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***Abstract*—To improve the bandwidth efficiency of free-space optical (FSO) systems and at the same time to reduce the impact of the background noise, we propose a differential M-ary pulse-amplitude modulation (M-PAM) signaling scheme that uses two laser transmitters. We *first* consider the condition that the receiver perfectly knows the instantaneous channel coefficient and compare the performance of the proposed differential PAM with the conventional PAM signaling and show the improved performance when the background noise level is relatively high. *Second*, we consider the practical situation where the receiver has to estimate the channel for signal detection. We propose an estimation scheme based on the characteristics of the differential PAM signaling while requiring no pilot symbol transmission. The proposed data-aided channel estimation is performed on a sequence of received PAM symbols. We show that for a sufficiently large observation window, the proposed estimation method allows achieving a performance close to the perfect channel knowledge.**

I. Introduction

A. Background

Free space optical (FSO) communications have recently attracted a great deal of attention due to offering a very large bandwidth, low implementation cost, high transmission security and robustness to electromagnetic interference, for instance [1]. Under clear sky conditions, the reliability and performance of these links can be severely affected by impairments such as atmospheric turbulence and pointing errors [1]. Furthermore, background radiations due mainly to sunlight can degrade the performance of the FSO links [2].

In order to deal with the effect of the background noise, a differential signaling (DS) technique was proposed in [3] using two lasers with close wavelengths at the transmitter when using intensity modulation with direct detection (IM/DD) based on ON-OFF keying (OOK) or pulse-position modulation (PPM). This way, the background noise is significantly reduced at the receiver through differential detection. Whereas [3] assumed almost identical fading coefficients for the two wavelengths, a more detailed analysis was later done in [4], [5], [6] for an OOK-based link taking into account fading correlation between the two underlying channels. Also, pointing error mitigation was considered in [7] based on a similar DS scheme.

Here, we consider IM/DD signaling based on M-ary pulse-amplitude modulation (M-PAM) [8], where OOK is its simplest form with $M = 2$. Indeed, it is well known that PPM is optimal in terms of energy efficiency among pulsed modulations but it suffers from relatively poor bandwidth (BW) efficiency [9], [10]. Although in optical communications we have a huge BW available, the increased synchronization complexity and required speed of opto-electronics are the main limiting factors that reduce the interest of PPM in high data-rate FSO systems. Meanwhile, one advantage of using PPM is that there is no need to threshold adjustment at the receiver for signal demodulation. PAM, on the other hand, offers a better BW efficiency but at the cost of increased peak-to-average power ratio (PAPR) and the requirement to adaptive threshold setting under channel fading conditions [8], [11], [12], [13], [14], [15], [16], [17], [18], [19]. Furthermore, compared to subcarrier IM schemes [20], PAM offers a better power efficiency since the former need a DC bias to be added to the signal to insure positive amplitude before IM.

For OOK signaling, maximum-likelihood sequence detection (MLSD) and its generalizations were considered in [21], [22], [23], [24] for the purpose of signal detection at the receiver. However, these methods involve computationally complex integral calculations that increases with M when applied to M-PAM signaling. As an alternative to the MLSD-based methods, channel estimation prior to data detection was considered in [25], [26], [27], for instance. This way, the channel is first estimated based on some pilot signals, which is then used to adjust the detection threshold. Given the incurred loss in the effective data throughput by pilot insertion, solutions avoiding such a pilot overhead are highly preferable. The case is still more important for PAM signaling where the data detection performance is highly dependent on the accuracy of channel estimation at the receiver, especially for large M.

B. Contributions

In this paper, to improve the BW efficiency of an FSO system while mitigating the impact of the background noise, we propose a differential PAM signaling scheme using two laser transmitters. We firstly consider the ideal condition where the receiver perfectly knows the instantaneous channel coefficient and show the performance improvement by in the presence of background noise. Next, we consider the practical case where the receiver has no a prior information on the channel and estimates it from the received differential PAM signals without requiring any pilot symbol. In other words, we propose a data-aided channel estimation method by exploiting the property of DS over a sequence of received symbols based on the ML criterion. We show that the differential PAM signaling with the proposed channel estimator can achieve performance very close to the perfect channel knowledge case, provided that the observation window is sufficiently large.

