



HAL
open science

Historical tasks to foster problematization

Bruna Moustapha-Corrêa, Aline Bernardes, Victor Giraldo

► **To cite this version:**

Bruna Moustapha-Corrêa, Aline Bernardes, Victor Giraldo. Historical tasks to foster problematization. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02421885

HAL Id: hal-02421885

<https://hal.science/hal-02421885>

Submitted on 20 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Historical tasks to foster problematization

Bruna Moustapha-Corrêa¹ Aline Bernardes² Victor Giraldo³

¹Federal University of the State of Rio de Janeiro (UNIRIO) and Federal University of Rio de Janeiro (UFRJ), Brazil, bruna.correa@uniriotec.br

²UNIRIO, Brazil, aline.bernardes@uniriotec.br

³UFRJ, victor.giraldo@ufrj.br

In this paper, we present a proposal for problematizing activities organized in task rounds, each one consisting of three stages: a triggering task, a historical task, and reflections on teaching. This model is part of a doctoral research project and it is being tested in a module of a professional master's program for mathematics teachers. We draw from the commognition framework (Sfard, 2008), combining the mathtask model (Biza, Nardi, & Zachariades, 2007) and Kjeldsen & Petersen's approaches (2014). More specifically, we discuss how we used history to design the historical task of the first task round, based on the so-called Aha problems from Ahmes' Papyrus. History of mathematics is used in the problematizing activities to move teachers out of their comfort zone and to legitimate different ways of producing mathematical knowledge. Some episodes are presented only to illustrate our discussion.

Keywords: Commognitive conflict, history in teacher education, mathtask, Ahmes' Papyrus.

Introduction

How do teachers deal with their students' unexpected solutions? We believe that acknowledging and legitimating different kinds of mathematical reasoning in all educational levels is a crucial aspect of teachers' knowledge and professional activity, and, therefore, this discussion cannot be put aside in teacher education programs. With this motivation, we have investigated how teachers deal with students' different solutions for mathematical problems in lower and upper secondary schools.

The experience reported in this paper is part of the doctoral research of the first author, jointly supervised by the other two authors. The aim is to design activities to problematize teachers' understandings of mathematical knowledge production, in particular their own mathematical knowledge production, and how they legitimate (or not) students' mathematical knowledge production. We understand problematizing as considering different alternatives for something that is usually taken for granted, which opens ways to explore questions that are usually not even regarded as relevant. In this paper, we focus on the use of historical problems and contexts to trigger problematization. For this, we consider recent trends of the historiography of mathematics, which builds upon the perspective of social and cultural contexts within which mathematical practices are produced. Our research combines frameworks on the integration between history of mathematics and teaching of mathematics and on teachers' education.

From the historical framework, we draw from Kjeldsen and colleagues' (e.g. Kjeldsen & Petersen, 2014) work, which suggests that the history of mathematics is a source of discourses governed by

different metarules. We believe that history can be used to legitimate unusual practices in specific contexts. From the teacher education framework, we consider the importance of a kind of knowledge which is developed through school practice (Tardif, 2000). For this reason, the participants in our study are experienced teachers, who were taking a professional master's program¹. Our field work took place in a module, which was part of their degree. In the sessions, participants were invited to analyse solutions for mathematical problems from Brazilian school syllabuses, including students' strategies and historical approaches. This was done by combining tasks inspired by Biza et al. (2007) and Kjeldsen and Petersen's history approach (2014). The field work's methodology was designed in the light of commognitive framework (Sfard, 2008), which will also be used in data analysis. The aim of this paper is to discuss the use of history in the design of these tasks. Episodes from the sessions are reported only to illustrate our task design.

Theoretical Background

According to Sfard, learning is understood as participating in a discourse and mathematics is a kind of discourse. For the author, discourse is a "special type of communication [...] distinguishable by their vocabularies, visual mediators, routines, and endorsed narratives" (Sfard, 2008, p. 297). She adds: "In mathematics, endorsed narratives are those that constitute mathematical theories." (idem, p. 298) However, Sfard does not reduce narratives to formal written texts. For her, narratives correspond to a series of spoken or written utterances, whilst routines grasp a set of metarules. These rules are related to the actions of *discursants* and are historically established, changing through the different discourses. Sfard (2008, p. 260) also points out that commognitive conflict

appears when one encounters a discourse incommensurable with one's own – when familiar routines are confronted with other people's alternative ways of implementing the same discursive tasks, grounded in different metarules.

