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# Comparing a priori analyses

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*The present research study is a theoretical reflection on the notion of a priori analysis in two different theories: the Theory of Didactical Situations (TDS) and the theory of Abstraction in Context (AiC). For both theories the epistemological perspective is of importance. The a priori analyses offered by the two theories are different. This difference reflects the different priorities of focus of analysis of both theories. The study demonstrates that, in their effort of networking theories, researchers of both theories will benefit of comparing their a priori analyses.*

*Keywords: A priori analysis, a posteriori analysis, Abstraction in Context (AiC), networking theories, Theory of Didactical Situations (TDS).*

## Introduction

The most recent Working Group on theory discussed the importance of developing networking with regard to the design dimension. As we will see in the following, the notion of a priori analysis is closely related to the phase of design. The present paper offers a theoretical reflection on the notion of a priori analysis in two different theories. It investigates the differences in the a priori analyses of the two theories and the influence of the different a priori analyses on the networking of theories. Dealing with a priori analyses, the focus of the paper is on methodology.

The theories discussed in this paper are the Theory of Didactical Situations (TDS) and the theory of Abstraction in Context (AiC). There are two main reasons for the choice of these theories. The first one is that this paper is based on my networking experience with these theories (Kidron, Lenfant, Bikner-Ahsbahs, Artigue, & Dreyfus, 2008; Kidron, Artigue, Bosch, Dreyfus, & Haspekian, 2014). The second reason is that the a priori analyses take into account the mathematical epistemology of the given domain and for both theories the epistemological perspective is of importance. The two theories consider the epistemological dimension in different ways. TDS combines epistemological, cognitive, and didactical perspectives. TDS focuses on the epistemological potential of didactical situations. AiC analysis is essentially cognitive and focuses on the students' reasoning; mathematical meaning resides in the verticality of the knowledge constructing process and the added depth of the resulting constructs. An epistemological stance is underlying this idea of vertical reorganization (this term will be explained in more details in a next section). For both theories the epistemological dimension has an important role. In the literature, the important role of epistemology is discussed in details in Artigue (1990, 1995). Kidron (2016) analyses the connection between epistemology and networking of theories.

In the next two sections, I will present shortly each theory and deal with the question what is an a priori analysis for each theory. Then, I will deal with the differences of the a priori analyses and demonstrate that, in their effort of networking theories, researchers of both theories benefit of comparing their a priori analyses.

## **TDS a priori analysis**

A short introduction to TDS is offered by Artigue, Haspekian and Corblin-Lenfant (2014). This well-established theory belongs to the French school. It began to develop in the 1960s in France, initiated by Guy Brousseau (1997). Artigue et al. (2014, p. 48) wrote;

... we focus on three characteristics that create the specific lens through which TDS considers the teaching and learning of mathematics: the systemic nature of teaching and learning; the epistemology of mathematical knowledge; and the vision of learning as a combination of adaptation and acculturation. These characteristics determine the questions that TDS raises and tries to answer, as well as the methodologies it privileges.

The first characteristic, the systemic perspective of TDS is expressed by the central object of the theory, the idea of situation, which is itself a system. A special attention is given to the second characteristic, the epistemology of mathematical knowledge, while discussing TDS a priori analyses. The third characteristic relates to the cognitive dimension.

The theory is structured around the notions of a-didactical situation (in a-didactical situation there is no didactical intentions and the teacher refrains from interfering) and didactical situations, and includes concepts relevant for teaching and learning in mathematics classrooms. The social dimension also has an important role in TDS. In essence, the central object of the theory, the situation, incorporates the idea of social interaction.

The systemic view led to the concept of *didactical engineering* (Artigue, 1989, 2013) which is explained in Artigue et al. (2014, p. 50):

It is a methodology which is structured around a phase of preliminary analysis combining epistemological, cognitive, and didactical perspectives, and aiming at the understanding of the conditions and constraints to which the didactical system considered is submitted, a phase of design and a priori analysis of situations reflecting its optimization ambition; and, after the implementation, a phase of a posteriori analysis and validation.