The rest of the paper is organized as follows. We present the formulation of the conventional PAM signaling in Section II. Differential PAM signaling and the BER analysis under perfect CSI knowledge are described in Section III. Next, we present in Section IV differential PAM with imperfect CSI where we propose a data-aided channel estimation solution and study its efficiency through numerical results. Lastly, Section V concludes the paper.

II. Conventional PAM Signaling

We consider an FSO link using M-PAM modulation over an atmospheric turbulence channel. We assume clear weather conditions and that the transmitter and the receiver are perfectly aligned. PAM signaling is done based on the discrete set of amplitudes $\{0, 1, ..., M-1\}$ with Gray bit-symbol mapping. Denoting the transmitted signal by s, the received signal r corresponding a given symbol interval can be written as

$$r = RhP^c_{t,\min}s + RP_b + n, \tag{1}$$

where R is the responsivity of the photo detector, h denotes the instantaneous channel attenuation, P_b is the background noise power, and $P^c_{t,\min}$ denotes the transmit power corresponding to the lowest non-zero PAM level. Denoting the average transmit power by $\bar{P}_t$, we have $\bar{P}_t = \frac{1}{M}\sum_{j=0}^{M-1} jP^c_{t,\min} = \frac{M-1}{2}P^c_{t,\min}$. Also, n is the sum of two zero-mean Gaussian random processes n_{th} and n_b, which represent thermal and background noises with variances σ^2_{th} and σ^2_b, respectively. Concerning n_b, we assume that the background radiations level is high enough to approximate the related Poisson distribution by a Gaussian, the mean of which is assumed to be rejected by the ac-coupled circuitry of the receiver [3]. We define the parameter $k = \sigma^2_b/\sigma^2_{th}$ for later use, similar to [3].

For conventional M-PAM signaling, and assuming that the receiver perfectly knows the CSI, the average link BER is

$$\mathbb{P}_{e\mathrm{PAM}} = \int_0^\infty P_{e\mathrm{PAM}|h}(h)f_h(h)dh, \tag{2}$$

where $\mathbb{P}_{e\mathrm{PAM}|h}(h)$ is the BER of the conventional M-PAM at the instantaneous channel fading coefficient which is equal to [28]:

$$\mathbb{P}_{e\mathrm{PAM}|h}(h) = \frac{2(M-1)}{M\log_2(M)}Q\left(\sqrt{\frac{h^2d^2_c}{4\sigma^2_{th}(1+k)}}\right), \tag{3}$$

where Q is the well known Q-function and $d_c = RP^c_{t,\min}$ is Euclidean distance between two neighboring M-PAM constellation points. The average received electrical energy per symbol is

$$E^C_s = \frac{1}{M}\sum_{j=0}^{M-1} j^2d^2_c = \frac{(M-1)(2M-1)}{6}d^2_c. \tag{4}$$

Denoting the average received electrical energy per bit by E^C_b, we have $E^C_b = E^C_s/\log_2(M)$. We define the electrical SNR per bit as $\Gamma_b = E^C_b/\sigma^2_{th}(k+1)$ and rewrite (3) in term of Γ_b as

$$\mathbb{P}_{e\mathrm{PAM}|h}(h) = \frac{2(M-1)}{M\log_2(M)}Q\left(\sqrt{\frac{3\log_2(M)h^2\Gamma_b}{2(M-1)(2M-1)}}\right). \tag{5}$$

Lastly, we consider the well-known gamma-gamma distribution for modeling the atmospheric turbulence. This way, the PDF of the normalized channel coefficient h is given by [29],

$$f_h(h) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}h^{\frac{\alpha+\beta}{2}-1}k_{\alpha-\beta}(2\sqrt{\alpha\beta h}), \tag{6}$$

where $\Gamma(.)$ is the gamma function, $k_m(.)$ is the modified Bessel function of second kind of order m and $1/\beta$ and $1/\alpha$ are respectively the variances of the small and large scale eddies and can be calculated directly from the Rytov variance σ^2_R.

III. Differential PAM Signaling

A. Transmission scheme and formulation

Figure 1 depicts the proposed system block diagram with D-PAM signaling. At the transmitter, the M-PAM input signal $s \in \{0, 1, ..., M-1\}$ and its complement $\bar{s} = M-1-s$ are used to drive the two optical sources (OSs) working on different wavelengths λ_1 and λ_2. We assume that OSs consist of laser diodes although the transmission scheme remains the same in the case of using light-emitting diodes as well). The outputs of the OSs are passed through a beam combiner (BC) before sending over the FSO channel. At the receiver, the optical signal is passed through optical filters (OFs) with center wavelengths of λ_1 and λ_2 to separate the received optical signals of the two lasers.

