



**HAL**  
open science

## Boundary crossing by design(ing): a design principle for linking mathematics and mathematics education in preservice teacher training

Erik Hanke, Angelika Bikner-Ahsbabs

### ► To cite this version:

Erik Hanke, Angelika Bikner-Ahsbabs. Boundary crossing by design(ing): a design principle for linking mathematics and mathematics education in preservice teacher training. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02417396

**HAL Id: hal-02417396**

**<https://hal.science/hal-02417396>**

Submitted on 18 Dec 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Boundary crossing by design(ing): a design principle for linking mathematics and mathematics education in preservice teacher training

Erik Hanke<sup>1</sup> and Angelika Bikner-Ahsbabs<sup>2</sup>

<sup>1</sup>University of Bremen, Germany; [erik.hanke@uni-bremen.de](mailto:erik.hanke@uni-bremen.de)

<sup>2</sup>University of Bremen, Germany; [bikner@math.uni-bremen.de](mailto:bikner@math.uni-bremen.de)

*Preservice teachers experience a fragmentation in their study program between two academic areas, university mathematics and mathematics education. To reduce this problem, we conduct a design study linking mathematics and mathematics education on two levels: First, the “dovetailing” of mathematics (here the course on complex analysis) and mathematics education, and second, students’ “interlinking” of content and pedagogical content knowledge in the development of learning materials for talented pupils. This paper contributes to the topic on theory and methodology by introducing the design principle “boundary crossing by design(ing)” in “nested design cycles”.*

*Keywords: Boundary crossing, boundary object, complex analysis, reflection, teacher education.*

## Introduction

Many preservice teachers in mathematics experience two types of discontinuities in mathematics teacher training. First, in their views, much of the mathematics they learned in school does not seem very helpful for the rigorous proof-oriented style in university mathematics courses. Second, preservice teachers may not see how the mathematics in their university courses is related to their practice as a teacher (Hefendehl-Hebeker, 2013; Prediger, 2013). Several attempts to minimize the perceived gap between university and school mathematics have been developed, for example with special exercises, or through capstone courses (e.g., Bauer & Partheil, 2009; Winsløw & Grønbaek, 2014). Furthermore, empirical studies show that the combination of mathematical knowledge and pedagogical content knowledge for teaching and learning mathematics (i.e. in the sense or similar to that of Ball, Thames, and Phelps (2008)) is important for evaluating situations in the classroom (e.g., Blömeke et al., 2014). Mehlmann and Bikner-Ahsbabs (2018) give an overview of recent trends in German mathematics preservice teacher education around the “double discontinuity” and the general criticism of preservice teachers towards their study programs which they experience as fragmented. Because there is barely any explicit research on how preservice teachers may link the scientific disciplines of mathematics and mathematics education<sup>1</sup> concerned with school mathematics, we ask how this fragmentation between the mathematical discipline at university and mathematics education in preservice teacher training can be overcome at least partially by making changes in the curriculum, and how preservice teachers establish interlinkages of the two areas. At the University of Bremen, we conduct the design study “Spotlight-Y” on these issues. Two out of

---

<sup>1</sup> We understand mathematics education as the scientific discipline about the teaching and learning of mathematics. In German preservice teacher training, mathematics education is mainly concerned with school mathematics.

three design cycles have been completed so far, and as a theoretical result from our first design cycles, we will establish the design principle *boundary crossing by design(ing)* in a *nested design cycle* to initiate the overcoming of the fragmentation in preservice teacher training. We will show how this design principle is interwoven with the methodology and theory of our study. At the end, examples of empirical findings round off the paper.

## **Linking mathematics and mathematics education in a two-fold way**

Linking mathematics and mathematics education can be viewed as a two-fold phenomenon in higher education: On the institutional level it is a matter of curriculum and we call the linkage of the domains on this level *dovetailing* (German: Verzahnen). On the students' level we call the linkage of the domains in students' thinking and acting *interlinking* (German: Vernetzen). This distinction corresponds to a distinction between "curriculum to be taught" and "learned curriculum". Our design study is devoted to an empirically grounded description of these two notions and their connections. Thus, with this terminology our general research questions in the project can be rephrased as: How can dovetailing be realized to initiate interlinking? Which circumstances foster or hinder students' interlinking and which interlinking strategies can be identified? To interlink the two scientific areas, our students have to cross the boundaries between them.

