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The use of *problem* in upper-primary and lower-secondary textbooks of the Republic of Cyprus

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Research suggests that transitions from primary to secondary education typically have negative influences on pupils' mathematical performance, motivation, and self-efficacy. In acknowledgement of these issues, I examine how the term 'problem' is used in the upper-primary and lower-secondary national maths textbooks in the Republic of Cyprus, in an attempt to explore the coherence between the instructional materials pupils are exposed to in the two school levels. More specifically, all tasks explicitly labelled as 'problems' in the two textbook series were identified. For analysing the tasks, an adapted version of Borasi's (1986) framework about the structural characteristics of problems was utilised. Findings conclude that coherence between the two textbook series is limited, as in the primary series the term 'problem' is extensively used in a particular way, while in the secondary series, a significant decline of the number of tasks labelled as problems is observed.

Keywords: Transitions, mathematical problem, textbooks, primary, secondary

Transitioning from primary to secondary school

Many pupils preparing to transition from primary to secondary school appear to have predetermined ideas about the challenges and difficulties of mathematics at the next school level (Attard, 2010). As a consequence, there is a general decline of pupils' engagement with mathematics as they move from primary to secondary education (Martin, Way, Bobis, & Anderson, 2015), a decline of their self-efficacy beliefs, motivation, and performance (Bouffard, Boileau, & Vezeau, 2001), and the reinforcement of stereotypes regarding gender and mathematics performance (Denner, Laursen, Dickson, & Hartl, 2018). While research evidence suggests that the observed differences could partly be attributed to teachers' self-efficacy beliefs (Midgley, Feldlaufer, & Eccles, 1989), as well as to teachers' and parents' emphases on goal (Friedel, Cortina, Turner, Midgley, 2010) across the two school levels, little is known about the impact of the use of instructional materials by primary and secondary mathematics teachers (Howard, Perry, & Tracey, 1997), and more specifically, how similarly or differently textbooks are used at these two school levels (Fan, Zhu, & Miao, 2013).

Here, I focus on the national textbooks of a highly centralised educational system, that of the Republic of Cyprus. The mathematics curriculum of Cyprus (as with all school subjects) is prescribed centrally by the Ministry of Education and Culture - MoEC (Mullis, Martin, Goh, & Cotter, 2016). In order to bridge the gap between the previous primary and secondary mathematics curriculum, the curriculum in effect, introduced in 2010, is built around the same five general topics, from pre-primary education up to the last year of upper-secondary education (MoEC, 2010). These five topics are numbers, algebra, geometry, measurement, and statistics-probability. Furthermore, this unified mathematics curriculum claims to be built on four principles, with principle 2 stating that the curriculum places emphasis on problem solving (MoEC, 2010). Yet, no

further clarifications are provided as to what a problem is, what problem solving means, and what kind of related skills are desirable.

Both in primary and secondary education, the vast majority of mathematics teachers' instruction methodology depends on the respective national textbooks, prepared by the MoEC (Xenofontos, 2014; Xenofontos & Papadopoulos, 2015). Although I do not, in this study, examine textbook use by teachers, I do explore how the concept of *problem* is presented in the upper-primary (grade 6) and lower-secondary (grade 7) textbooks, which were introduced after the initiation of the reform in 2010. Such an approach sees textbooks as a *potentially implemented curriculum* (Schmidt, McKnight, & Raizen, 1997), or, as Mesa (2004, p. 255–256) puts it, “a hypothetical enterprise: What would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?”

Why *problem*?

In the mathematics education literature, the term *problem* is one of the most widely used; yet, there does not seem to be an agreement as to what it means. In general, my views coincide with Schoenfeld's (1985, p. 74) statement that “being a ‘problem’ is not a property inherent in a mathematical task. It is a particular relationship between the individual and the task that makes it a problem for that person”. For the purposes of this study, however, I do not want to use any particular definition of the term, as my goal is to examine how *problem* is conceptualised and formed in the national textbooks of Cyprus. Such a deliberate choice may serve as a proxy to how problems and problem solving are conceptualised at the level of policy-making, that is, the intended curriculum (Mullis et al., 2016; Schmidt et al., 1997). Also, I acknowledge that a number of comparative studies have concluded that *problem* and *problem-solving* are perceived differently across educational systems (Cai, 1995; Xenofontos & Andrews, 2014).

