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On the historical development of algorithms - hidden in technical devices?

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The content of Adam Ries' book (1574) with the instructions to written arithmetic fostered an enlightening for citizens at this time since they learned how to calculate themselves making it obsolete to pay a "Rechenmeister" for this task. In the early modern times algorithms entered the industrial world by mechanical automatisms intended to take away the routines from people. Nowadays algorithms hidden in many technical devices have become a part of our daily procedures often staying unaware for people. The paper discusses the historical development of the concept of algorithm and its potential for mathematics and general education today.

Keywords: Algorithms, enlightening, fundamental ideas, mathematics education

A historical view on the mathematical aspects of algorithms

The history of algorithms as part of mathematics started in the very early days since the systematic solving of mathematical problems is part of doing mathematics. For example, during the Old Greek times Euclid (300 v. Chr.) gave a rule to calculate the greatest common divisor of two given natural numbers. Another well-known algorithm is the sieve of Eratosthenes, a step-by-step procedure giving prime numbers up to a certain given natural number. Several hundred years later it was Al-Khwarizmi who presented a number of mathematical applications for traders. These were translated into Latin with the title "Algorithmi" containing the artificial word "arithmos" that is derived from "number" and the name of the mathematician (cf. Chabert, 1999).

Hundreds of years later Adam Ries (1574) wrote a booklet on basic arithmetic operations which had great influence on the German citizenship at that time. In the year 1614 the Scottish John Napier published a book on logarithms which made him look like the inventor of logarithms. Leonardo da Vinci (1452-1519) as another significant personality of the Renaissance era demonstrated with the "Vitruvian Man" his understanding of proportions blending geometric descriptions with the Renaissance art. Another hundred years later Leibniz (1646-1716) used the binary code and algorithms on that basis.

In 1815 Augusta Ada Lovelace was the first woman to program for the (technologically underdeveloped) calculator of Charles Babbage (1791-1871). 1931 Gödel destroyed the dream of many mathematicians that all mathematical theorems can be proved by algorithms by his Incompleteness Theorems:

They imply that there is no algorithm with a finite number of steps that can decide if an arithmetic statement is true or false in all cases imaginable.

Furthermore, it is impossible to prove consistency for a formal system whose complexity is the same as that of the natural numbers.

In this context we also must accept an incompleteness of arithmetic:

There is no (finite) algorithm that proves the valid statements of arithmetic on the whole. (Ziegenbalg, 2010; translated by the authors).

Later on during the 20th century von Neumann and Turing provided the mathematical basis for computing with which they started the new era of computer science (Chabert, 1999, p. 457).

During the last decades the development of “artificial intelligence” progressed so rapidly that the nature of algorithms became in immense multiple ways complex (e.g. voice recognition, decision making systems). This development was possible because of rapidly advanced technical conditions which as a consequence offered an improvement in calculating speed and storage space. As an organizational form cloud computing brought enhancements.

Concluding this very short historical view on the successive appearances of algorithms through the centuries it becomes apparent that in former days they were documented in a written form by books with the aim of becoming public to scientists and citizens. Even in the case of military purposes, namely cryptography, the secrecy of the underlying algorithm was not considered appropriate. Only the "key" that gives details of a general – even public – encryption process had to be hidden, which is the subject of Kerckhoff's principle (Kerckhoff, 1883, p. 12). Nowadays the algorithms behind apps and programs often stay hidden because they are part of a business secret often with the idea to gain money. The question arises when exactly the balance turned from transparency to its opposite and how we can deal with this new situation. One can expect insight from sociological and historical analyses that are not yet been finally concluded.

Today's mathematical exactification and its impact on education

During centuries an algorithm was understood as a series of instructions. The development algorithm's understanding led to distinguished properties.

Steps towards formalization

For the longest algorithms consist of a finite series of instructions to solve a given problem like it was apparent with the sieve of Eratosthenes. Generally accepted are also the following properties which led to a general view and that can be understood as postulates for algorithms (Saake et al., 2010):

- (1) The algorithmic procedure must end up with the same outcome in case of the same preliminaries.
- (2) Any next step to take is uniquely determined.
- (3) The description of an algorithm implies a finite length of the source text which needs to consist of a limited number of signs.
- (4) An algorithm is requested to have a limited storage space at any time during its procedure.
- (5) An algorithm stops after a finite number of steps.

For didactical purposes it is therefore clear what solving procedures fall under the concept of an algorithm. Like with the concept of sets there is a naïve version of it. And after being confronted with logical difficulties (Russell, 1903) the need of exactification became apparent and was solved in form of axiomatization. The naïve perception of algorithms has its limits, exactified by the work of Turing and especially of Gödel's Incompleteness Theorems. Both topics, sets and algorithms, finally lead to questions of cardinality and self-reference.

