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Construction of Critical Thinking Skills by the infusion approach in “Probability and Statistics in Daily Life”

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This paper presents a teaching experiment that combined the explicit teaching of critical thinking with the content of an existing mathematics unit called “Probability and Statistics in Daily Life”. The original unit was designed to teach probability and statistics through real-life scenarios at the high-school level. I took the original mathematical content and “infused” it with a progression of critical thinking skills, so that both the mathematics and the critical thinking competencies developed hierarchically, growing more and more complex as the unit progressed. This paper illustrates the mutual benefits of this “infusion” to both the teaching of statistics and of critical thinking. The paper discusses the problem of transfer of critical thinking skills and shows some promising elements in that sense, deriving from the analysis. Finally, it also discusses some educational implications of the work done, the limitations of the unit’s first run and the improvements that may yet be made.

Keywords: Critical thinking, probability, statistics in daily life.

Introduction

This paper reports a research project undertaken into the utilisation of the mathematics classroom for the specific development of critical thinking skills through probability instruction (Aizikovitsh-Udi, 2012). In this paper, critical thinking skills are taken as representative of the aspirations of the majority of mathematics curricula. I suggest that the mathematics education community needs to investigate why the consensus regarding the importance of higher order thinking skills in mathematics has not led to greater consistency and success in its classroom implementation. This research represents one attempt to give classroom reality to this curricular aspiration and examined the processes of construction of critical thinking skills (e.g., identifying variables, suspension of judgment, referring to sources, searching for alternatives) during the study of the “Probability in Daily Life” learning unit in an infusion approach. In particular, I closely reviewed the contents of the learning unit for connection to relevant thinking skills. The skills that were found relevant were: (a) identifying variables; (b) referring to sources; (c) identifying assumptions; (d) evaluation of statements; (e) suspending judgment; (f) offering alternatives.

Theoretical Background

The term “critical thinking” (CT) has been used in academic circles for less than a century, but evidence of the relevance of this concept in education is far older, spanning several forms of human endeavor and 2500 years of human history since the Greek Antique. The universal applicability of these “ancient” skills is perhaps more relevant than ever in today’s complex and ceaselessly changing reality, which requires independent decision-making on a daily basis. Fostering and developing students’ ability to think critically, to be capable of engaging in inquiry and evaluation based on rational considerations regarding the various messages they are exposed to in different

areas of life, is therefore a particularly important part of their education (Bryan, 1987; Feuerstein, 2002; Glaser, 1972). While this need to focus on the promotion of CT skills has long been widely recognized by educators (Ku, 2009), they have not yet reached a similar consensus regarding *how best this should be done*. This on-going debate raises the question of whether critical thinking should be taught as a topic in its own right, or integrated into a topic already present in the school curriculum. This question in turn raises additional questions – for instance, if critical thinking *is* integrated, which subjects should it be integrated with? Furthermore, when teaching CT skills in conjunction with another topic, should these skills be taught *implicitly*, as a hidden component ‘immersed’ in the primary material, or should they be an *explicit* part of the learning experience, in which the students’ attention is drawn to their presence as an additional component ‘infused’ into the material through which it is being taught?

Amongst those who believe that critical thinking should be taught in conjunction with other subjects, one of the topics suggested for this purpose is mathematics. In the field of education, mathematics has traditionally been considered a branch of knowledge particularly suited to the teaching and learning of higher-order thinking skills such as critical thinking. Mathematics curricula all over the world, including Israel, identify the acquisition of these skills as one of their goals. The idea that mathematics is a discipline suited to teaching critical thinking also appears in the research literature in a more or less explicit way (Elder & Paul, 1994; Paul, Elder, & Bartell, 1997).

