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# Time series analysis: Moving averages as an approach to analysing textbooks

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*In this study we introduce time series analysis, specifically moving averages, as a novel strategy for analysing mathematics textbooks. Such analyses show how different topics or mathematical processes are emphasised over different time periods, whether at the level of the lesson, the week, the month or year. In this paper, by way of example, we show how one of the eight categories of foundational number sense (Andrews & Sayers, 2015), namely simple addition and subtraction, is distributed and sequenced across three English, year one, textbooks. The analyses are compared empirically with four other methods found in the literature to show how time series analysis using moving averages helps address the shortcomings of these different approaches.*

*Keywords: comparative textbook analysis, foundational number sense, moving average, time series.*

## Introduction

For many teachers of mathematics, irrespective of where they work, the textbook they use is not only the major resource for lesson planning and the provision of tasks for students but also the means by which the curriculum within which they work is realised (Stein & Kim, 2009; Tarr, Chaves, Reys & Reys, 2006; Stacey & Vincent, 2009). However, textbook analysis is in some way probabilistic in the sense that teachers make decisions as to how they use any book, leaving the analytical question “what would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?” (Mesa, 2004, pp. 255–256). That is, researchers are typically interested in understanding the “likely impact of the text on the users (teachers and students)” (Stacey & Vincent, 2009, p.276). Moreover, in those cultures in which textbooks are unregulated, often leading to a plethora of choice for teachers, students may receive very different opportunities to learn (Charalambous, Delaney, Hsu & Mesa, 2010; Huntley & Terrell, 2014; Tarr et al., 2006).

Much of the world’s textbook-related research has employed some form of qualitative description, typically aligned with an analytical framework related to the topic under scrutiny, supplemented by frequency analyses. That is, researchers often count and compare the number of occurrences of particular forms of task in different textbooks. Such studies usually focus on either mathematical content knowledge or mathematical processes, although there are exceptions such as Borba and Selva’s (2013) study of calculator use in Brazilian primary school textbooks. Research focused on mathematical content knowledge has included, for example, studies of primary school textbooks’ treatment of fractions in Kuwait, Japan, and the USA (Alajmi, 2012), Cyprus, Ireland, and Taiwan (Charalambous et al., 2010), division of fractions in grades six and seven in Chinese, Japanese and US textbooks (Li, Chen & An, 2009), multiplication and division of fractions in grades five and six in the textbooks of the USA and Korea (Son & Senk, 2010), inverse relations in US and Chinese primary textbooks (Ding, 2016), functions in the middle school textbooks of 15 countries (Mesa,

2004); geometry in English and Japanese grade eight textbooks (Jones & Fujita, 2013), number sense in year one textbooks in England and Sweden (Löwenhielm, Marshall, Sayers & Andrews, 2017) the distributive property (Ding & Li, 2010) and so on.

Textbook-related studies on mathematical processes has included, for example, examinations of mathematical problem solving (Brehmer, Ryve & Van Stenbrugge, 2016; Fan & Zhu, 2007), mathematical problem posing (Cai & Jiang, 2017), general perspectives on reasoning and proof in Australia (Stacey & Vincent, 2009) and the USA (Stylianides, 2009), geometry-related proof for students in the grades 6-9 in the USA (Otten, Gilbertson, Males, & Clark, 2014) and France and Japan (Miyakawa, 2017), calculus-related reasoning and proof for upper secondary students in Finland and Sweden (Bergwall & Hemmi, 2017).

Other studies have attempted to identify and demonstrate the different emphases within a textbook by going beyond simple quantification. For example, Tarr et al., (2006), in their analysis of American teachers' use of different types of textbooks, introduced what they called the emphasis index, which was defined as the number of lessons taught by a teacher on a topic divided by the number of lessons in the textbook on the same topic. Interestingly, a similar procedure could be applied uniquely to a textbook, where the emphasis index would be the number of lessons on a particular topic in the book divided by the total number of lessons.