Commognitive conflicts are at the core of our research. Thus, to lead participants to discuss and reflect on different ways that mathematics can be produced, we expose them to different discourses in two ways: using historical sources, and through tasks inspired by *mathtasks* (Biza et al., 2007). Biza et al. (2007) propose *mathtask* as a situation-specific task designed to engage teachers with classroom situations, to which they are invited to discuss attitudes they would take in the actual classroom. These situations are usually hypothetical, yet data grounded and likely to occur in the actual practice. The connection between history of mathematics and the commognitive framework is inspired by Kjeldsen's work. Since history is a source of discourses governed by different metarules (e.g. Kjeldsen & Petersen, 2014), it provides different ways of approaching problems and their solutions.

Arcavi and Isoda (2007, p. 112) highlight the importance of teachers' listening capabilities, that is, "giving careful attention to hearing what students say (and to see what they do), trying to understand it and its possible sources and entailments". This is close to our goal, since we aim to problematize teachers' criteria to legitimate (or not) students' production of mathematical

¹ In Brazil, legal certification to teach at school is granted by an undergraduate degree. So, this master program is a professional development and is also a way to increase public school teachers' income.

knowledge. In our work, history is used to reach this problematization by discussing historical solutions to mathematical problems, which may differ from the usual strategies teachers are familiar with. Thus, the *problematizing activities* we propose are designed to use historical sources that move teachers out of their comfort zones, by revealing consistent practices within specific social contexts.

The Problematizing Activities Design

The *problematizing activities* we propose are organized in *task rounds*, each one consisting of three stages: *triggering task*, *historical task*, and *reflections on teaching*. Our field work was conducted in a module of a professional master’s program for mathematics teachers. Participants were 12 mathematics experienced teachers (who teach students with ages range from 11 to 18) who were taking this module. The module comprised thirteen 3-hour sessions (organized in three task rounds), including students’ assessment. In this paper, we discuss how we used history, in articulation with frameworks on teachers’ education, to design the activities, with a special focus on the historical task. We address the case of one of the three task rounds – based on the so-called *Aha* problems from Ahmes’ Papyrus. Some episodes are presented, in order to illustrate our discussion. It is not an aim of this paper to rigorously analyse this empirical data.

The task rounds’ structure

Each round starts with a *triggering task*, in which participants are given hypothetical student solutions to a mathematical problem. The aim is to bring about the ways they deal with students’ responses, and to which extent they are open to unusual solutions. This is followed by a *historical task*, in which participants are presented to excerpts from original sources. The aim is to shake participants’ comfort zones formed by the kind of solutions they are familiar with, since the historical discourses are usually different from the ones used in the classroom. In the last stage of each round, *reflections on teaching*, participants are again given hypothetical student solutions, which now also draw upon historical solutions.

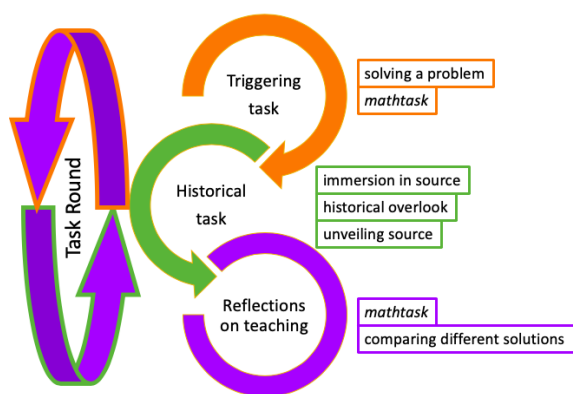


Figure 1: Structure of task rounds

The structure of the *triggering task* and the *reflections on teaching* stages are inspired by Biza et al. (2007) *mathtask* model, namely: firstly, participants are asked to solve a mathematical problem; then they are asked to analyse student solutions to this problem, and to describe the feedback they would give to students in an actual classroom situation. In each of the three stages, participants are

first asked to write down individual responses, and then invited to collective discussion. Either by bringing historical excerpts, or by using *mathtasks*, we try to promote a commognitive conflict (Sfard, 2008). Our goal is to encourage the participants to reflect and discuss on: i) their own *routines* regarding their classroom practices, ii) how they understand mathematics, and also iii) the criteria they apply when judging their students' mathematical solutions and reasonings. For the choice of each task round's topic we take into account: potential to allow different solutions; relevance to the participants' practice; and availability of primary and secondary historical sources.