The notion of “milieu” is an important construct in TDS. The a-didactic milieu was initially defined by Brousseau as the system with which the student interacts in the a-didactic game. In the design of learning situations, there is a special attention to the constituents of the milieu organized for the learner.

In her chapter “Perspectives on design research: the case of didactical engineering”, Artigue (2015) presents the evolution of Didactical Engineering (DE) in the last three decades and explains its links with TDS. She also presents its characteristics as a research methodology. In this chapter we read that design has always played a fundamental role in the French school. We also read how design is connected to the a priori analysis.

In Artigue et al. (2014, pp. 54–60), we have a detailed example that explains the components of the TDS a priori analysis and the requests of the a priori analysis for developing the systemic analysis typical for TDS. For example, the need for information of the mathematical knowledge of the

students, of the particular situation at stake, of the teacher's expectations regarding this situation. The methodology for analysis is described in the following sentence:

We developed thus our analysis using the usual techniques of TDS, that is to say, preparing an a priori analysis focusing on the determination of the cognitive potential of an a-didactic interaction with the milieu, for a *generic* and *epistemic* student, that is, a student accepting the a-didactical game and able to invest in it the mathematical and instrumental knowledge supposed by the teacher. (Artigue et al., 2014, p. 63)

In TDS a priori analysis, the researchers make assumptions about the supposed mathematical knowledge of the students which is necessary for a productive interaction with the "milieu". The a priori analysis must then play its role of reference as well as its role of revealing the didactic phenomena. Then the a posteriori analysis is compared to the a priori analysis and sometimes the hypotheses which were done in the a priori analysis are not in accord with the a posteriori analysis of the collected data. This comparison of the a priori analysis and the a posteriori analysis is important for the TDS researchers in order to deeply understand the functioning of the "situation".

### **AiC a priori analysis**

Dreyfus & Kidron (2014) offers a short introduction to AiC. The theory is explained in more details in (Schwarz et al., 2009). AiC has been developed over the past 18 years with the purpose of providing a theoretical and methodological approach for researching, at the micro-level, learning processes in which learners construct deep structural mathematical knowledge. Methodologically (and this is the focus of the present study), the AiC researchers are offered tools that allow them to observe and analyze students' thinking processes. A detailed treatment of the methodology is offered by Dreyfus, Hershkowitz and Schwarz (2015). AiC view of abstraction is grounded in the works of Davydov (1990) and Freudenthal (1991). In the introduction part, I wrote that AiC focuses on the students' reasoning and that mathematical meaning resides in the verticality of the knowledge constructing process and the added depth of the resulting constructs. Freudenthal ideas led his collaborators to the idea of "vertical mathematization". This idea is explained by Dreyfus et al. (2015, p. 186–187):

Vertical mathematization points to a process that typically consists of the reorganization of previous mathematical constructs within mathematics and by mathematical means by which students construct a new abstract construct. As researchers in mathematics education, we preferred the expression "vertical reorganization" to the expression "Vertical mathematization" to discern between what is intended by the teacher - the mathematization, and what often happens - a reorganization....In vertical reorganization, previous constructs serve as building blocks in the process of constructing.

Thus, AiC defines abstraction as a process of vertically reorganizing some of the learner's previous mathematical constructs within mathematics and by mathematical means in order to lead to a new construct (for the learner). For the convenience of the readers I will report some more details about AiC written in Kidron (2016, p. 154). The process of abstraction has three stages: the need for a new construct, the emergence of the new construct and the consolidation of this new construct. The second stage, the emergence of the new construct. is analyzed by means of three observable

epistemic actions: **Recognizing**, **Building-With** and **Constructing**. Recognizing takes place when the learner recognizes that a specific knowledge construct is relevant to the problem she or he is dealing with. Building-with is an action comprising the combination of recognized knowledge elements, in order to achieve a localized goal, such as the actualization of a strategy, or a justification, or the solution of a problem. Constructing consists of integrating previous constructs by vertical mathematization to produce a new construct.