## **Theoretical background: Boundary crossing**

The idea of boundary crossing has origins and applications in the fields of general education and social theory (Akkerman & Bakker, 2011). Boundaries can be described as "sociocultural differences that give rise to discontinuities in interaction and action" (Akkerman & Bakker, 2011, p. 139) and boundary crossing as the "efforts by individuals or groups at boundaries to establish or restore continuity in action or interaction across practices" (Bakker & Akkerman, 2014, p. 225). Means to achieve this are boundary objects which "are artifacts that articulate meaning and address multiple perspectives" (Akkerman & Bakker, 2011, p. 140), e.g., a concrete task or solution. Aside from this initial understanding of a boundary object, it can also be conceptualized as a "shared problem space" (Akkerman & Bakker, 2011, p. 147) where motives of different domains act together and "materiality derives from action, not from [...] 'thing'-ness" (Star, 2010, p. 603).

It is the interpretation of the members of the communities that give objects their meaning as boundary objects in social contexts (Star, 2010). Consequently, even if teachers or researchers intend to implement objects as boundary objects in their teaching or research, it may not be the case that the people who work with the objects understand them as such. However, because the aim is "to integrate different types of knowledge typically developed in different practices" (Bakker & Akkerman, 2014, pp. 224–225), boundary objects "may provide learning opportunities" (Akkerman & Bakker, 2011, p. 141), and students working with boundary objects may carry out interlinking actions or even practices, probably implicitly. Interesting for our purpose are three out of four mechanisms for the process of boundary crossing identified by Akkerman and Bakker (2011) when considering learning in terms of boundary crossing in the broadest sense. *Identification* is about finding out particularities of the social practices and seeing the relevance of different practices which meet at a certain border. *Coordination* means to mediate between social worlds. *Reflection* differs from identification in the sense that particularities of one community's practice are made

explicit for another community (*perspective making*) or are regarded from the viewpoint of the other community (*perspective taking*). In Spotlight-Y the notion of reflection is much broader than the boundary crossing mechanism of reflection (see section “*Boundary crossing by design(ing) through a nested design approach*”), but perspective making and taking can be part of it.

## **Methodology**

### **Curricular linkage: Dovetailing in the lecture on complex analysis**

The project “Spotlight-Y” consists of three cycles of developing the course on complex analysis with the aim to overcome fragmentation (see Hanke & Schäfer 2018a, 2018b; Mehlmann & Bikner-Ahsbals, 2018, for details). Each cycle takes a year with a lecture in complex analysis being split into a branch for future teachers and future mathematicians after around nine weeks of the lecture. While the future mathematicians continue with more advanced complex analysis, the preservice teachers prepare learning materials (exercise sheets and means to explore and visualize with GeoGebra) for talented secondary school pupils about a mathematical topic they identified in the lecture (e.g., power series expansion, spherical geometry, or differentiation as dilation-rotation) and implement this on a day for experimental mathematics for classes of school students visiting university (XMaSII). In parallel with the complex analysis lecture, the preservice teachers participated in a seminar in mathematics education which also covered the design of tasks. Our assumption is that through boundary crossing in preparing and implementing of this practical teaching, the preservice teachers will overcome fragmentation by showing interlinkage between the knowledge from both areas. So far, 35 students participated in the teacher branch in complex analysis during the last two years.