However, while frequency analyses have the potential to offer insight into the topics privileged by a textbook's authors they typically offer little with respect to how mathematical ideas are sequenced. Indeed, even though some scholars have conceded that they "examined only proportions of various types of opportunities and did not attend to the sequencing of activities, which may be important with regard to students' learning" (Otten et al., 2014, p.75), few have examined the distribution of codes within a text. Of those studies that have attempted explicitly to examine the distribution of codes, four methods stand out. Firstly, method 1, researchers have identified, in the sequence of all tasks, the position of the first task of interest (Fujita, 2001). Secondly, method 2, researchers have analysed the proportion of a textbook covered before an occurrence of interest (Alajmi, 2012; Li et al., 2009). Thirdly, method 3, studies, particularly those analysing series of textbooks that cross several grade boundaries, have displayed relative percentages across grades (Borba & Selva, 2013; Ding, 2016). Moreover, Ding displayed results in a line diagram, where the distribution of a code per grade level in one book series could be compared with the distribution of the same code per grade level in another book series. This takes us to our final method and the inspiration for our contribution. Fourthly, method 4, studies have exploited timeline analyses, whereby coded observations were presented as dots on a horizontal axis representing the sequence of all tasks through the textbooks. With such an approach, "large swaths of these textbooks were not coded", allowing the reader to see, at a glance, how topics under scrutiny are both located and emphasised (Huntley & Terrell, 2014, p. 758). One advantage of such diagrams is that they give a picture that is fine-grained down to occurrence of individual codes. However, this is also its draw-back. For larger data sets, the graphic in the time-line diagram might get too jammed making it difficult to discern any useful information in the diagram. In this paper we present a comparison of several of these methods before offering a novel methodological perspective on the analysis of textbooks that goes

beyond what others have achieved. In so doing, we aim to make a meaningful contribution to the methods of comparative textbook analysis.

## **The context of this study**

This paper is a preliminary account of textbook analyses conducted as part of the Foundational Number Sense (FoNS) project, which is an ongoing Swedish Research Council-funded study of how FoNS, a set of eight core number-related competences, is acquired by year one children in England and Sweden. These core competences, which research has shown to be essential for later mathematical success, derived from a constant comparison analysis of literature from mathematics education, psychology, special educational needs and generic education (Andrews & Sayers, 2015). While it is important to understand how textbooks structure opportunities for learning, England and Sweden are of particular interest as both systems, due to perceptions of systemic failure on international tests of achievement like TIMSS and PISA, have begun importing and adapting textbooks used in high-achieving countries. In Sweden, textbooks from Finland and Singapore have been imported, while in England, textbooks from Singapore and Shanghai.

In the study presented here, which is principally a methodological contribution, we examine aspects of three textbooks used with English grade 1 classes, Abacus, Inspire Maths (hereafter, Inspire), and Maths - No Problem (hereafter, MNP). These were chosen because Abacus is a long-standing series written by a well-known primary mathematics education researcher. The other two, drawing on societal perceptions that English mathematics teaching would be improved by the adoption of Singaporean practices, were English-authored adaptations of Singapore textbook series.

## **Comparing different display methods on the same data**

Many educational research analyses yield codes of zeros and ones for the absence or presence of specific phenomena, particularly analyses of the tasks found in textbooks. Thus, depending on whether analysed properties are present in each task, one outcome of this process might be a sequence  $\{0, 0, 1, 0, 1, 1, \dots\}$  of ones and zeros. The manner in which the results from such analyses are displayed necessarily depends on the research question and could be the presentation of frequencies or the distribution of such codes throughout the data. When comparing textbooks, differences in the frequencies of particular codes can be compared by means of, say,  $\chi^2$  tests. However, when the research goal is to focus on the sequential distribution of codes, the focus of this paper, other methods are needed. In this paper, as a response to this, we offer time series analysis as a novel method for comparing the sequential distribution of particular task properties in textbooks. In so doing, we compare this novel approach with others found in the literature.

