



**HAL**  
open science

# Mathematical practices of teachers in technology-enhanced classrooms: A case of teaching slope concept

Seçil Yemen Karpuzcu, Mine İşiksal Bostan

► **To cite this version:**

Seçil Yemen Karpuzcu, Mine İşiksal Bostan. Mathematical practices of teachers in technology-enhanced classrooms: A case of teaching slope concept. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02417085

**HAL Id: hal-02417085**

**<https://hal.science/hal-02417085v1>**

Submitted on 18 Dec 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Mathematical practices of teachers in technology-enhanced classrooms: A case of teaching slope concept

Seçil Yemen Karpuzcu<sup>1</sup> and Mine Işıksal Bostan<sup>2</sup>

<sup>1</sup>Middle East Technical University, Faculty of Education, Ankara, Turkey; secilyemen@gmail.com

<sup>2</sup>Middle East Technical University, Faculty of Education, Ankara, Turkey; misiksal@metu.edu.tr

*While it is obvious that teachers need to have mathematical knowledge under different domains, few studies put forward teaching practices associating them with teacher moves in the background of the mathematical knowledge used in teaching in a technology enhanced classroom environment. Moreover, characterizing those practices can show a practical understanding of the knowledge for its usefulness in teaching in which tools and dynamic technologies are deemed necessary. In this study, we examined the characteristics of teachers' mathematical practices in a classroom teaching where the students engaged in a sequence of GeoGebra-integrated tasks designed to support students' conceptualizations of slope of an object. The results suggest teacher moves for the GeoGebra-integrated tasks for conceptualization and suggestions for teaching practices from pedagogical perspectives.*

*Keywords: Middle school mathematics teachers, GeoGebra, mathematical practices, slope.*

## Introduction

We all accepted that there are prominent frameworks on mathematics teaching. However, while these frameworks provide tremendous contributions, how teachers' knowledge is used and developed in classroom teaching practice for specific learning areas need to progress when it comes to combining theory and practice (Doerr, 2004; Wasserman, 2015). Specifically, investigating teachers' mathematical practices in slope teaching in school algebra requires having a distinctive spectacle. Although slope is one of the big ideas in curricula, we do not know teachers' mathematical practices in slope teaching during the implementation of an instructional sequence (Cheng, Star, & Chapin, 2013). In this regard, research on knowledge of algebra teaching (McCrorry, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) can enlighten this process which includes mathematical practices in connection with the content knowledge. Therefore, the mathematical practices dimension can be used and developed for investigating particular mathematical practices of teachers as their didactical performance.

Dynamic mathematics software (e.g. GeoGebra) is seen as the indispensable part of mathematics teaching technology in 21st century. However, although there are research studies that integrate curricula including new approaches and technology into algebra teaching in the classroom (Bozkurt & Ruthven, 2017; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010), studying the practice of slope teaching within the technological environments need further elaborations to reveal mathematical practices of teachers. Thus, interpreting mathematical practices of teachers in slope teaching from the perspective of algebra teaching can provide in depth insight about the mathematical practices in the work of mathematics teaching in technology enhanced classroom environment.

## **Framework of the Study**

The mathematical practices are moves or actions which are seen as the ways for using content knowledge in teaching (McCrorry et al., 2012; Wasserman, 2015). That is, they involve teacher's actions with specific goals and perspectives on particular mathematical ideas that are centralized through the interactions with students in classroom teaching. In terms of algebra strand, McCrorry et al. (2012) characterize Knowledge of Algebra Teaching (KAT) framework that is developed for assessing teachers' mathematical knowledge on algebra teaching. In the framework, considering three domains of knowledge (i.e. knowledge of school algebra, knowledge of advanced mathematics, and mathematics-for-teaching knowledge), teachers' mathematical work as mathematical practices is emphasized under three categories: bridging, trimming, and decompressing. By means of bridging, McCrorry et al. (2012) emphasize ways of making connections across topics, concepts, representations, and domains. In the framework, trimming practices are described as ways of removing complexity of mathematical ideas while holding mathematical integrity (McCrorry et al., 2012). Introducing slope in linear functions graphs in consideration with the future mathematics (instantaneous rate of change in calculus and analysis) can be given as an example for trimming practice (McCrorry et al., 2012). However, making an unfortunate explanation of "multiplying makes bigger" in teaching multiplication of whole number is an example of 'over' trimming when considered the multiplication of fractions and rational numbers (McCrorry et al., 2012; Wasserman, 2015). At last, decompressing is described as making explicit complexity of mathematical ideas in ways that make them comprehensible for students (McCrorry et al., 2012). McCrorry et al. (2012) gave examples of unpacking the algorithm for solving equations and meaning of identities.