Didactical perspectives for math classes

The idea of an algorithm appears several times in school in mathematics classes. On the elementary level the four basic written arithmetic calculations have algorithmic nature and are named as such in mathematics literature. In our daily practice we do not observe a very detailed reflective attitude; neither teachers nor pupils seem to reflect on their algorithmic nature. At the beginning of the secondary I level the well-known Euclidean algorithm is subject matter in class. Very often this is the first time that pupils get aware of the concept of an algorithm. It is regarded as a prototype of an algorithm since it is named like it.

Following the spiral principle other examples of algorithms occur on the secondary I level (in the German system meaning classes 5 – 10: the methods used to solve linear and quadratic equations as well as systems of linear equations. Typical examples on the secondary II level occur with curve sketching, finding extrema, points of inflection and other properties of functions. At this stage the systems of linear equations are again subject matter with an even more algorithmic procedure (Gauß algorithm).

In school algorithms also occur using a calculator or tablets including software for educational purposes. That is for example the dynamic geometry software (DGS; GeoGebra, Cinderella) and Computer Algebra systems (CAS). Nowadays Witzke (2018) uses 3D-Plotters in math classes on different levels to not only visualize mathematical objects but to materialize graphs and geometric solids. This means that algorithms generate didactical tools not only iconical, but even haptical.

Comparing all the mentioned school examples above very few are labeled explicitly as algorithms through the names as the Euclidean and the Gauß algorithm and are therefore more obvious to the pupils. The algorithmic character of other parts of the curriculum is also not easily recognized by the pupils. Even with algorithms whose names prove to be programmatic the teaching normally emphasizes the arithmetic acting and often fails to reflect upon the concept and the meaning of an algorithm. Furthermore, the use of the recently introduced tools in mathematics classes serve didactically as technical devices to manage some type of mathematical problems. However, several of them create some kind of didactical bubble. Some calculators for the secondary level are meaningful only for math classes.

Algorithms as fundamental ideas – alibi or truth?

In 1905 Felix Klein (1849-1925) claimed functional thinking as a guiding principle for mathematics education which was implemented in the German curriculum through the last decades. Already in the seventies of the last century the importance of algorithmic thinking was claimed (Engel, 1977, p. 5). This was expressed in the light of the situation 40 years ago. Nowadays, there should be no

doubt that the situation became much more prominent. The fact that the relevance of algorithms is still not in the center of educational issues is apparent with the “German Bildungsstandards” (Ständige Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland 2004, 2012): On the primary level algorithms are not mentioned at all and on the secondary I and II levels some of the well-known procedures are classified as algorithms but mostly because of their repetitive character and not in terms of their entire meaning.

We observe a simplified and shortened understanding of the notion on the part of the pupils and teachers as well, in particular relating to its potential in science, society, economy and culture in general. The latter is only recently a subject of scientific research. In the last decades algorithms and their different mechanical realizations have infiltrated our whole life in various and subtle ways. They are omnipresent in every angle of our daily routines. Social networks (like Facebook) and search engines (like Google or Yahoo) are selecting, ordering and evaluating our data, our inputs and their results. Especially the selecting part based on evaluation is a sensitive issue that the companies and institutions have not made transparent, let alone to make an enlightening out of it. On the contrary, many companies admit that they hide the conditions of their procedures, and one often gets the impression of a conscious deception of the public. Advertisements are individualized simulating or pretending that some kind of care plays a vital role. However the companies gain money by using or selling our data and neutrality and objectivity became illusions.

The role of algorithms for general (math) education

Since our daily life is influenced by algorithms in the ways described above the question arises whether mathematics education can play an enlightening role. The aim here is not only the listing of typical, historically well-known examples, but the task is to elucidate about the significance of the impact on all facets of society. Heymann (2013) emphasized general education in the light of fundamental mathematical ideas. He aimed at showing that the fundamental ideas indicate the universality of mathematics and their relevance for the “entire culture” (Heymann, 2013, p. 158) and this needs to become apparent for the pupils during their school life.

Here we follow the structure of Heymann’s frame of reference. The criteria include their relevance for the mathematical development and show the essential nature of mathematics:

- (1) Preparation of life;
- (2) cultural coherence;
- (3) global orientation;
- (4) critical use of reason;
- (5) sense of responsibility;
- (6) communication and cooperation.

There can be no doubt that algorithms affect our daily life in an increasing manner. Therefore, school and especially math classes need to prepare for this fact. Although it is apparent that different countries answer with different attitudes: Europe, primarily the European Union, looks for governmental regulations while the USA care less about rules. The Far East, that is predominantly

China, responds to the situation with even stronger governmental rules. In this light cultural coherence as an aim seems to be out-of-date, and, at the same time, the developed global view is an inevitable consequence of it. Hence the critical use of reason becomes more urgent than ever before. The sense of responsibility needs to be revised and also discussed and reflected in school. And this even more since the situation became much more complex and it is not easily apparent what is for the benefit of humanity and what will harm. Of course communication and cooperation stay important in this light.