In view of AiC essential cognitive perspective, the focus is on the students' processes of construction of knowledge. In the AiC approach, contextual aspects are considered to be integral factors of the learning process. Context is regarded in a wide sense, comprising historical, physical and social context. Historical context includes students' prior learning history, physical context includes artefacts such as computers and software, and social context refers to interaction with peers, teachers and others.

The importance of design is considerable in AiC. This is in accord with the epistemological stance which is underlying the idea of vertical reorganization. The design is accompanied by its epistemological aspects. As a part of the AiC methodology, an effort is made to foresee students' expected processes of construction of knowledge and an a priori analysis of the activities is carried out.

The AiC a priori analysis consists first on assumptions about the previous mathematical knowledge of the students, in particular, previous constructs which have been constructed in the past and that may be helpful in the present task. Then, the AiC a priori analysis consists of intended constructs that are required in the given task. For each intended construct, the AiC researchers give an operational definition. The operational definition will help the researchers in their decision if the student did express the intended construct. It will offer a criterium for evidence if the intended construct has been constructed. Different researchers in the team conduct separately their a priori analyses. Then the a priori analyses are compared and discussed until there is agreement between the researchers.

Comparing the a priori analysis and the a posteriori analysis, the AiC researchers note that sometimes the students achieve new constructs which were not expected in the AiC a priori analyses. This fact is an important and interesting stage in the research. Sometimes, students only achieve constructs that partially match a corresponding intended construct in the a priori analysis (Ron, Dreyfus, & Hershkowitz, 2010)

The AiC a priori analysis is not only a list of intended constructs. It is more a structure of intended constructs with some interactions between the different constructs. Some constructs are contained in others. Some intended constructs might be a prerequisite for others. Sometimes, possible paths of thinking are taken into account. This is relevant, for example, for a priori analyses of justification tasks. Justification is a specific case of construction of knowledge. Each itinerary of thinking towards the justification might be in itself a kind of construction of knowledge and different itineraries of thinking, each with a structure of intended constructs will appear in the a priori analysis.

## **Comparing the a priori analyses and the benefit for networking**

For both theories, TDS and AiC, the epistemological perspective is of importance but their a priori analyses have a different focus. In AiC the focus of the a priori analysis is on the learner's construction of knowledge. The a priori analysis reveals hypotheses about constructs that might be observed during the construction process. For AiC, processes of abstraction are inseparable from the context in which they occur. The notion of context is very wide in AiC. The context has many components. For example, the task, the computer, the teacher, the social interaction between students are considered as part of the context. The AiC a priori analysis with its focus on the learner's processes of construction of knowledge cannot take explicitly into account all the contextual factors. In a later phase, the researchers will analyze the influence of the context on the construction processes that were observed in the analysis of data. For example, Kidron & Dreyfus (2010) analyzed the influence of the computer on the construction processes observed in the analysis of data. They describe how instrumentation led to cognitive constructions and how the roles of the learner and the CAS intertwine. But there is an essential difference if you analyze your data taking into account in advance the contextual factors or if you first analyze the data and the processes of construction of knowledge and only then you analyze the influence of the contextual factors on the construction processes.

For TDS, the situation is different. The focus is on didactical systems. TDS observes the entire situation and not only the student and the mathematical activity. For example, TDS is interested in relations between systems and the teacher is an element of the system. As a consequence, TDS considers already in the a priori analysis the role of the teacher and how he/she may extend the results of the a-didactical situation. As a consequence of the different foci between TDS and AiC, context is not theorized and treated in the same way in the different theories. This fact has an important consequence on the differences of the a priori analyses.

AiC a posteriori analysis might be influenced by the fact that some contextual factors are not taken explicitly into account in the a priori analysis. As a consequence, some excerpts which might add direct knowledge in the analysis of the cognitive processes might be missed if one focuses first on the cognitive processes and only then analyzes the influence of other parts of the context.

Kidron et al. (2014) refer to a networking case that links three theories. The issue of context is compared and contrasted in the three theories. The analyses from the different perspectives refer to a set of data from a video recording that show a session from the group-work of two students, during a teaching experiment on the exponential function in secondary school.