In the following, as indicated above, we draw on FoNS-related analyses of three English year one textbook series, in which every task that explicitly expected a student response was coded for its FoNS-related opportunities. Thus, every task could be represented as set of eight ones and zeros ranging from  $(0, 0, 0, 0, 0, 0, 0, 0)$ , in which no FoNS-related codes were identified, to  $(1, 1, 1, 1, 1, 1, 1, 1)$ , in which all FoNS codes were identified (Löwenhielm et al., 2017). However, with the objective of comparing methodologies, space prevents an analysis of all eight FoNS categories, so we restrict this presentation to just one, in order to exemplify the advantages and disadvantages of different analytical approaches. Thus, the following focuses solely on FoNS category seven, simple

addition and subtraction within the number range 0 – 20 due, in part, to the significance of such competence in later mathematical learning.

The figures of table 1 show the results of the two methods employed by Fujita (2001) and Alajmi (2012) respectively. The first, based on the position of the first occurrence of the code in question (Fujita, 2001), shows substantial differences in the number of tasks completed before simple arithmetical operations are introduced. However, due to differences in the total number of tasks in each series, it is not necessarily straightforward, without additional calculation, to discern whether the positions of the first occurrence are comparable across the three books. In this respect, the second method, based on the percentage of all tasks presented before the first occurrence (Alajmi, 2012), gives a better indication of how deep into the book the first occurrence of each code occurs and, of course, their relative positions. Thus, it can be seen that the introduction of simple arithmetic in the two textbooks based on the Singaporean tradition, Inspire and MNP, occurs not only substantially later than in the English textbook, Abacus, but also in different positions relative to each other.

Book	FoNS category	Position of first occurrence	Percentage of tasks before first occurrence
Abacus	7	22nd of 1522	1%
Inspire	7	168th of 907	18%
MNP	7	175th of 1955	9%

**Table 1. Method 1 (1st occurrence) and method 2 (% tasks before first occurrence)**

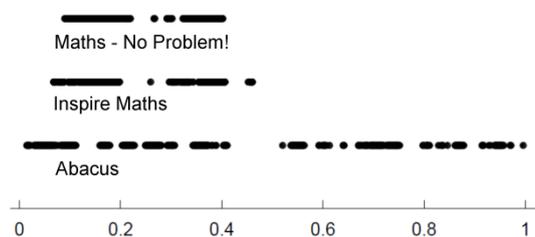
The third method, represented in Table 2, shows the relative percentages across the various workbooks in each series (Borba & Selva, 2013). Here, each series is partitioned into a different number of booklets; Abacus is in three booklets, Inspire in four and MNP in two. What is clear, interestingly, is the variation in the distribution of tasks focused on code 7, simple addition or subtraction, across the three sets of workbooks; Abacus incorporates substantial proportions of such material across all three booklets, while both Inspire and MNP incorporate such tasks only in the first half of their respective series. Of course, presenting the same data as line diagrams in the manner of Ding (2016) may have offered a clearer picture, but little by way of additional insight into the actual sequencing of the tasks.

The fourth method, Figure 1, shows a timeline with the position of each task coded for simple addition or subtraction, code 7, shown as a dot (Huntley & Terrell, 2014). As with method 3, this highlights well differences between Abacus and the two Singapore-based books, Inspire and MNP. The former shows simple arithmetic tasks distributed throughout the workbooks, while the latter confirms that such tasks only occur within the first half of the series. They also show a not dissimilar pattern within the two Singapore-based books, with periods of extended opportunity,

followed by a gap with occasional revisitations before a second extended opportunity. That being said, such graphs offer insight only into the sequencing of activities within a textbook and not necessarily an indication how children’s day-to-day learning may be structured.