The outputs of the OFs are converted to the electrical signals r_1 and r_2 by the photo-detectors (PD)s. We have

$$\begin{cases} r_1 = & Rh_1P^d_{t,\min}s + n_1, \\ r_2 = & Rh_2P^d_{t,\min}\bar{s} + n_2. \end{cases} \tag{7}$$

Here, $P^d_{t,\min}$ is the minimum signal power of each of the transmitters and is set as $P^d_{t,\min} = P^c_{t,\min}/2$ in order to ensure the same average total transmit power $\bar{P}_t$ as for the conventional PAM signaling considered before. Also, $n_1 = n_{th,1} + n_{b,1}$ and $n_2 = n_{th,2} + n_{b,2}$, where $n_{th,1}$ and $n_{th,2}$ represent thermal noise components with variance σ^2_{th}, and $n_{b,1}$ and $n_{b,2}$ represents background noise components with variance σ^2_b. Afterwards, we subtract r_2 from r_1 to obtain r_d. We have

$$r_d = RP^d_{t,\min}(h_1s - h_2\bar{s}) + n_{th,1} - n_{th,2} + n_{b,1} - n_{b,2}, \tag{8}$$

Obviously, for signal demodulation, the receiver requires the CSI knowledge to adjust the detection thresholds. For instance, Fig. 2 shows the received signal constellation for the differential PAM (D branch in Fig. 1) and the related detection thresholds τ_is for $M = 8$. As we will describe in more detail in the next section, we use the feature that $s + \bar{s} = M-1$ for channel estimation purpose. For this, in addition to P_d, we obtain P_s by adding $P_{r,1}$ to $P_{r,2}$. We have

$$r_s = RP^d_{t,\min}(h_1s + h_2\bar{s}) + n_s, \tag{9}$$

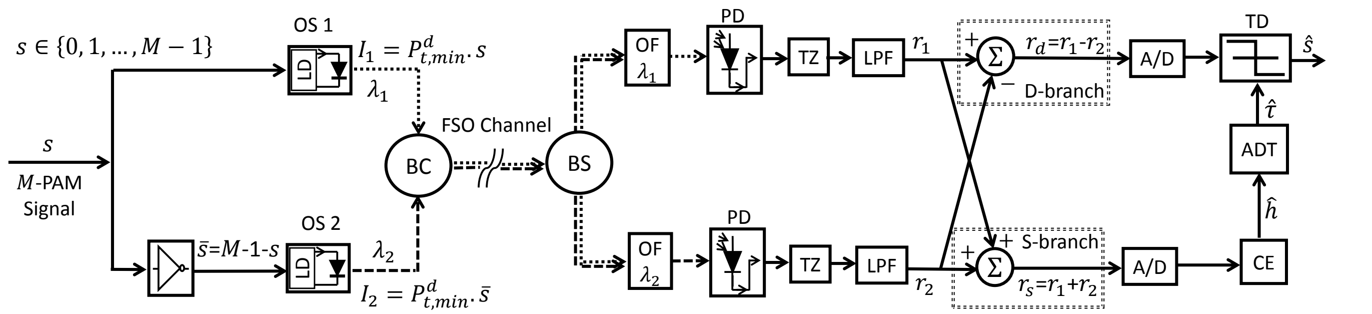

Fig. 1: Block diagram of proposed differential PAM signaling. LD, OS, BC, BS, OF, PD, CE, TZ, LPF, A/D, ADT and TD are laser diode, optical source, beam combiner, beam splitter, optical filter, photo detector, channel estimator, transimpedance circuitry, low-pass filter, analog-to-digital converter, adjust detection threshold, threshold detection.