The first task round: context and structure

Our first task round is built around the *simple false position rule*, according to which the solution is reached by using an “experimental number” that is adjusted by a factor. Thus, the process of finding the solution is not based on a representation for the solution, as in the case in the contemporary algebraic methods. We choose the context of the ancient Egyptian *Aha* problems (e.g. Roque, 2012; Chace et al., 1927) from Ahmes Papyrus to explore different solutions to problems modelled by linear equations nowadays, as a potential trigger of commognitive conflicts. We could consider many metarules regarding the discourse from *Aha* problems (e.g. the ones related to the Egyptian symbols, the ones related to the structure of the *Aha* problems, the ones related to the Egyptian mathematical practices), but we focus on the metarule related to the assumption of an “experimental number” to start the *Aha* problem – which considerably differs from representing the unknown quantity by a letter and operating with it. Thus, we use the simple false position rule as a means of problematizing algebraic solutions commonly used at school. The choice of this activity is also justified by the fact that the false position rule is grounded in the notion of proportionality. Despite this is a key concept in school mathematics, it is sometimes underestimated in the teacher education programs (at least in the Brazilian context). The *triggering task* consists of a *mathtask* with two hypothetical solutions to the following problem:

Marina likes to make problems for her father inspired from what she learns at school. One night she said: “Dad, find out how many Reais² I have! The tip is: if I add a quarter of what I have to what I have, I'll get R\$15,00.”

Ten participants attended this session, all of which presented an algebraic solution, as expected. A *mathtask* with two students' solutions to the problem was then discussed. The *mathtask*'s context was an introductory lecture on algebra at elementary school for 12/13 years old students. The first solution uses a diagram to split the whole in 4 parts, and then add a fourth one. So, 15 is divided by 5, and the result is multiplied by 4. The second solution is a “trial and error” strategy. Aiming to set up commognitive conflicts, both student solutions were designed considering *routines* different from the algebraic solutions. The metarules behind the first solution's routine are related to the use of fraction bars, whilst the ones in the second solution routine are related to the use of a trial and error strategy.

² Brazilian currency.

Designing the historical task: our methodological option

The historical task concerns two well defined moments: an immersion in original sources and a historical overlook. These approach were inspired by Arcavi and Isoda (2007)'s hermeneutic method. The authors propose a sequence of tasks aiming to

parsing the source, posing questions to oneself (or to a peer) around it, paraphrasing parts of the text in our words and notations, summarizing partial understandings, locating and verbalizing what is still to be clarified, and contrasting different pieces for coherence. (Arcavi & Isoda, 2007, p.116)

Thus, immerging in a source is the opportunity to analyse and understand different mathematical practices, contrasting them with the participants' own ones. In the historical overlook, we discuss (mathematical and cultural) contexts of the source and some aspects of the Egyptian mathematics. We adopt this approach as a means to lead participants to experiencing "other perspectives", as Arcavi and Isoda (2007) posit, which, in the context of the history of mathematics, may be the ones from the primary sources, as we used in this research. As these authors suggest, in the context of the classroom, the "other perspective" is the students' ones. So, they argue that trying to understand historical sources may lead teacher to be more receptive to students' productions, specially the unexpected ones.

The aim of the first round was to explore Problem 25 of the Ahmes' Papyrus, which states: "*A quantity whose half is added to it becomes 16.*" Firstly, we invited participants to analyse Problem 25 in Hieratic and Hieroglyphic (Figure 2), as it appears in the Ahmes' Papyrus (Chace et al., 1927). Next, they were presented to Problem 25, as it appears in Figure 3, with translation to English and to (our) Portuguese. After presenting a brief contextualization on Ancient Egypt, the role of the scribes in the society organization, and what Ahmes' Papyrus and, particularly, the *Aha* problems are about, we presented Problem 25 again as it appears in Figure 3, and invited participants to explore it. We then proposed tasks from Arcavi and Isoda's work, including: the elaboration of a dictionary of Egyptian symbols, especially numbers; a problem with Egyptian multiplication; and a set of tasks exploring Problem 24³ (Arcavi & Isoda, 2007) in stages. Finally, we discussed Ancient Egyptian mathematics, especially number system, fractions and operations, using Roque (2012) as the main reference.