In Kidron et al. (2014, p. 175), the authors noted that

An interesting, and also revealing, point is the fact that, in the analysis, AiC researchers focus on the autonomous work of the students, while TDS researchers pay more attention to the episode where the students interact with the teacher...

The a posteriori analyses of the two theories are influenced by the differences in the a priori analyses and their different priorities in their focus of analysis. Different units of analysis are taken

into account and as a consequence of the focus of analysis, as demonstrated in the a priori analysis, each theory shapes the kind of data that is appropriate to this focus. As pointed by Radford (2008):

... it is through a methodological design that data is first produced; then the methodology helps the researcher to “select” some data among the data that was produced but also helps the researcher to “forget” or to leave some other data unattended. (p. 321)

As a consequence, the different a posteriori analyses conducted within the two theories complement each other. Each analysis highlights a specific view which reflects the focus of research of the given theory. AiC analysis, with its specific tools, offers a fine grained analysis of the students constructing processes. TDS, with its different focus, analyses the entire situation and, in particular, the interaction between the teacher and students. For example, in Kidron et al. (2014, p. 172) we read how TDS analyses the role of the teacher:

TDS complements the AiC analysis in analyzing how the teacher extends the outcomes of the a-didactical interaction. The TDS analysis seems to start where the AiC analysis stops.

The different a priori analyses result from the different priorities of the theories with regard to the focus of analysis. Investigating these differences in the a priori analyses might lead to a better understanding of the different a posteriori analyses and to the insights offered by one theory to the other one in the networking process.

## **Concluding remarks**

Reflecting on the role of a priori analysis in both theories, TDS and AiC, we realize its importance and why it is necessary towards a better understanding of the a posteriori analysis of the collected data. For both theories, the a priori analysis plays a role of reference while comparing the a priori and a posteriori analyses. For TDS, it plays the role of revealing the didactic phenomena and helps to deeply understand the functioning of the “situation”. For AiC, it offers a structure of intended constructs that are required in a given task as well as possible paths of thinking. We also realize the importance that each theory keeps the specific characteristics of its a priori analysis. I wrote in a previous section that there is an essential difference if you analyze your data taking into account in advance the contextual factors or if you first analyze the data and the processes of construction of knowledge, and only then you analyze the influence of the contextual factors on the construction processes. This essential difference is tightly connected to the specific characteristics of the different a priori analyses for AiC and TDS. In Kidron et al. (2008, p. 262), we read that:

In networking, we want to retain the specificity of each theoretical framework with its basic assumptions, and at the same time profit from combining the different theoretical lenses. What we aim at is to develop meta-theoretical tools able to support the communication between different theoretical languages, which enable researchers to benefit from their complementarities.

Comparing a priori analyses might enable to support the communication between theories:

Realizing some common points in the a priori analyses enables the beginning of a dialogue between the theories. In the case of networking between TDS and AiC, for example, the common points in the epistemological dimension help towards the beginning of the dialogue (as demonstrated in Kidron, Artigue, Bosch, Dreyfus, & Haspekian, 2014). This idea could be used for other theories

and other cases of networking: Some other common points in the a priori analyses, for example on the social dimension, might help towards the beginning of the dialogue.

Realizing the differences in the a priori analyses, we better understand the choices of data (as well as the “data which was left unattended”) that researchers of each theory select for their a posteriori analyses. Sometimes, the data which was left unattended by one theory might add direct knowledge in the analysis of this theory and, as a consequence, some cognitive processes, for example, might be missed. The complementary insights which are missing might be offered by means of networking theories. This situation might happen in different cases of networking theories.

This paper offers a reflection on a priori analyses, epistemology and networking. It contributes to the TWG discussion on the reciprocity of theory and methodology for research and design. The notion of a priori analysis is closely related to the phase of design. It allows making epistemological assumptions explicit. The TWG discussions addressed importance to the notions of mathematical epistemology and design. The common points in the epistemological dimension which enable the beginning of the networking between AiC and TDS contributed to these discussions.

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