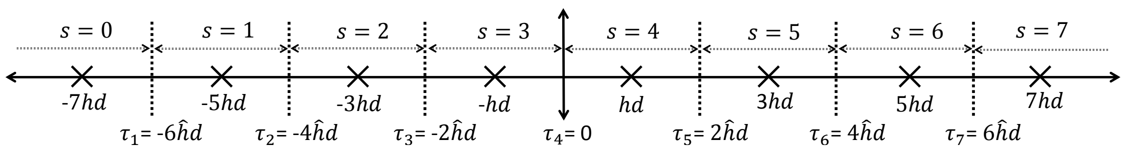

Fig. 2: Average received constellations of the proposed differential M-PAM signaling at the given h and their related detection thresholds τ_is for $M = 8$.

where $n_s = n_1 + n_2$. As proved in [3], assuming that λ_1 and λ_2 are very close (e.g., on the order of several tens of nanometers), $n_{b,1}$ and $n_{b,2}$ are highly correlated, i.e., we have $n_{b,1} \approx n_{b,2}$. Indeed, with differential detection described by (8), background radiations are practically suppressed and we can neglect their effect on r_d. On the other hand, given that λ_1 and λ_2 are very close, we rationally assume that the corresponding channel attenuations are the same, i.e., $h_1 \approx h_2 = h$, as well as the corresponding photo-detector sensitivities.[1] With these assumptions, we can rewrite (8) and (9), as

$$r_d = d_d s_d h + n_{th,1} - n_{th,2}, \tag{10}$$

$$r_s = d_d(M-1)h + n_{th,1} + n_{th,2} + 2n_b, \tag{11}$$

where $s_d = 2j - M - 1$ for $s = j-1$, $j \in \{1, 2, ..., M\}$ and $d_d = RP_{t,\min}^d = \frac{d_c}{2}$.

Note that concerning the D-branch, the background noise is suppressed before A/D conversion. Hence, the receiver dynamic-range limitation is dictated by the PD, TZ, and LPF (see Fig. 1), which should be much less constraining than the A/D. However, For the S-branch, the A/D input may saturate if the background noise level is too high.

B. BER analysis under perfect CSI

Let us first show the BER performance of differential PAM when the receiver has perfect knowledge of the CSI. Based on (10) and aforementioned assumptions, $p(r_d|s = i-1, h)$ for $i\{1, 2, ..., M\}$ is given by

$$p(r_d|s = i-1, h) = \frac{1}{\sqrt{4\pi\sigma_{th}^2}} \times \exp\left(-\frac{(r_d - d_d(M-1+2i)h)^2}{4\sigma_{th}^2}\right). \tag{12}$$

Based on (12) and similar to the conventional PAM case, the BER of differential M-PAM conditioned to h is

$$\mathbb{P}_{e\mathrm{DPAM}|h}(h) = \frac{2(M-1)}{M\log_2(M)} Q\left(\sqrt{\frac{h^2 d_d^2}{2\sigma_{th}^2}}\right). \tag{13}$$

The average BER is obtained by substituting (13) in (2). Let us denote by E_s^D the average received electrical energy per symbol for differential M-PAM.

$$E_s^D = \sum_{j=0}^{M-1} (-M-1+2j)^2 d_d^2 = \frac{M^2-1}{3} d_d^2, \tag{14}$$

which is related to the average received energy per bit E_b^D through $E_b^D = E_s^D / \log_2(M)$. The SNR per bit is hence $\Gamma_b = E_b^D / 2\sigma_{th}^2$, and we can rewrite (13) as a function of Γ_b as

$$\mathbb{P}_{e\mathrm{DPAM}|h}(h) = \frac{2(M-1)}{M\log_2(M)} Q\left(\sqrt{\frac{3\log_2(M)h^2}{(M^2-1)}\Gamma_b}\right). \tag{15}$$

In order to evaluate the performance of the differential PAM scheme compared to conventional PAM, we have provided plots of BER versus Γ_b in Figs. 3a and 3b corresponding to $k = 1$ and 50, respectively, and for different modulation orders M. We have set $\bar{P}_t = 1\,\mathrm{W}$ and the gamma-gamma model parameters $\alpha = 11.6$ and $\beta = 10.2$ ($\sigma_R^2 = 1$). As expected, the differential scheme outperforms the conventional one, especially for relatively high background noise levels, i.e.,

[1]When λ_1 and λ_2 are very different, the performance of the proposed scheme could be affected considerably and needs to be studied in detail, which is the beyond the scope of this paper.