However, in spite of this assumption, very few empirical studies to date have engaged with the question of whether the study of school mathematics indeed develops or even requires this mode of thinking. The answer to this question is far from being clear. The study upon which this paper is based attempts an approach to teaching critical thinking through mathematics that *explicitly* integrates the topic into a learning unit designed to teach high-school level probability and statistics. This topic is, I believe, particularly well-suited for the acquisition and practical application of critical thinking skills. Our model is based on the combination of two theoretical elements: the hierarchical model of critical thinking skills presented in Ennis’ (1985, 1987a, 1987b) taxonomy and the “infusion approach” to teaching posited by Swartz & Parks (1994). To create the new learning unit, whose classroom implementation is presented in this paper, I took the mathematical content of an existing learning unit called “Probability and Statistics in Daily Life” (Lieberman & Tversky, 2001) and (in collaboration with one of this unit’s co-creators) I “infused” it with a hierarchical progression of critical thinking skills according to Ennis’ taxonomy (Ennis, 2002). In this paper, I present, in order of increasing complexity, a series of three samples from the classroom implementation of the unit. These illustrate a) how the two topics develop hierarchically together, and b) how each lesson combines them anew, calling on the students to draw on both their mathematical and their critical thinking skills to solve problems based on daily life.

Methodology

The “Probability and Statistics in Daily Life” learning unit as a basis for teaching CT

“Probability and Statistics in Daily Life” is a preexisting learning unit developed by Lieberman and Tversky (2001), which was expanded and modified for the purposes of this research. I selected this unit as a basis upon which to build the teaching experiment reported in this paper because it was

designed to teach mathematical content using problems and stories from daily life, and because its rationale already alluded to “elements of critical thinking”, citing these among the “issues relevant to daily life” that it hoped to teach (Lieberman & Tversky, 2001, p. 3).

The original unit was based on Tversky and Kahneman’s well-known work on making decisions in conditions of uncertainty (Tversky & Kahneman, 1974; Kahneman, Slovic, & Tversky, 1982). It covers topics in statistics and probability in hierarchical order, connecting each to daily-life scenarios and decisions. The purpose of the original unit was to turn students into ‘intelligent’ consumers of information by introducing them to modes of thinking that went beyond the mechanics of mathematical calculation. The use of problems from daily life exposed the students to various additional fields such as medicine, economics, and law, illustrating the practical applications of probability and statistics to these fields and showing the students the ways that statistical considerations are inextricably woven into our lives. Furthermore, the unit’s composition required them to analyze problems, raise questions and think critically about the numerical data and the information placed before them. Faced with problems that did not necessarily have one correct, clear-cut answer, the students learned not to be satisfied with arriving at a numerical solution, but to assess the validity of data, and to assess the problems before them in a qualitative – and not just a calculative – manner. I composed a revised version (Aizikovitsh-Udi, 2012). The unit combines the hierarchy of topics in probability and statistics with a corresponding hierarchy of topics in critical thinking, so that, as the students progressed in the former, they would also progress in the latter.

Setting, population and data collection

Results presented here are from a subgroup of one class taken out of a larger population of six classes (147 students in all, three experiment groups [70] and three control groups [77]). The larger population was used for deriving quantitative results regarding the experiment’s efficacy, which are not presented here. In this paper, I will present two groups: 18 students were taught by the researcher in high school 1, 20 students were also taught by the researcher in high school 2 in central Israel.

The experiment consisted of 15 sessions (90 minutes each) during the course of the academic year and served as the “probability and statistics” section of the students’ mathematics curriculum for that year. The group whose sessions are described here was the one taught by me, in my capacity as these students’ regular mathematics teacher. Data collection was conducted by way of triangulation between the following sources: (i) The students’ written products, including exams, in-class papers, and homework were collected. (ii) Sessions were recorded, transcribed and analyzed (paying special attention to their relation to CT skills). The teacher kept a log on every session. In general, data were processed by means of qualitative methods, which enabled me to follow the students’ patterns of thinking. (iii) Personal interviews: 27 students were randomly chosen (four from each of the seven experimental classes) and interviewed at the end of the first and second semesters, in the middle and at the end of the unit. Personal interviews were conducted in order to reveal changes in the students’ attitudes towards critical thinking throughout the academic year. The interviews were of two kinds: closed/structured interviews, based on questions chosen in advance, and open/semi-structured interviews, where only part of the questions chosen in advance was asked (possibly in