Towards an implementation of algorithms in math classes

Both concepts, the set and the algorithm, are simple enough to be presented on the primary level: a set as a Venn diagram (as understood within naïve set theory) and an algorithm as a series of instructions as in a cooking recipe. Therefore both can be represented on the enactive level (Bruner 1974) and are accessible for the iconic one. Their difference lies in the fact that sets can be regarded foremost as a static phenomenon simultaneously due to time, while temporal processing is in the nature of algorithms. In this sense they are complementary. On top of it the concept of sets plays a vital role in the succession of math classes (spiral principle) as the notion of algorithms could and should also (Bruner, 1976).

Closely connected is a theory of conceptual progression of mathematical concepts (Vollrath, 1987). Vollrath presents a didactical theory of learning and teaching mathematical terms. He outlines in general what kind of various steps lead to a certain understanding of mathematical notions:

- (1) Intuitive level;
- (2) content level;
- (3) integrated level;
- (4) formal level.

On the intuitive level the pupils have an idea of a series of steps as a first access to algorithms. This may be represented by game rules or cooking recipes. On the second level they explore typical properties of algorithms. Actually on the third level a network should be recognized on the part of the pupils. In the case of an algorithm we observe that the notion itself provides a conceptual network. We distinguish two levels of formulation: Firstly, there is an exact description of certain mathematical issues through algorithms represented by flow charts or simple programming languages. Secondly, the notion of the concept of an algorithm itself is to be formulized.

It is apparent that the exactification makes the idea of an algorithm accessible for pupils already during early schooling. It became apparent that the idea of an algorithm is rich enough to be taught on all levels with a shifting emphasis. Therefore mathematics education is obliged to prove again its self-established claim for general education. This needs to be done on the conditions of the presence and how they have developed.

Analysis on consequences for the teaching and learning of algorithms

In early historical times algorithms started as a scientific insight on a mathematical problem and additionally gave information for a systemized solving procedure. Paul Ernest (2004, p. 5) describes it like this:

An example is provided by algorithms. These denote precisely specified sequences of actions, procedures which are as concrete as the terms they operate on. They establish connections between the objects they operate on, and their products. They are a part of the rich structure that interconnects, and thus helps to implicitly define, the terms, and hence the objects of mathematics. This must be relativized to the solver, for what is routine for one person may require a novel approach from another. It is also relative to a mathematics curriculum, which specifies a set of routines and algorithms.

At the very end of the middle ages when Adam Ries, standing at the brink of the abacus using concrete tokens and the decimal number system using symbols on paper, the arithmetic operations in form of a booklet had an enlightening impact on the citizens; an enlightening event that is surely not often recognized in history (Deschauer, 1992). It is an example of a practice through all these times when it was mostly the aim to inform, to get insight and to support people's solution practices.

In order to simplify the solving practices and to avoid mistakes, Babbage, for instance, designed a mechanical machine (1823). And, as soon as machines came into place, the questions also involved avoiding counting errors. Later on the development continued with electronic devices aiming at more and better efficiency which first was the storage place and later the shortening of time.

We observe that the chances of an enlightening process in today's society got at least diminished as soon as commercial ambitions entered the global scene and the hiding of information was part of the business practices. And now our so called informational society is confronted with many different "black boxes".

Here, mathematics and its didactics have a long enough history to feel responsible to undertake the task to clarify the situations and to offer enlightening insights. A view into history confirms once again that we do not observe totally new phenomena. Babbage dealt with a similar situation during the early 19th century. In his "Reflections on the Decline of Science in England", first published in 1830 (Babbage, 2013), he complained about scientific mishaps: He observed that scientific results were whitewashed: by trimming (levelling of irregularities), cooking (quoting of results which fit to a theory, omitting results that contradict) and forging (inventing of scientific results which fit a conventional meaning or can be fitting to a desired doctrine).

Concluding our observations on the development of algorithms and our analyses on their position today we notice that they bear only minor enlightening character anymore compared to former times. In order to answer to this situation there needs to be a focus in math classes on prototypes of today's computers. These elementary simplifications could make apparent that the programmed algorithms are combined with money-making commercial ambitions which have to do with

evaluating, selecting and, as a result, manipulating. This goes along with Freudenthal's idea of a guided reinvention within his concept of "mathematics as a human activity" (Freudenthal, 1983).

