offered by teachers and particularly researchers together. Our evidence here reinforces the emphasis on the role of "knowledgeable others" in other models of LS (cf Takahashi & McDougal, 2014). We suggest that this role is crucial and has significant implications for any attempt to "cascade" the implementation of LS in contexts where experience with professional collaboration on the one hand, and focused lesson engagement on the other, are new. Cascading down, inevitably dilutes such roles. The

[1] A detailed and in-depth report on this research can be found in "A case of Lesson Study in South Africa", Adler & Alshwaikh (in press).

important implication for the practice of LS in 'less resourced' contexts is that the human resources of knowledgeable others should not be side-stepped nor undervalued. This is a learned role. It is not a role that officials in a system can necessarily carry out without being inducted into LS activity over time (cf. Jita et al., 2008).

Theme 2: Changes are a function of the MTF framework and so theoretically informed

The vignette also shows how discussion was shaped by attention being drawn to the MTF framework, and particularly the example set. The example set was initially informed and shaped by principles of variation, as were the changes. The additional examples in the example set kept the numbers and/or terms in all the examples invariant while the signs were changed, or the task was shifted from expansion to insertion of brackets. Here we see the deep inter-relation between theory and practice constituted in LS reflection. It was the teachers' practice-based concerns that influenced the ultimate joint decisions. As in Kullberg's (2017) learning study, the role of variation was a theoretical resource in the LS work.

Concluding remarks

In this paper we have illustrated part of a LS cycle on "simplifying algebraic expressions with brackets in different positions" with Grade 10 learners in the context of a professional development project in South Africa. Structured by a framework developed in the project to support lesson planning and reflection, we described a key change moment in the reflection following the first enactment of the plan where Thembi expressed her concern with the example set in Activity 3; and how this shaped the subsequent lesson plan, its enactment and the reflection following. Through this we have made a case for explicit focus on exemplification and example sets in LS. Examples are critical for mathematics teaching and learning. Deliberate attention to how they are selected and sequenced and embedded in appropriate tasks is important in professional activity like LS. We zoomed in on one LS cycle with one LS cluster, indeed our most sustained and participative cluster. This was to illustrate the possible, which of course then could have broader purchase.

A related concern here could be that our focus on example sets and what they offer for teacher learning might be relevant to SA but not extend to lessons that are more problem based. We suggest (following Wood (2018)) that different solutions or strategies to a set of problems are themselves examples. How and what is variant and invariant in these would shape what is possible to learn. Attention in LS reflection to the different examples of possible solutions would hold similar possibilities and particularly if framed by principles of variation.

Furthermore, changes to the example set, and so exemplification – the additions, deletions, changing representations, and changing task demands – evolved as a function of the collective enterprise of the LS group - teachers and teachers educators or knowledgeable others. The implications for this in LS across contexts are serious. There are cost implications if knowledgeable others are to be included in any expanded form. Without this, however, LS will not function optimally, whatever the context.

Finally, we have pointed to the impact and the value of structured and theoretically informed observation and reflection. Planning, teaching and reflection were structured and shaped by the

WMCS theoretical approach, the MTF tool used with teachers in our LS. Our work thus adds impetus to arguments for theoretically informed LS, and can hopefully stimulate further research along these lines.

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