³ Problem 24 is quite similar to Problem 25 and states "*A quantity whose seventh is added to it becomes 19.*"

PROBLEM 25



Figure 2: Hieratic and Hieroglyphic solutions of Problem 25

Problem 25

'b'	2 · f	br-f	bpr · f m 16	[Uma quantidade, 1/2 dela adicionada a ela, torna-se 16.]
1	2			
2	1			
3	3			
4	6			
5	12			
6	2			
7	1			
8	5 3			
9	10 3			

br-t my bpr	
The doing as it occurs. [O fazer como ocorre.]	
'b'	10 3
The quantity [A quantidade]	
2	5 3
dmd	16.
Total [Total]	

Figure 3: Modern translation of the Problem 25

We then resumed Problem 24’s solution and encouraged the participants to understand the solution as a whole. As Arcavi and Isoda suggest, we asked them to try to present a solution using a different experimental number. Moreover, we invited them to try to compare the solutions for Problems 24 and 25. The solution was written in modern terms and the expression “false position rule” was presented. Finally, the questions (inspired by Winicki, 2000) were posed to guide the discussion: 1) In modern terms, to which kind of equations do the *Aha* problems correspond? 2) Present a justification for the false position rule. 3) In the false position rule, does the result depend on the experimental number? 4) Does the false position rule work for all kinds of *Aha* problems? Why? 5) Why do you think such a rule was invented? 6) Why is it anachronistic to claim that the Egyptian “solved equations”?

To conclude the historical part, we asked the teachers about simple false-position-rule rationale, that is, to explain why the method is valid. After a brief discussion, we presented a modern justification, using a linear function’s graphic in order to show the proportionality between the experimental number (false position) and the solution. The *reflections on teaching* stage was guided by a *mathtask* presenting a situation in which a 12-year-old boy that doesn’t know algebra yet solves Problem 25 by trial. His father wants to convince him that it is easier to solve the problem through the use of an equation. The boy tells the story to his teacher and asks her why does his dad insist on the equation. The participants were asked to explain how they would answer the boy, if they were his teacher. By the end of the task round, we had 5 different types of solutions for a linear equation: i) algebraic one, ii) using diagrams (fraction bars), iii) trial and error, iv) simple false position, and v) another one by trial presented by the boy (i.e., 3 solutions by trial). We invited the participants to compare these solutions.

During the whole first task round, we tried to promote commognitive conflicts, since the discourse of the historical Egyptian source, the discourse of the solutions presented in the *mathtasks* are modelled by different metarules (Sfard, 2008; Kjeldsen & Petersen, 2014) and are (supposed to be) different from the teachers own’s discourses. This strategy aimed to stimulate the participants to reflect about the way they deal with their students’ different solutions and reasonings.

The first task round: what happened

The first task round took a bit more than two sessions, guided by the first two authors, and with the participation of 12 teachers (four of which attended all of the three meetings). During the sessions, we repeatedly asked the participants about how they deal with different reasonings and solutions brought by their students. Almost all participants have long-term experience in teaching mathematics at public and private schools. The sessions were audio and video recorded. There were separated audio recordings for each small group of 3 or 4 teachers, that discussed each task. The individual written answers to the *mathstasks* and to the historical task were collected. After each session, the participants were asked to write individual reflective diaries, in which they recorded their impressions of the session. Those data will be analysed in a next stage of the doctoral research project.

Impressions from the first activity's round implementation

In general, the participants engaged with the tasks and discussions conducted in the first task round. We could discuss the approaches to equations and algebra teaching they use in classroom. When asked about how they deal with unusual solutions brought by their students, they acknowledge that they recognize them, sometimes sharing them with the rest of the class. However, although we tried to relativize the supremacy of algebra over other approaches, their discourses suggest that their main goal is to develop students' algebraic reasoning and writing. That is, even though they recognized some value on other approaches, they kept pursuing their goal, namely to lead students to understand that algebra makes solutions faster. For example, when we discussed solutions by trial, they repeatedly highlighted that if the numbers were bigger, they would have spent much more time, whilst algebraic solutions take the same amount of time regardless of how big the numbers are. None of them seemed to consider that experimenting with some examples could lead to an understanding of the problem, in such a way that testing all numbers would not be needed. One of the reasons for the teachers keep pursuing the algebra in classrooms might be the important role played by the algebra in the school syllabus in Brazil.