Abacus (3 booklets)	Inspire (4 booklets)	MNP (2 booklets)
56%	55%	40%
24%	35%	0%
35%	0%	
	0%	

**Table 2. Method 3 (% occurrences per booklet) for code 7**



**Figure 1. Method 4 (timeline dot plot) for code 7**

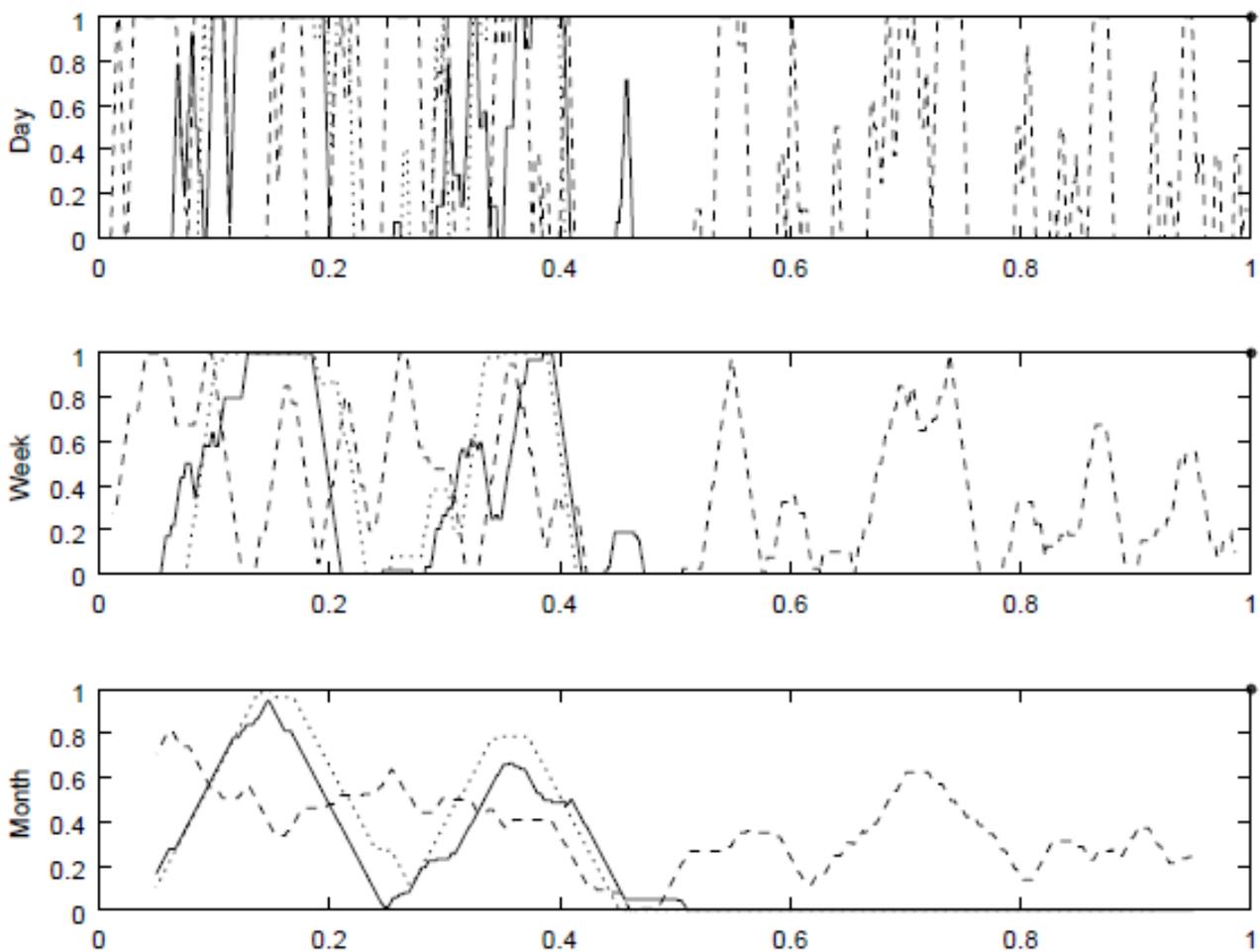
### Moving averages

The fifth method, which is our contribution to the debate, involves moving averages as an approach to time series analysis in which data are logged at equally spaced points in time. Typically, these are undertaken to “understand the underlying dynamics, forecast future events, and control future events via intervention” of stochastic processes (Fan & Yao, 2003, p. 9). Now, while a school mathematics textbook is not stochastic, the use of time series, whereby data are successive tasks, should offer a clear indication of a textbook’s sequential emphases over time. However, the graphs yielded by time series analyses are typically extremely noisy, as with daily records of air temperature (Wakaura & Ogata, 2007), a problem that can be overcome by means of moving averages. Here, rather than report single data points, moving averages are based on a sequence of overlapping sets of data or moving windows. This process smoothes out short-term fluctuations in time series so that longer-term patterns become more visible and the influence of outliers is eliminated. Mathematically, a moving average means substituting a single data point  $(t_k, y_k)$  with  $(t_k, \hat{y}_k)$ , where  $\hat{y}_k$  is the arithmetic mean of its neighbouring data points  $y_j$  as in equation 1.

$$\text{(Equation 1)} \quad \hat{y}_k = \sum_{j=k-n}^{k+n} \frac{1}{2n+1} y_j$$

Of interest here, is the size of the divisor,  $2n + 1$ , which represents the total number of data points included in the calculation and is dependent on the time period chosen for the calculation. That is,  $2n + 1$  refers to the original point,  $y_k$ , and its  $2n$  neighbouring data points,  $n$  before and  $n$  after. In

the context of a mathematics textbook, the width  $2n + 1$  of this window could be the number of tasks that an average student is expected to cover each day, or each week or each month and this choice depends on the research question. Thus, one choice of the width of the moving average window could be  $\frac{\text{all items in a book series for one year}}{40 \text{ school weeks}}$ , roughly corresponding to a single week's