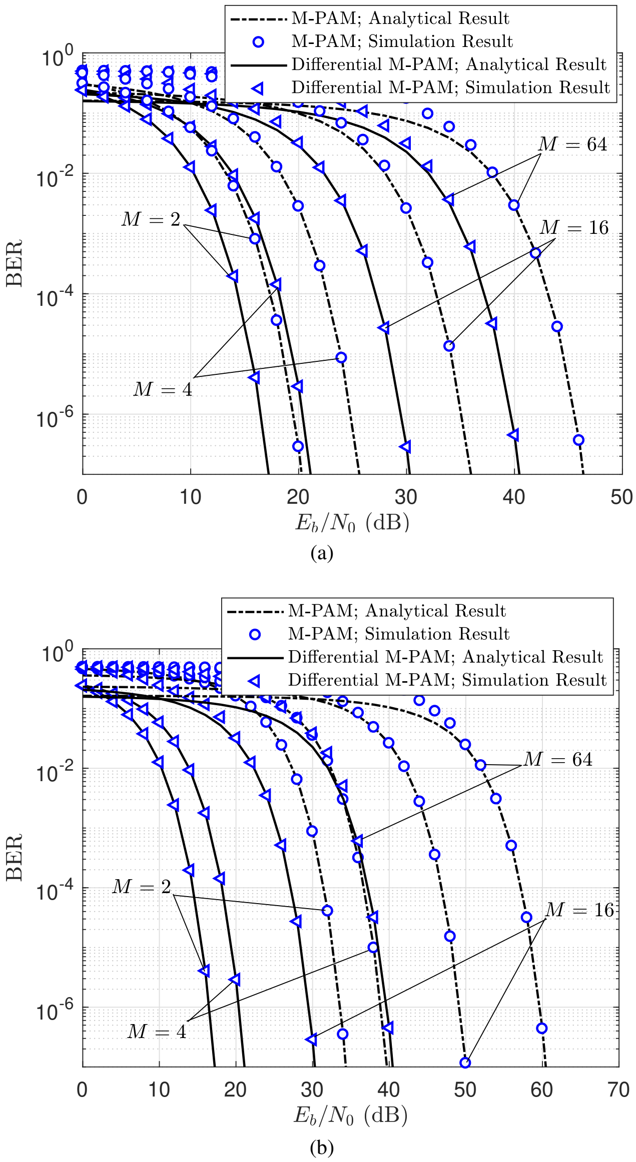

Fig. 3: BER performance comparison of the proposed differential M-PAM and the conventional M-PAM signaling for different values of M and for (a) $k = 1$, and (b) $k = 50$. Perfect CSI at the receiver is assumed.

large k. Moreover, for BERs lower than 10^{-3}, we notice a perfect match between the analytical and simulation-based results, which testifies the accuracy of the presented formulation.

IV. Differential PAM Signaling Under Imperfect CSI

In practice, in order to adjust the detection thresholds for PAM demodulation, the receiver has to estimate continuously the channel attenuation coefficient. The classical pilot-based channel estimation method could incur a non-negligible pilot overhead, especially for relatively large M. Here, we show that with the proposed differential PAM scheme and with the aid of the second (S) branch at the receiver (see Fig. 1), we can estimate the channel without requiring any training symbol.

A. Data-aided channel estimation

Given that $s + \bar{s} = M - 1$, at the receiver, the S-branch output signal can be effectively used as a pilot. Notice that, in contrary with the D-branch where the background noise is suppressed before A/D, for the S-branch, r_s includes the background noise $2\,n_b$. We assume here that the factor k is not too high to result in the saturation of the A/D input for the S-branch. In other words, in practice, the estimation method that we propose here can work for a not-too-high background radiation level. According to (11), we have

$$p(r_s|h) = \frac{1}{\sqrt{2\pi\sigma_{th}^2(2+4k)}} \times \exp\left(-\frac{\big(r_s - d_d(M-1)h\big)^2}{2\sigma_{th}^2(2+4k)}\right). \tag{16}$$

The maximum likelihood estimation of h is formulated as follows.

$$\begin{aligned}\hat{h} &= \arg\max_h \left\{ \ln p(r_s, h) \right\} \\ &= \arg\max_h \left\{ \ln p(r_s|h) f_h(h) \right\} \\ &= \arg\max_h \left\{ \ln\big(p(r_s|h)\big) + \ln\big(f_h(h)\big) \right\} \\ &= \arg\max_h \left\{ -\frac{\big(r_s - d_d(M-1)h\big)^2}{2\sigma_{th}^2(2+4k)} + \ln\big(f_h(h)\big) \right\}. \end{aligned} \tag{17}$$