modified form) according to the interviewees' answers. The interview questions were: (a) What do you think about the importance of critical thinking ability? In which fields/activities is it important? (b) Can you give an example of a situation (from school, everyday life, etc.) where critical thinking is necessary? Have you used it? Did it help you? (c) In your opinion, is it possible to develop/improve critical thinking ability? How? Do you have any suggestions for improving it? (d) In your opinion, is it possible to change dispositions for critical thinking? How? What influences the dispositions? Do you have any suggestions for improving them? (e) Did your studies in the other disciplines improve your ability for critical thinking? If yes, in what course and in what way? If not at all, why not? (f) How would you evaluate yourself in the area of critical thinking? The intended aims of the six questions were: (1) To ascertain the degree of students' awareness of the nature of critical thinking (mainly questions a, b, e, f, through the pertinence of the examples and the answers). (2) To identify students' ideas about possible fields of application for critical thinking (a, b), and in particular to identify suggested fields far from those proposed during the course (as a sign of interiorization and a premise for a possible transfer). (3) To get feedback about students' perception of which aspects/moments of the course had an impact on their CT skills and dispositions (c, d), as a way to assess students' awareness of the aims of the course.

Results

Analysing the Findings by two lessons

In what follows, I present two lessons, taken from three progressive points within the learning unit. For each, I present the daily-life topic upon which the lesson was based and show how first the mathematical content and then the thinking skills were integrated into it. In addition to exemplifying how the two topics were tied together, the succession of the lessons shows how both the mathematical content and the critical thinking skills built upon themselves hierarchically as the unit progressed, involving students in meaningful activities related to the aims of the learning unit (development of both CT and mathematical competencies). Each of the three selected samples also highlights a different element of the program. The first lesson, in which the students are sent outside of the classroom to gather data for themselves, highlights the centrality of the practical "daily life" element in the study unit. As this example shows, each daily life story or problem places the students in a position where they must draw on both the mathematics and the critical thinking skills to gather the information they need to make their decision. The second lesson is the most elaborated and detailed of the three examples, and it shows (through a long excerpt and the related analysis), how the melding working hypothesis was implemented (thus showing its feasibility). It also shows how by that point in the learning unit specific features of CT had already been interiorized by students, and how specific CT expressions entered students' language and were used by them in an appropriate way (see Discussion). The third example stresses the four-step format of each lesson, and emphasizes the function served by each: It shows how the daily life scenario triggered the students' intuitive response, which was then informed and modified by the mathematics before being rethought and adjusted a second time by further application of CT. This problem shows the unit's lessons at their final and most advanced stage, where the students see that CT can be used not only to support conclusions based in mathematics, but also to look beyond them.

Example #1: The shoe size problem as an introduction to Simpson's Paradox¹

This lesson (Aizikovitsh & Amit, 2008), taken from the point of transition between chapters one and two, introduced the mathematical concept of the “mediating factor” (see below). In terms of critical thinking, it engaged the students in: a) identification of relevant questions and variables; b) locating the source of information; c) evaluating/analyzing the validity of statements. The students were given the following dialog called “Shoes and Mathematics”:

Avi: “There is a connection between shoe size and level of mathematical knowledge.”

Beni: “Can't be.”

Avi: “Go to the school in the next building and see for yourself.”

Beni: “You are right, the kids who wear bigger shoes really know math better!”

Why is this phenomenon true? What do you think about the conclusion?