### **Approach for designing the lecture**

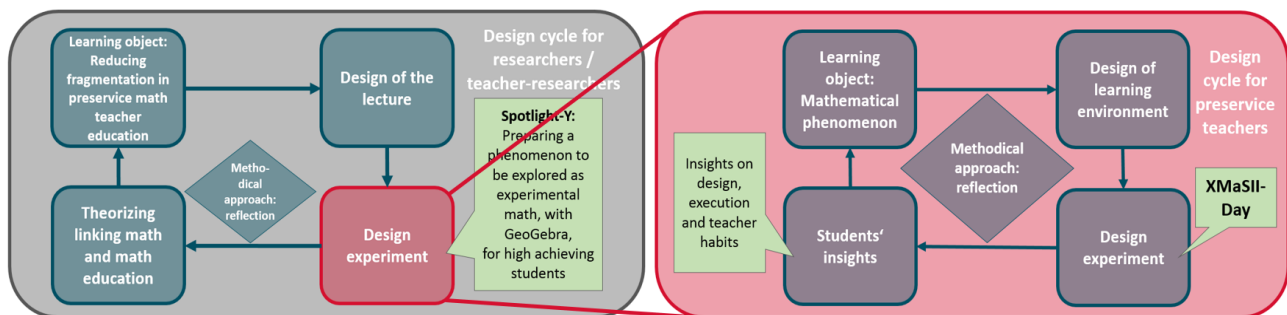
To develop the teaching of complex analysis as described above we conduct a design study (Gravemeijer & Cobb, 2006). It aims at achieving a design as well as local theoretical knowledge about how the design works, clarifying circumstances under which the fragmentation in teacher training can be alleviated, and helping to identify obstructions. Each design cycle consists of four elements (Prediger et al., 2012; see Figure 1, left): First, a learning object or phenomenon has to be restructured. This leads to the development of a (local) design<sup>2</sup> based on a specific aim. Then, the design is executed in a design experiment. Insights from these steps are used to theorize the phenomenon, and these insights allow for a better description of the learning object or phenomenon, changes in the curricular implementation, design of the lecture, and execution. The cycle can then start over again. The core idea of our methodological frame is the nature of the design experiment. It contains an embedded cycle of the same structure that initiates “boundary crossing” in students’ work (Figure 1, right).

---

<sup>2</sup> The word *local* is used here to distinguish the overall design in the design study “Spotlight-Y” from the designs developed in it: In the outer research cycle it is the course design of complex analysis including teaching strategies, and in the inner cycle it stands for the designs the students develop for their learning arrangement (see Figure 2).

## Boundary crossing by design(ing) through nested design cycles

In our design study, two cycles are nested (Figure 1). The outer cycle (grey, left) is the methodical frame for research and teaching: The learning object for us as researchers is dovetailing and interlinking university mathematics and mathematics education in preservice teacher education. The general design of the lecture was described in the section “Curricular linkage: Dovetailing in the lecture on complex analysis” and the design experiment is the course work in the branch for preservice teachers in the lecture on complex analysis. The inner circle (red, right) frames the development and implementation of the learning materials by our students. Here, the learning objects (for students and pupils) are the mathematical phenomena the students choose from the lecture.



**Figure 1: Nested design cycles in Spotlight-Y**

The process is accompanied with three types of written reflective activities. We understand *reflection* as anticipatory as well as retrospective (re-)consideration of own work, including experiences of various kinds, thoughts about personal relationships to the two disciplines (mathematics and mathematics education), the own role as a future teacher, and possibilities for the improvement of local designs of learning arrangements. In a *pre-reflection*, the preservice teachers describe the mathematical phenomenon and anticipate difficulties at the beginning of their planning, in *ad-hoc-notes*, they reflect on observations right after the execution of XMaSII, and in a final *portfolio*<sup>3</sup> they reflect on the full process and their professional development, each with guiding stimuli (Hanke & Schäfer, 2018a, 2018b). This is in accordance with the notion of reflection-on-action as proposed by Schön (1983) and reflection-pre-action by Bikner-Ahsbals (2017).