By differentiating (17) with respect to h and setting the result equal to zero, we obtain

$$\frac{2d_d(M-1)\big(r_s - d_d(M-1)h\big)}{2\sigma_{th}^2(2+4k)} - \frac{f'(h)}{f_h(h)} = 0. \tag{18}$$

To find the optimum $\hat{h}$, (18) can be solved numerically, which is rather computationally complex. Here, given that $f'_h(h)/f_h(h)$ is close to zero for various levels of the Rytov variance according to [27], we neglect this term in (18), which gives

$$\hat{h} = \frac{r_s}{d_d(M-1)}. \tag{19}$$

According to (11), we can rewrite (19) as

$$\hat{h} = h + \frac{n_{th,1} + n_{th,2} + 2n_b}{d_d(M-1)}. \tag{20}$$

Since (20) relies on only one sample for channel estimation, the resulting estimation error can be quite important. Therefore, we consider using a sequence of the received signal to estimate the channel coefficient. We consider an observation window of length L, $\underline{r_s} = \{r_s[1], r_s[2], ..., r_s[L]\}$, during which the channel is assumed to remain unchanged. This is a quite reasonable assumption given the quasi static nature of the FSO channel.

The ML estimation of h can be readily obtained as

$$\hat{h} = \frac{1}{d_d(M-1)L} \sum_{m=1}^{L} r_s[m]. \tag{21}$$

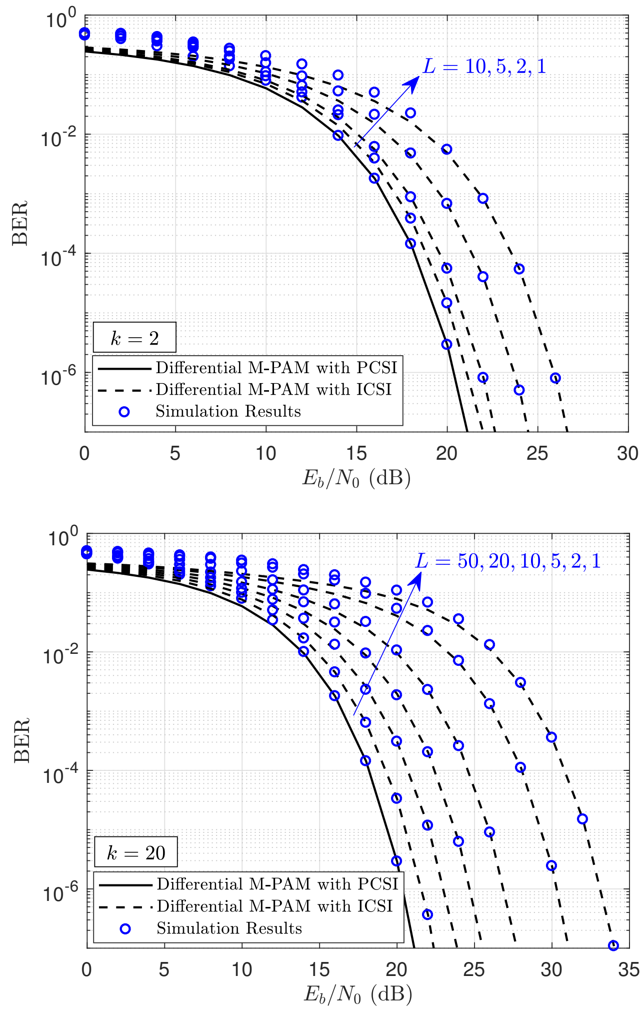

Fig. 4: BER performance of the proposed differential 4-PAM signaling with perfect CSI and with the channel estimation of (21) for (a) $k = 2$, and (b) $k = 20$.