workload across the year. This means that wherever the moving average diagram shows 'above zero', then the pupil would have met that coded property during that week. Figure 2 shows the results, for code 7, simple addition or subtraction, for different time periods. The vertical axis refers to the average code score, taking values from 0 to 1, for any given time interval, and the horizontal axis the year for which the textbook is intended for use. The solid line refers to MNP, the dotted line to Inspire and the dashed line to Abacus.



**Figure 2. Method 5 (moving average) for code 7**

As can be seen, analysing the daily opportunities for code 7 produces a noisy diagram that is difficult to interpret, particularly with the three textbooks are presented together. However, as seen in the weekly graphs, a longer time period begins to smooth out much of this short-term variation and expose patterns of emphasis. Therefore, in such analyses it is important to select a window that is sensitive to the context of the data source in to obtain a graph that shows neither too little nor too much of the data's details. In the context of this study, the monthly graph is probably the most illuminating. The two Singapore books are virtually indistinguishable but Abacus, shows continued

exposure, albeit at different levels of intensity, to tasks focused on simple addition and subtraction, opportunities, we know from several of the analyses above, missing in the second half of the two Singapore-inspired series.

In conclusion, we believe that graphs presenting moving averages offer a novel but methodologically important tool in the analysis of textbooks. In fact, it can be argued that it represents a generalisation of the four methods compared above, albeit dependent on the choice of time interval. For example, where the moving average curve in figure 2 rises above zero, it shows where the first occurrence of a code occurs, which is the information provided by methods 1 and 2 in table 1. Moreover, it shows where in a series of books a code occurs, as yielded by method 3 in table 2, corresponding to a non-overlapping (disjoint) average, while moving averages do overlap. Finally, moving averages offer a direct generalisation of strategy 4, since a moving average with a window of length one would reproduce the diagram in Figure 1.

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