Instances of interlinking mathematics and mathematics education can be identified in students' reflections from which we extract interlinking strategies (see results section). Such a strategy may be a certain local mechanism a student implemented, or a personal insight. Since the reflections are written after a teaching episode with classes of school students (the design experiment), we might also reconstruct interlinking strategies which have not appeared in action, but which have been thought about by the students. The reflections have two functions: First, they serve to reconstruct and typify the interlinkages between university mathematics and mathematics education, and

<sup>3</sup> A portfolio is a collection of all written work, this includes the exercise sheets, written planning and description as well as answers to final reflective questions (see Hanke & Schäfer, 2018a, 2018b, for details).

second, students experience reflection as a means to learn from own practical experience (planning, executing, assessing) and be aware of interlinking activities (Mehlmann & Bikner-Ahsbahs, 2018).

In the inner cycle, in the branch for the preservice teachers, the boundary of the scientific lecture and mathematics education is crossed by referring to each other. The *initial boundary object* is the student task to create the XMaSII-day learning arrangement. This task can be interpreted from the mathematical as well as the mathematics education side. The specificities of the implementations and reflections indicate how the students interlink both areas. The work of the students leads to a design process which initiates boundary crossing. All data, in the form of exercise sheets, computer arrangements and different kinds of written reflections shape a *dynamic boundary object* which is assembled by the students' work at the boundary of mathematics and mathematics education and which, at the same time, is subject to change. We call this design principle (van den Akker, 1999) *boundary crossing by design(ing)*. "By design" refers to the curricular implementation, and "by designing" refers to the processes the students engage in. The boundary crossing or interlinking mechanisms we observe stand for the interlinking practices of students and can be further used for the next cycle of the branch for preservice teachers in the lecture on complex analysis. In that sense, the design principle *boundary crossing by design(ing)* serves as our epistemic means to describe to which results acting at the boundary of mathematics and mathematics education may lead. In other words, *dovetailing* is the attempt to initiate boundary crossing and the students' boundary crossing may lead to thinking and acting between mathematics and mathematics education, i.e. *interlinking*. Thus, boundary crossing is regarded as a mediator between dovetailing and interlinking.

The novelty of *boundary crossing by design(ing)* is not the fact that mathematics and mathematics education are asked to be combined—this is expected practice of any (future) teacher. It is that the linkage we try to establish is between the two scientific domains, mathematics at university and mathematics education for preservice students' teaching (cf. Introduction).

### **Exemplary results of the first two cycles**

Having developed the methodology and the design principle, we conjecture that implementing the design principle into the third cycle will initiate students' interlinking between complex analysis and mathematics education, and we ask whether we will find the same or new interlinking strategies of the students. But how do these interlinkings show up and what conditions foster or hinder them? Until now, this is answered by analyzing the data of the first two cycles, and here, we give some examples.

Results indicate that interlinking is an additional demand (concerning mathematical expertise and/or time) which is often not accomplished by our preservice teachers, and happens in microscopic ways. Besides others, the students were asked to *describe which elements of the course on complex analysis changed their views on the scientific discipline of mathematics and mathematics as a school subject*. This question aimed at a retrospection on the course as a whole. One of the students wrote (we note that the learning arrangement of this student did not deal with extension of number sets but with differentiation):

The extension of number sets with the set of complex numbers is very demanding at first. Pupils who encounter negative numbers for the first time might feel similarly. The experiences I made

in this course can help me to empathize with pupils and understand why the extension of numbers with the set of irrational numbers can be difficult for them and why they may not consider the number  $\pi$  as a number.

In addition, the experience of the lacking imagination of the graphical representation of complex functions can help to understand the difficulties pupils have with the visualization of graphs of functions. (translated from German)

This description expresses the sensitization for pupils' difficulties by analogy. The example in the quote shows an experience-based perspective on problems with extensions of number sets and diagrams which can be regarded as a coordination mechanism with the potential for a transformation mechanism (see: Akkerman & Bakker, 2011). For example, a transformation could occur if the student presented a suggestion for how to deal with these issues in class. We may conclude that in this case the student's own experience with subject matter sensitizes for learning processes of pupils. What is not important is that the content in the student's learning process is beyond school but that what is learnt, in each case, is new for the respective learner (student and pupil).