In attempts to analyse the types of problems found in mathematics textbooks, various colleagues have used well-defined frameworks. For instance, in their work with two widely used textbook series in Singapore, Fan and Zhu (2000) distinguish between routine and non-routine problems, with various other categories falling under the latter (namely problem-posing problem, puzzle problem, project, journal task). In subsequent work, Fan and Zhu (2007) identified various similarities and differences in the promotion of problem-solving strategies in Chinese, Singaporean, and US textbooks, again, using a very structured, predetermined framework related to how the researchers understood the concepts of problem and problem-solving. In the same spirit, Xin (2007) examined the distribution of seven types of word problems (namely: multiplicative comparison-compared, multiplicative comparison-referent, multiplicative comparison-scalar, rate times a quantity, fair share or measurement division, and proportion problem type) in Chinese and US middle school textbooks, noting a more balanced distribution in the former than the latter. In turn, Son & Kim (2015) investigated how teachers select problems from textbooks and present them in class; however, the researchers “use problems and tasks interchangeably” (p. 493), meaning that they did not utilise a specific definition of what a *problem* might be.

Methodology

This study is based on the following research question:

How similar/different are the characteristics of the tasks explicitly labelled as problems in the upper-primary and the lower-secondary national textbooks of Cyprus?

This question is of particular significance, mainly because in its rhetoric the MoEC declares that the new unified curriculum aspires to promote a smooth transition from one school level to another (MoEC, 2010).

Instead of utilising a predetermined definition of the term *problem*, I chose to focus only on those tasks explicitly labelled as problems in the two textbook series. By *explicitly*, I refer to a presentation of the term ‘problem’ in the instructions provided, i.e. “solve the following problems”. Nonetheless, to enable comparisons to be made, I used an adapted framework based on Borasi’s (1986) ideas about the structural characteristics of problems. In her work, Borasi acknowledges the difficulties in deciding whether a specific task *is* a problem or not, the same way Schoenfeld (1985) recognises the role of the individual solver in the labeling of a task as a problem. Borasi, therefore, proposes four structural characteristics of tasks, which are independent of the solver. The first is the *formulation*, and refers to how instructions about what needs to be done are presented. These can take three forms: a question, a statement, or no presentation of instructions at all (open for the solver to decide). The second is the *context* in which the task is presented, and can take two forms: purely mathematical or applied (Blum & Niss, 1991). The third is the *set of the acceptable solutions*. There may be no acceptable solution to a problem, one and only solution, or more than one. Finally, the fourth characteristic has to do with the *methods of approach*. In respect to this, Polya (1981) classified problems in four categories: *one rule under your nose* (when the problem can be solved by simply applying an algorithm just presented), *application with some choice* (when the suitable algorithm must be selected among others previously studied), *choice of a combination* (when in order to reach the solution some of the algorithms previously learnt must be suitably combined), and *approaching research level* (when the elaboration of a new algorithm is required or when the task cannot be solved algorithmically). The term ‘approaching research level’ is used by Polya himself to refer to tasks that are genuinely problematic, and perhaps is closer to Schoenfeld’s (1985) views about what a problem is.

As explained earlier, this paper focuses on the tasks explicitly labelled as problems in the upper-primary (grade 6) and lower-secondary (grade 7) national mathematics textbooks of Cyprus. Each task was described in terms of the four structural characteristics (formulation, context, set of acceptable solutions, methods of approach). For grade 6, the textbooks are organised in six parts, while for grade 7 they are organised in two parts.

Findings

In grade 6 textbooks, 228 tasks were identified as being explicitly labeled as *problems*, while in grade 7 textbooks, only two tasks were identified. Although the total number of tasks (labelled as problems or not) in the two series was not counted, a quick scan through the textbooks shows that in each grade (6 and 7) there are more than 1000 tasks.

Below, figure 1 presents an example from grade 6, and how it was coded with the use of Borasi’s (1986) four structural characteristics. Figure 2 demonstrates another example from the textbooks of grade 6, which, contrary to the algorithmic task of figure 1, was identified as approaching research

level, as there is no standard algorithm that could be applied for its resolution. In figure 3, one of the two tasks identified in grade 7 textbooks is presented. In fact, both tasks were coded under “approaching research level”, due to their high complexity and lack of any particular algorithm to be applied.