One of the participants, who we will refer to by the pseudonym Ulisses (who were present in all sessions) caught our attention in two different ways. Firstly, he told us that he used Ancient Egyptian multiplication algorithm (which had been discussed in the task round) with his students. He reported that, as his students were facing difficulties with the standard multiplication algorithm, he decided to show the Ancient Egyptian method. The students understood it and claimed that it was easier. We remark that we did not make any suggestions for the participants to apply specific approaches in their classrooms, and Ulisses did so by his own initiative. This kind of attitude is relevant to our work, since this teacher allowed himself to experiment in his own classroom with historical practices discussed during the sessions. During the discussion about Problem 25's solution, Ulisses once more surprised us: he thought of non algebraic solutions. Even though he wrote down the equation, he tried out different values. He noted that $10+5=15$ and $12+6=18$. Looking to $10+5=15$, he noticed that 1 more was necessary, so he split 1 in three pieces $\frac{1}{3}$, $\frac{1}{3}$ and $\frac{1}{3}$, of which two should be with 10 and one with 5 – and he reached the solution.

Arcavi and Isoda (2007, p. 125) posit that

the intention to nurture attentive listening by creating a gap between our method and the Egyptian had a slightly counterproductive effect for some: the symbolic method is so powerful and efficient why bother to even consider a complicated alternative so difficult to understand.

Contrary to this, in our case, the teachers considered all methods – of simple false position and the ones that were used in the *mathtasks* – correct and valid.

Final Remarks

The continuation of our field work included two more task rounds. The second one explores Euclid's approach to areas without numbers (quadratures) and the role of the Pythagorean Theorem's in Euclidian tradition. The aim is to problematize the teaching of areas, which is mainly based on formulas in Brazil. The third task round is designed around the development of the concept of function, and some of its historical definitions, aiming to foster the reflection of image of mathematics as an immutable science.

The episodes reported above indicate that our proposal combining *mathtasks* and historical tasks seems to be a promising way to problematize teachers' understandings of mathematical knowledge production, in particular their students' mathematical knowledge production. When data analysis from our field work is completed, we will have more evidence to deepen the understanding of our proposal's potential. Nevertheless, the episodes from the first task round indicate that at least this group have been receptive to the use of the history of mathematics. We highlight that Ulisses spontaneously tried the Egyptian multiplication out in one of his classrooms. This indicates that he problematized, at least to some extent, not only his own and his students' mathematical knowledge productions, but also his teaching practices.

Acknowledgments

We acknowledge CAPES for the scholarship for a doctoral internship at Rutgers University under the supervision of Prof. Arthur Powell. We are grateful to Prof Arthur Powell for the partnership, especially in the design of the field work methodology and to Prof. Elena Nardi and Dr. Irene Biza for the opportunity of working together and exchanging experience through the MathTASK project.

References

- Arcavi, A., & Isoda, M. (2007). Learning to listen: From historical sources to classroom practice [Special Issue]. *Educational Studies in Mathematics*, 66, 111–129. doi: 10.1007/s10649-006-9075-8.
- Biza, I., Nardi, E., & Zachariades, T. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education*, 10(4–6), 301-309.
- Imhausen, A. (2007). Egyptian Mathematics. In V. Katz (Ed.), *The Mathematics Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* (pp. 7–56). New Jersey: Princeton University Press.
- Chace, A.B., Manning, H.P., & Archibald, R.C. (1927). *The Rhind Mathematical Papyrus: British Museum 10057 and 10058* (Vol. 1). Oberlin, OH: Mathematical Association of America.

- Chace, A.B., Bull, L., Manning, H.P., & Archibald, R.C. (1929). *The Rhind Mathematical Papyrus: British Museum 10057 and 10058* (Vol. 2). Oberlin, OH: Mathematical Association of America.
- Kjeldsen, T.H., & Petersen, P. H. (2014). Bridging history of the concept of a function with learning of mathematics: Students' metadiscursive rules, concept formation and historical awareness. *Science & Education*, 23, 29–45.
- Miatelo, L. (2008). The difference $5\frac{1}{2}$ in a problem of rations from the Thind Mathematical Papyrus. *Historia Mathematica*, 35 (4), 277–284.
- Roque, T. (2012). *História da Matemática: Uma visão crítica, desfazendo mitos e lendas*. Rio de Janeiro: Zahar.
- Sfard, A. (2008). *Thinking as Communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.
- Tardif, M. (2000). Saberes profissionais dos professores e conhecimentos universitários: elementos para uma epistemologia da prática profissional dos professores e suas consequências em relação à formação para o magistério. *Revista Brasileira de Educação*, 13. Retrieved from http://www.anped.org.br/sites/default/files/rbe/files/rbe_13.pdf.
- Winicki G. (2000). The Abalysis of Regula Falsi as an Instance for Professional Development of Elementary School Teachers. In V. Katz (Ed.), *Using history to teach mathematics—an international perspective*, MAA notes (Vol. 51, pp. 129–133). Washington, DC: MAA.