To the stimulus "*What did you learn personally? Do you feel assured in your role as a teacher? How and why (not)?*" another student expressed (his learning arrangement was about power series),

So I find the problem to prepare a very complex and alien (German: fremd) topic for pupils very interesting and instructive because I, myself, had to cope with this topic intensively once again.

Unfortunately, I had to realize that exactly in this transformation very much time and work had to be invested which lacked for example for the precise planning and execution. At any time, the subject specific understanding (German: das fachliche Verständnis) was the most important aspect of the planning. (translated from German)

This utterance shows that the creation of a learning arrangement about a demanding mathematical topic requires resources in order to engage deeply with the subject matter. Here, the need for these resources sidelined the planning actions for the actual planning and implementation.

## **Discussion and reflection**

Reflecting on the two design cycles at the university level we went through, the steps of how the design principle developed becomes clear. Due to the fragmentation in preservice teacher education, our first design principle was to *link university mathematics and mathematics education through providing practical experience for our students in one mathematics lecture*. The method seemed clear; the students identify a mathematical phenomenon from the lecture, plan and implement learning materials for talented pupils, and thus, overcome fragmentation. But this did not happen with most of the students. Therefore, after the completion of the first design cycle, we clarified the relationship between this method and our research design. That is, our students were actually performing a design cycle similar to ours on the research level. Thus we obtained a more precise methodology, the *nested design approach*. Since we were not able to rely on previous research, we had to bridge the distinction between *dovetailing* and *interlinking* theoretically. The theory of *boundary crossing* serves this purpose. The concise link between theory and methodology was

achieved after the second cycle and on this basis we obtained the current design principle. *Boundary crossing by design(ing)* condenses the theoretical approach of boundary crossing and links it to the task of creating and implementing learning materials while taking into account two scientific domains. This design principle addresses two different kinds of design: *design research* at the university level for course development in which the students *design learning arrangements*. These threads together can lead to insights into the reduction of the fragmentation in teacher training we strive for. Ultimately, our project exemplifies that a suitable design principle does not necessarily stand at the beginning of a design study but can be achieved through development and confluence of methodology and theory.

Our students valued the practical experience in the project. Even when they did not manage to produce explicit interlinkages of the two domains, in some cases of interlinking the two domains we observed a kind of coordinating, a mechanism of boundary crossing. Hence, we should ask what kind of conditions hindered the preservice teachers to experience interlinking mathematics and mathematics education (of school mathematics). We next aim at developing a scaffolding procedure. For example, cognitive overload might be avoided by a concrete example of how a phenomenon of complex analysis could be transposed into a learning arrangement for talented pupils or by a more detailed structure for the task to create a learning arrangement that assists preservice teachers in their boundary crossing.

Last, we highlight an important epistemic obstruction resulting from our methodological approach. The contents and depths in the students' reflections limit the depths of our insights. Also, reflections mainly on a descriptive level are at most an indicator for students' weak interlinking. This means we also have to reconsider the stimuli which we ask our students to consider.

### **Acknowledgment**

This project is part of the "Qualitätsoffensive Lehrerbildung", a joint initiative of the Federal Government and the *Länder* which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research (no. 01JA1612). The authors are responsible for the content of this publication.

### **References**

- Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169.
- Bakker, A., & Akkerman, S. F. (2014). A boundary-crossing approach to support students' integration of statistical and work-related knowledge. *Educational Studies in Mathematics, 86*, 223–237.
- Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. What makes it special? *Journal of Teacher Education, 59*(5), 389–407.
- Bauer, T., & Partheil, U. (2009). Schnittstellenmodule in der Lehramtsausbildung im Fach Mathematik. *Mathematische Semesterberichte, 56*(1), 85–103.