Table 1 illustrates the distribution of these tasks for each form of a structural characteristic.

The Grade 6 pupils are $\frac{1}{6}$ of the total pupil population in a school. There are 258 pupils in total. Find how many pupils are in Grade 6.

(α) Οι μαθητές της ΣΤ΄ τάξης αποτελούν το $\frac{1}{6}$ των μαθητών ενός σχολείου. Όλοι οι μαθητές του σχολείου είναι 258. Να υπολογίσετε τον αριθμό των μαθητών της ΣΤ΄ τάξης.

formulation: statement
 context: applied
 acceptable solutions: one
 methods of approach: one rule under your nose

Figure 1: An example of a task (grade 6, part B, page 89)

24. Να επιλύσετε τα προβλήματα.

(α) Η Μαργαρίτα ξεκίνησε να περπατά από το σημείο Α. Περπάτησε $\frac{1}{2}$ km νότια, μετά $\frac{3}{4}$ km ανατολικά, μετά $\frac{2}{5}$ km βόρεια και στο τέλος $\frac{3}{4}$ km δυτικά και έφτασε στο σημείο Β. Να βρείτε πόσα χιλιόμετρα απέχει (σε ευθεία γραμμή) το σημείο Α από το σημείο Β.

formulation: statement
 context: applied
 acceptable solutions: one
 methods of approach: approaching research level

Figure 2: An example of a task (grade 6, part B, page 106)

In both grades’ textbooks, certain tasks were identified, in which the term problem appeared in the instructions. However, these were classified as problem-posing tasks, as they invited pupils to write a problem that would meet particular criteria (i.e. ‘translate’ a symbolic expression/representation into a real world situation). In the textbooks of grade 6, eleven such tasks were identified, while in the textbooks of grade 7, there were four tasks of this type. Below, figures 4 and 5 demonstrate an example from the primary and secondary textbooks, respectively.

One of the problems in number theory says:

11. Ένα από τα προβλήματα της θεωρίας αριθμών λέει:
Μετά από πολλά χρόνια συναντούνται δύο συμμαθητές, ο Πυθαγόρας και η Υπατία, που είχαν ιδιαίτερη αγάπη στα μαθηματικά και διεξάγεται ο πιο κάτω διάλογος:

Π: Παντρεμένος. Έχεις παιδιά; Πόσες; Ποιας ηλικίας;
Υ: Ναι, τρία και το γινόμενο των ηλικιών τους είναι 36.
Π: (ίστερα από σκέψη) Δεν μπορώ να βρω τις ηλικίες τους, τα στοιχεία που μου έδωσες δεν είναι αρκετά.
Υ: Σωστά, και αν σου πω ότι το άθροισμα των ηλικιών των παιδιών μου είναι το ίδιο με τον αριθμό του σπιτιού σου.
Π: Ούτε και τώρα μπορώ να βρω τις τρεις ηλικίες. Θέλω και άλλη βοήθεια...
Υ: Σωστά! Και αν σου πω πως το μεγαλύτερο παιδί έχει ξανθά μαλλιά...
Π: Τώρα ναι! Μπορώ να σου πω τις ηλικίες των παιδιών σου, χωρίς καμιά αμφιβολία!

Ποιες είναι οι ηλικίες των παιδιών και τι σκέψεις έκανε ο Πυθαγόρας, για να φτάσει στην απάντηση;

After many years, two old friends, Pythagoras and Hypatia, met. They both loved mathematics. Below, is presented the discussion between them:

P: Are you married? Do you have kids? How many? How old are they?

H: Yes, I have three, and the product of their ages is 36.

P: (after some thought) I can't find their ages, the information you gave me isn't sufficient.

H: You're right, but what if I tell you that the sum of their ages is the same as the number of your house?

P: I still can't find the age of each. I need another clue...

H: You're right! What if I tell you that the eldest has blond hair?

P: Oh, yes! I can tell you the ages of your kids, without a doubt!

formulation: question
context: applied
acceptable solutions: one
methods of approach: approaching research level

What are the children's ages and what thoughts did Pythagoras have to reach the answer?