The purpose of this lesson was to show the students that while causal connections *are* predicated on the presence of statistical connections, the presence of a statistical connection does *not* necessarily mean that a causal connection exists. Most of the students had a strong initial intuition that the causal connection was impossible, but they did not yet know how to explain and support it mathematically. After the preliminary discussion of the students' intuitions, several of them were sent out to ‘gather evidence’ at the nearby elementary school, stopping random students and asking them: a) their shoe size, and b) what mathematical topics they were familiar with. This exercise introduced a critical element into the lesson, with the students validating the reliability of their source by gathering the (seemingly) corroborative data themselves. The ultimate purpose of this lesson is that the students recognize the logical fallacy in the connection suggested here between shoe size and mathematical knowledge by realizing that this connection is generated by a mediating element (the child's age). The mathematical content of the lesson consisted in the translation of the information into a mathematical format (set theory) and the use of a two-dimensional matrix. I also performed new calculations that included the mediating factor C. I found that statistical connections exist between A (shoe size) and C (age), as well as between B (mathematical knowledge) and C (age). This, I argued, is the reason for the statistical connection between A and B. By means of this process and its illustration through the real-life example of shoe size, the students were introduced to Simpson's Paradox and the concept of the mediating factor. The critical thinking portion of this lesson consisted primarily in the students learning to evaluate and question the validity of their information, even when they had obtained that information themselves, in recognizing the fallacious connection and in drawing an alternative, valid conclusion.

¹ Simpson's Paradox refers to the situation where normalizing data from different ways of partitioning the same population will provide incompatible conclusions about the associations that hold in the total population. For example, a partition by gender might indicate that both males and females fared worse when provided with a new treatment, while a partition of the same population by age indicated that patients under fifty, and patients fifty and older both fared better given the new treatment. The relevant element for this paper is the role of the mediating variable (age or gender).

Discussion & Conclusion

As the examples above show, the mathematical content of statistics and probability is well suited to being 'infused' with instruction in critical thinking skills. The hierarchical patterns in which both the content and the skills can be taught complement one another structurally, with more elements of critical thinking coming into play as the mathematics becomes progressively more complex. The two instructional goals (statistics and critical thinking) are also mutually beneficial in terms of content: The mathematics provides the students with a means to engage in critical thought and pursue independent challenges and confirmations to the information they are given. The "Probability in Daily Life" unit provides the mathematics with a practical 'real-world' context that can help the students comprehend the material more concretely as more than the abstract manipulation of numbers on a page. The various critical thinking skills could then be taught explicitly in the context of using mathematics to solve problems in the real-world situations. The addition of critical thinking provides an added reflective dimension to the mathematical treatment rendering both the approach and the results more meaningful to students.

Analyzing the findings, I have arrived at the following generalizations regarding the process of critical thinking skills construction and teaching: (1) It seems that critical thinking skills do not develop spontaneously and that even good students acquire them by means of explicit instruction. This finding is in direct opposition to Tennyson & Rasch (1988) claim that learning skills and learning strategies develop in the student spontaneously, without direct instruction. (2) To a large extent, the construction and teaching of critical thinking skills are determined by specific contents and tasks the teacher uses. In this research, the skills were chosen with respect to the contents and the increasing difficulty level of the learning unit. (3) It is possible to significantly influence and change the mathematical discourse of the classroom and the students' language of critical thinking, by providing appropriate conditions and using appropriate instruction methods. This type of learning emphasizes the development of skills in the process of solving mathematical problems.

In much of the literature, critical thinking development is referred to as an important goal of the educational system. This research may contribute to the public discourse of the mathematical education community on this issue. It also raises the public awareness of the need to develop critical thinking in the framework of mathematical education, which may enable future examination and promotion of critical thinking development through mathematics teaching in a fuller and more informed way. To conclude, the main contribution of this research lies in revealing the connection between critical thinking and the teaching of mathematics. It should be possible to strengthen the status of the study of mathematics in imparting higher-order thinking skills, both in parallel with and beyond the formal education program. This paper has reported an investigation into the use of the infusion approach to teach critical thinking skills while also teaching conventional probability and statistics content. The students acquired critical thinking skills that they came to value.

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