To calculate the BER, we need to calculate $\mathbb{P}^{\text{symb}}_{D\text{-PAM}}(e|s = j-1, h, \hat{h})$ which is the error probability due to the transmitted $s = j-1$ at the given h and under the channel estimation of (21). By substituting (11) in (21), we have

$$\hat{h} = h + \frac{1}{d_d(M-1)L}\sum_{m=1}^{L} n_s[m]. \tag{22}$$

Based on (22), the detection thresholds for the differential M-PAM scheme are

$$\begin{aligned}\tau_j &= (-M+2j)\hat{h}d_d \\ &= \frac{(-M+2j)}{(M-1)L}\sum_{m=1}^{L}\left(n_{th,1}[m] + n_{th,2}[m] + 2n_b[m]\right) \\ &\quad + (-M+2j)hd_d, \qquad \text{for } j \in \{1, ..., M-1\}. \end{aligned} \tag{23}$$

The symbol error rate (SER) of the differential M-PAM conditioned to h can be obtained as

$$\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h}}(h) = \frac{1}{M}\sum_{j=1}^{M}\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=j-1}(h), \tag{24}$$

and $\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=j-1}(h)$ is the error probability due to the transmitted $s = j-1$ at the given h and with the channel estimation of (22). We have,

$$\begin{aligned}&\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=0}(h) = \mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=M-1}(h) \\ &\quad = \text{Prob}\{r_d - \tau_{M-1} < 0 | s = M-1\} \\ &\quad = \text{Prob}\Bigg\{(M-1)Lhd_d < 2(M-2)\sum_{m=1}^{L} n_b[m] \\ &\qquad + (M-2-(M-1)L)\, n_{th,1}[m'] \\ &\qquad + (M-2+(M-1)L)\, n_{th,2}[m'] \\ &\qquad + (M-2)\sum_{\substack{m=1 \\ m\neq m'}}^{L}(n_{th,1}[m] + n_{th,2}[m])\Bigg\} \\ &\quad = Q\left(\sqrt{\frac{L(M-1)^2h^2d_d^2}{2\Big((M-2)^2(1+2k) + L(M-1)^2\Big)\sigma_{th}^2}}\right) \\ &\quad = Q\left(\sqrt{\frac{3L\log_2(M)(M-1)h^2\Gamma_b}{(M+1)\Big((M-2)^2(1+2k) + L(M-1)^2\Big)}}\right). \end{aligned} \tag{25}$$

For $s = j-1$ and $j \in \{2, 3, ..., M-1\}$, $\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=j-1}(h)$ is derived as

$$\begin{aligned}&\mathbb{P}^{\text{symb}}_{eDPAM|h,\hat{h},s=j-1}(h) \\ &= \text{Prob}\Big\{r_d - \tau_{j-1} < 0 | s = j-1\Big\} \\ &\quad + \text{Prob}\Big\{r_d - \tau_j > 0 | s = j-1\Big\} \\ &= \text{Prob}\Bigg\{(M-1)Lhd_d > 2(2j-M-2)\sum_{m=1}^{L} n_b[m] \\ &\quad + (2j-M-2-(M-1)L)\, n_{th,1}[m'] \\ &\quad + (2j-M-2+(M-1)L)\, n_{th,2}[m'] \\ &\quad + (2j-M-2)\sum_{\substack{m=1 \\ m\neq m'}}^{L}(n_{th,1}[m] + n_{th,2}[m])\Bigg\} \\ &\quad + \text{Prob}\Bigg\{-(M-1)Lhd_d > 2(2j-M)\sum_{m=1}^{L} n_b[m] \\ &\quad + (2j-M-(M-1)L)\, n_{th,1}[m'] \\ &\quad + (2j-M+(M-1)L)\, n_{th,2}[m'] \\ &\quad + (2j-M)\sum_{\substack{m=1 \\ m\neq m'}}^{L}(n_{th,1}[m] + n_{th,2}[m])\Bigg\} \\ &= Q\left(\sqrt{\frac{L(M-1)^2h^2d_d^2}{2\Big(L(M-1)^2 + (2j-M-2)^2(1+2k)\Big)\sigma_{th}^2}}\right)\end{aligned}$$

$$+Q\left(\sqrt{\frac{L(M-1)^2h^2d_d^2}{2\Big(L(M-1)^2+(2j-M)^2(1+2k)\Big)\sigma_{th}^2}}\right)$$
$$=Q\left(\sqrt{\frac{3L\log_2(M)(M-1)\Gamma_b h^2}{(M+1)\Big(L(M-1)^2+(2j-M-2)^2(1+k)\Big)}}\right)$$
$$+Q\left(\sqrt{\frac{3L\log_2(M)(M-1)\Gamma_b h^2}{(M+1)\Big(L(M-1)^2+(2j-M)^2(1+k)\Big)}}\right). \quad (26)$$