We do not favour teaching algorithms as it developed chronologically. This might be an interesting experiment, but would be naïve to push the analogies between ontogenesis and history of science to the limit. Instead of it we support a refinement of historical awareness and instruct students to pay attention to the conditions that caused today's appearance of algorithms – one of several possible implementations of a few fundamental ideas of mankind. This is what Paul Ernest (2015) calls the "postmodern perspective". According to Lyotard (1984), postmodernism can be narrowed to scepticism towards meta-narratives. Since different fields focus on and deal with unlike sometimes divergent ideas and phenomena, their historically developed metanarratives differ enormously among each other. Therefore analysing (deconstructing) metanarratives means to retrace historical developments and the decisions made on the way. This endeavour reveals choices which were taken along the historical evolution within each field.

References

- Babbage, Charles (2013). *Reflections on the Decline of Science in England*. Cambridge: Cambridge University Press.
- Russell, Bertrand (1903). *The principles of Mathematics*. Cambridge 1903. (Chapter X online: <http://fair-use.org/bertrand-russell/the-principles-of-mathematics/s106>)
- Bruner, Jerome (1974). *Entwurf einer Unterrichtstheorie (Toward a theory of instruction)*. Berlin: Berlin Verlag.
- Bruner, Jerome (1976). *The process of education*. Harvard: Harvard University Press.
- Chabert, Jean-Luc (1999). Algorithms for arithmetic operations. In Chabert, Jean-Luc (Ed.) (1999): *A history of algorithms. From the pebble to the microchip*. Berlin, Heidelberg: Springer, 7-48.
- Deschauer, Stefan (1992). *Das zweite Rechenbuch von Adam Ries*. Braunschweig, Wiesbaden: Vieweg.
- Engel, Arthur (1977). *Elementarmathematik vom algorithmischen Standpunkt*. Stuttgart: Klett.
- Ernest, Paul (2004). *The Philosophy of Mathematics Education*. London: Taylor & Francis.
- Ernest, Paul (2015). Postmodern Mathematics. In *The Proceedings of the 12th International Congress on Mathematical Education*. Heidelberg, New York, Dordrecht, London: Springer, 605-608.
- Freudenthal, Hans (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: D. Reidel Publishing Company.
- Heymann, Hans Werner (2013). *Allgemeinbildung und Mathematik*. Weinheim, Basel: Belz.
- Kerckhoff, Auguste (1883): *La cryptographie militaire*. In *Journal des sciences militaires*, Volume 9, 5-38.
- Lyotard, Jean-François (1984). *The postmodern condition: A report on knowledge*. Manchester: Manchester University Press (French original published by Les Editions de Minuit, 1979).
- Napier, John (1614). *Mirifici logarithmorum canonis descriptio ejusque usus in utraque trigonometria etc*. Edinburgh 1614. (Online: <https://archive.org/details/mirificilogarit00napi>)

- Roberge, Jonathan & Seyfert, Robert (2017). Was sind Algorithuskulturen? In Seyfert, Robert & Roberge, Jonathan (Eds.), *Algorithuskulturen. Über die rechnerische Konstruktion der Wirklichkeit* (pp. 7-40). Bielefeld: Transcript Verlag.
- Saake, Gunter & Sattler, Kai-Uwe (2010). Algorithmen und Datenstrukturen. Heidelberg: dpunkt.verlag.
- Ständige Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland (Ed.) (2004). Beschlüsse der Kultusministerkonferenz: Bildungsstandards im Fach Mathematik für den Mittleren Bildungsabschluss. Beschluss vom 04.12.2003.
https://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/2003/2003_12_04-Bildungsstandards-Mathe-Mittleren-SA.pdf
- Ständige Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland (Ed.) (2004). Beschlüsse der Kultusministerkonferenz: Bildungsstandards im Fach Mathematik für den Primarbereich. Beschluss vom 15.10.2004.
https://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/2004/2004_10_15-Bildungsstandards-Mathe-Primar.pdf
- Ständige Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland (Ed.) (2012). Beschlüsse der Kultusministerkonferenz: Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife. Beschluss vom 18.10.2012.
https://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/2012/2012_10_18-Bildungsstandards-Mathe-Abi.pdf
- Vollrath, Hans-Joachim (1987). Begriffsbildung als schöpferisches Tun im Mathematikunterricht. In *Zentralblatt für Didaktik der Mathematik* 19, 123-127.
- Winter, Heinrich (1996). Mathematikunterricht und Allgemeinbildung. In *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, 61, 37-46.
- Witzke, Ingo & Hoffart, Eva (2018). 3D-Drucker: Eine Idee für den Mathematikunterricht? Mathematikdidaktische Perspektiven auf ein neues Medium für den Unterricht (Preprint).
- Ziegenbalg, Jochen & Ziegenbalg, Oliver & Ziegenbalg, Bernd (2010). Algorithmen von Hamurapi bis Gödel. Wiesbaden: Springer Spektrum.