- Bikner-Ahsbals, A. (2017). Design Research—ein Ansatz zum Forschenden Lernen. In S. Doff & R. Komoss (Eds.), *Making change happen. Wandel im Fachunterricht analysieren und gestalten* (pp. 87–110). Wiesbaden, Germany: Springer VS.
- Blömeke, S., König, J., Busse, A., Suhl, U., Benthien, J., Döhrmann, M., & Kaiser, G. (2014). Von der Lehrerausbildung in den Beruf – Fachbezogenes Wissen als Voraussetzung für Wahrnehmung, Interpretation und Handeln im Unterricht. *Zeitschrift für Erziehungswissenschaft*, 17(3), 509–542.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17–51). London: Routledge.
- Hanke, E., & Schäfer, I. (2018a). Learning complex analysis in different branches – Project Spotlight-Y for future teachers. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild, & N.M. Hogstad (Eds.), *Proceedings of the Second Conference of the International Network for Didactic Research in University Mathematics (INDRUM2018, 5-7 April 2018)* (pp. 54–63). Kristiansand, Norway: University of Agder and INDRUM.
- Hanke, E., & Schäfer, I. (2018b). Modellprojekt 2: Spotlight-Y (Mathematik). In Universität Bremen (Ed.), *Resonanz. Magazin für Studium und Lehre an der Universität Bremen. Sonderheft 2018: Schnittstellen gestalten. Das Zukunftskonzept für die Lehrerbildung an der Universität Bremen. Professionalisierung zum Reflective Practitioner* (pp. 70–74). Bremen, Germany: Universität.
- Hefendehl-Hebeker, L. (2013). Doppelte Diskontinuität oder die Chance der Brückenschläge. In Chr. Ableitinger, J. Kramer, & S. Prediger (Eds.), *Zur doppelten Diskontinuität in der Gymnasiallehrerbildung. Ansätze zu Verknüpfungen der fachinhaltlichen Ausbildung mit schulischen Vorerfahrungen und Erfordernissen* (pp. 151–168). Wiesbaden, Germany: Springer.
- Mehlmann, N., & Bikner-Ahsbals, A. (2018). Spotlights Lehre – Ein Ansatz zur Vernetzung von Fachwissenschaft und Fachdidaktik an der Universität Bremen. In I. Glowinski, A. Borowski, J. Gillen, S. Schanze, & J. von Meien (Eds.), *Kohärenz in der universitären Lehrerbildung. Vernetzung von Fachwissenschaft, Fachdidaktik und Bildungswissenschaften* (pp. 77–102). Potsdam, Germany: Universitätsverlag Potsdam.
- Prediger, S. (2013). Unterrichtsmomente als explizite Lernanlässe in fachinhaltlichen Veranstaltungen. In Chr. Ableitinger, J. Kramer, & S. Prediger (Eds.), *Zur doppelten Diskontinuität in der Gymnasiallehrerbildung. Ansätze zu Verknüpfungen der fachinhaltlichen Ausbildung mit schulischen Vorerfahrungen und Erfordernissen* (pp. 151–168). Wiesbaden, Germany: Springer.
- Prediger, S., Link, M., Hinz, R., Hußmann, S., Ralle, B., & Thiele, J. (2012). Lehr-Lernprozesse initiieren und erforschen—Fachdidaktische Entwicklungsforschung im Dortmunder Modell. *MNU*, 65(8), 452–457.
- Schön, D. (1983). *The reflective practitioner. How professionals think in action*. New York: Basic Books.

- Star, S. L. (2010). This is not a boundary object: Reflections on the origin of a concept. *Science, Technology, & Human Values*, 35(5), 610–617.
- van den Akker, J. (1999). Principles and methods of development research. In J. van den Akker, R. Branch, K. Gustafson, N. Nieveen, & T. Plomp (Eds.), *Design approaches and tools in education and training* (pp. 1–14). Dordrecht, the Netherlands: Kluwer Academic Publishers.
- Winsløw, C., & Grønbæk, N. (2014). The ecological Klein's double discontinuity revisited: contemporary challenges for universities preparing teachers to teach calculus. *Recherches En Didactique Des Mathématiques*, 34(1), 59–86.