Assuming Gray bit/symbol mapping, the equivalent BER of the proposed differential M-PAM, $\mathbb{P}^{\text{bit}}_{eD\text{PAM}|h,\hat{h}}(h)$, can be approximated as

$$\mathbb{P}^{\text{bit}}_{eD\text{PAM}|h,\hat{h}}(h)=\frac{1}{\log_2(M)}\mathbb{P}^{\text{symb}}_{eD\text{PAM}|h,\hat{h}}(h). \quad (27)$$

Then, the BER of proposed differential PAM with the channel estimation of (21) is obtained by substituting (25), (26), (24) and (27) in (2), that should be solved numerically.

In order to obtain a closed-form expression for the integral in (2), we express the $K_a(x)$ and $Q(x)$ in terms of the Meijer's G-function [30], i.e., $K_a(x)=\frac{1}{2}G^{2,0}_{0,2}\left[\frac{x^2}{4}\middle|{}^{-}_{a/2,-a/2}\right]$ and $Q(x)=\frac{1}{2\sqrt{\pi}}G^{2,0}_{1,2}\left[\frac{x^2}{2}\middle|{}^{1}_{0,1/2}\right]$. By using these expressions as Meijer's G-function and using [31, Eq. 21], the closed-form expression for the BER is obtained as in (28) (on the top of the next page).

Note that, in the simplest case where $M=2$, we have just one threshold level which is equal to zero. In this simplest case, we find the same formulation as for the case of OOK modulation, as presented in [3].

B. Numerical results

In Figs. 4a and 4b for $k=2$ and 20, respectively, we have compared the performance of the proposed differential PAM signaling under perfect CSI knowledge and using the ML estimator of (21) for different lengths L of the observation window. As expected, by increasing L, we obtain a better channel estimate, and consequently, the BER performance gets closer to the that with perfect CSI knowledge. For instance, as we notice that for $k=2$ and a target BER of 10^{-6}, the SNR gap between the cases of perfect CSI and estimated channel from (21) is reduced from 5.5 dB for $L=1$, to about 2.8, 1.3, and 0.6 dB for $L=2$, 5, and 10, respectively. For larger k, the receiver noise level is more important, and hence, we require a larger L to obtain the same performance. For instance, the SNR gap between the cases of estimated and perfect CSI is reduced from 13 dB for $L=1$, to about 10, 7, 4, and 1 dB for $L=2$, 5, 10, and 50, respectively.

V. Conclusion

To improve the bandwidth efficiency of FSO systems and to reduce the impact of the background noise, we proposed a differential M-PAM signaling scheme, which allows at the same time to estimate the channel efficiently without requiring any pilot symbol transmission. It is worth mentioning that the proposed channel estimator has a very low computational complexity although it requires a slight increase in the system implementation complexity due to the requirement of the S-branch in Fig. 1, i.e., a second PD and the corresponding electronics. This would be quite justified given the advantage of having a low-cost channel estimator of zero pilot overhead.

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$$\begin{aligned}\mathbb{P}_{eD\text{PAM}}^{\text{bit}} =& \frac{2^{\alpha+\beta-3}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)M\log_2(M)} \times \Bigg\{ 2G_{0,2}^{2,0}\left[\frac{3L\log_4(M)(M-1)\Gamma_b}{4(M+1)\Big(L(M-1)^2+(M-2)^2(1+k)\Big)}\middle|\begin{matrix}1, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}\\ 0, 0.5\end{matrix}\right] \\ &+ \sum_{j=2}^{M-1}\Bigg(G_{0,2}^{2,0}\left[\frac{3L\log_2(M)(M-1)\Gamma_b}{4(M+1)\Big(L(M-1)^2+(-M+2j-2)^2(1+k)\Big)}\middle|\begin{matrix}1, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}\\ 0, 0.5\end{matrix}\right] \\ &+ G_{0,2}^{2,0}\left[\frac{3L\log_2(M)(M-1)\Gamma_b}{4(M+1)\Big(L(M-1)^2+(-M+2j)^2(1+k)\Big)}\middle|\begin{matrix}1, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}\\ 0, 0.5\end{matrix}\right]\Bigg)\Bigg\}. \end{aligned} \tag{28}$$

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