Figure 3: An example of a task (grade 7, part A, page 94)

Structural characteristic	Forms of each structural characteristic	Upper-primary (grade 6)	Lower-secondary (grade7)
<i>Formulation</i>	Question	166	2
	Statement	62	0
	Open	0	0
<i>Context</i>	Purely mathematical	5	0
	Applied	223	2
<i>Acceptable solutions</i>	None	0	0
	One	228	1
	More than one	0	1
<i>Methods of approach</i>	One rule under your nose	110	0
	Application with some choice	37	0
	Choice of a combination	73	0
	Approaching research level	8	2

Table 1: The identified tasks and their structural characteristics

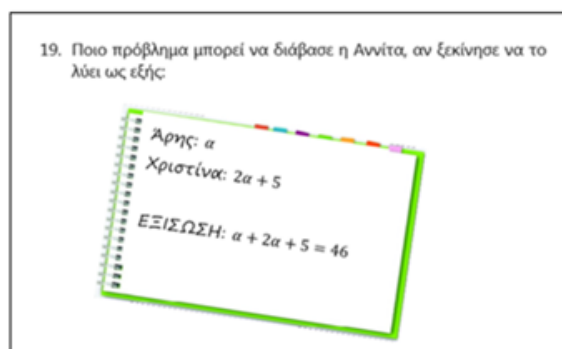
Write a problem for each of the following mathematical propositions (sic). Work in your exercise book.

7. Να γράψετε ένα πρόβλημα για καθεμιά από τις πιο κάτω μαθηματικές προτάσεις. Να εργαστείτε στο τετράδιό σας.

(α) $5 \cdot \frac{3}{8}$

(β) $\frac{3}{8} \cdot 5$

Figure 4: A problem-posing task (grade 6, part D, page 21)



Which problem might Annita have read, if she began solving it as follows:

Aris: a
Christina: $2a + 5$
EQUATION: $a + 2a + 5 = 46$

Figure 5: A problem-posing task (grade 7, part A, page 80)

Discussion

When approached from different perspectives, the findings of this study offer insight into at least two matters. Firstly, regarding transition from primary to secondary education, this paper suggests that pupils in grade 6 (upper primary) are significantly exposed to the term *problem* when interacting with their mathematics textbooks, compared to grade 7 (lower secondary). There appears to be a change of discourse, as we move from grade 6 to grade 7. This does not necessarily mean that children in grade 7 are exposed to less tasks that, according to Polya (1981), *approach research level*. Nonetheless, an abrupt disappearance of the term *problem* as children move from primary to secondary education is apparent. Although the latest educational reform intends to bridge the gap between the mathematics curricula at various transition points (MoEC, 2010), this does not seem to be happening in an effective manner. In fact, questions are raised about the extent to which the intended “cohesive and coherent curriculum from pre-primary to upper secondary education” (MoEC, 2010, p. 15) is, in reality, cohesive and coherent. Secondly, when these findings are taken as a whole, and especially when the problem-posing tasks in the two series are added to the equation, the study echoes my previous work examining pre-service teachers’ problem-solving related beliefs (Xenofontos, 2014; Xenofontos & Andrews, 2014). In the Republic of Cyprus, there seems to be a very particular, perhaps culturally specific, understanding of what a mathematical problem *is*:

A ‘problem’, for the educational system of Cyprus, seems to be a real-world task, in which the instructions regarding what needs to be done are mainly presented in the form of question, or, sometimes, with a statement. There is always one (and only) acceptable solution to this task, which can either be solved by applying a straight-forward method/algorithm or a combination of methods/algorithms, which in any case, are already known.

Such a cultural perception of mathematical problems is, I think, extremely problematic, especially when the new curricula claim, in their rhetoric, to be placing emphasis on problem solving (MoEC, 2010). Nevertheless, I do acknowledge that this study, as all studies, carries a set of limitations that has the potential to introduce new research avenues. As explained, in this paper I only examine those tasks explicitly labelled as problems. There appear to be other tasks in the textbooks that are not labelled as problems, the methods of approach of which, in Polya's (1981) terminology, approach research level. Furthermore, even though teachers in Cyprus seem to rely heavily on textbooks (Xenofontos, 2014; Xenofontos & Papadopoulos, 2015), what we don't know is *how* they use textbooks and other instructional materials in the classroom. Finally, future research could examine and compare primary and secondary in-service teachers' beliefs and practices in relation to problem-solving, so that we can have a complete picture of how to make the transition, as smoothly as possible, from one school level to another.

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