In this current study, the KAT framework was used for the analysis of teachers' mathematical practices in classroom teaching. In this regard, bridging refers to teachers' mathematical practices of connecting topics, concepts, representations, domains, and activities in teaching slope of an object (i.e. the slope concept in physical situations) in a technology enhanced classroom environment in which GeoGebra materials and concrete objects were used. Beyond, we characterized the mathematical work of bridging with an agglutination of using tools and technology by putting terms of concrete object and GeoGebra material. In detail, this category involves mathematical situations (e.g. position of a ladder as a physical situation), mathematical concepts (e.g. horizontal length, line segment, ratio, etc.), concrete objects as mathematical objects, and GeoGebra material for representing a mathematical idea. Since teachers can use various kinds of tools and technologies in classroom teaching, we cannot ignore the use of concrete objects and dynamic technology in such kind of mathematical work. Similar characterization of the mathematical work is also considered both for the practices of trimming and decompressing. Trimming refers to teachers' mathematical practices of removing the complexity of slope of an object while holding the mathematical integrity (e.g. by omitting/adding details, modifying level of rigor) in both eighth-grade mathematics curriculum and advanced mathematics in the technology enhanced classroom environment. In other words, we looked for the ways of mathematical work that teachers considered the complexity of slope in the context of physical situation, while maintaining the integrity of slope concept anticipating further slope conceptualizations. Therefore,

this kind of practice corresponds to trimming up or down of the mathematical idea for the essential content anticipating further mathematical ideas related with slope. Moreover, decompressing refers to unpacking the complexity of slope of an object in ways that make them comprehensible to eight-grade students in the technology enhanced classroom environment (e.g. looking for the ways of mathematical work that teachers unpack the computational algorithm of slope as the rate of change of vertical distance relative to horizontal distance of that object/feature).

This paper introduces teachers' actions in their mathematical work in specific to using GeoGebra materials and concrete objects while enacting an instructional sequence in the technology enhanced classroom environment that was designed for eighth graders based on the national curriculum. In this regard, characterizing such actions in teaching with the teacher's rationales for them and interrelations among them is the focus of this study. Therefore, the aim of this paper was to answer the following research questions: What are the characteristics of the teachers' mathematical practices while teaching the slope concept in technology enhanced classroom environment? How do those practices emerge and interrelated with each other in the moment of teaching?

## **Method**

This study was a part of a long term study and used design research method to understand teachers' knowledge base and expertise within the complexity of classroom teaching (Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008). In the light of Lesh and Kelly's (2000) multi-tier design experiment method, this study involved four tiers. While the tier of students developed mathematical ideas and computational algorithm on slope, the tier of teachers developed and implemented activity sheets involving GeoGebra-integrated tasks through an instructional sequence while examining students' mathematical understanding with the support of the first author. In addition, the tier of facilitator (the first author) developed sessions for teachers and the tier of researchers (two authors and a doctoral student) produced interpretations of teachers' and students' experiences. The whole process of this study involved a prototype study with a teacher and a main study with another teacher after the preliminary research was done and tentative design principles for sessions with teachers and products for data collection were determined. In this paper, we presented findings of the tier of the teacher from the main study which involved the topics of teaching slope, linear equations and their graphs.

## **Context and Participants**

The sequence of activities involving GeoGebra integrated tasks formed the basis for a two-week course for students in two eighth grade classes. The teacher had eight years of experience of teaching middle school students. There were 20 students in 8A class (12 girls, 8 boys) and 23 students in 8B class (12 girls, 11 boys). Throughout their work on the tasks, the teacher set the class seating plan considering group work (heterogeneous, 6-7 students) in collaboration. The teacher allowed the students to study and discuss within the group and between the groups. The groups completed the tasks using laptop computers for the groups, and the teacher's computer with a Bluetooth mouse in addition to the related activity materials. The teacher engaged in whole-class discussions that involved the students discussing their slope computation algorithm and how that slope changes when the attributes of the objects changed.

In this regard, the students were asked (1) to notice and describe slope as a measurement of steepness of a physical object, (2) to describe slope of an object being independent of the length of an object, (3) Relate vertical and horizontal distances to the slope of an object, (4) to structure the computation of slope of a given physical object, and (5) evaluate the effects of horizontal and vertical distances on the computation of the slope of a physical object as a ratio given in GeoGebra materials integrated activities. The teacher also wanted to have students use concrete objects for experiencing the slope of an object practically. In the beginning of the course, the students were given concrete objects (i.e. fixed length battens), tape measures, and Positions of Battens (PoB) activity sheet. In the activity, the students were expected to measure the horizontal/vertical lengths of the objects, to describe the slope as a measurement of steepness and being independent of the length of the object, and to collect data for relating the horizontal and vertical lengths to the slope of the object. The GeoGebra material of the activity was used to have students relate the horizontal/vertical lengths to the slope of the object in the dynamic environment. Then, the students were given Fire Truck (FT) activity sheet and the GeoGebra material. In this activity, the students were expected to compute the slope of a fire truck ladder in different positions with a constant horizontal length and to evaluate the change of the slope as the vertical length changed using GeoGebra. At last, the students were given Tent (Tt) activity sheet and the GeoGebra material. The students were expected to compute the slope of a tent rope in different positions with a constant vertical length and to evaluate the change of the slope as the horizontal length changed using GeoGebra (See materials on <https://ggbm.at/sbffsxt>). The lesson was structured as PoB for the first hour and FT and Tt for the second hour.

### **Data Sources and Analysis**

The data sources of this study included videotapes of all classroom sessions, written field notes and memos, students' activity sheets, students' computers screen records. Following the classroom teaching, there was an analyzing instruction session involving audio recorded semi-structured interview. In this process, the teacher reflected, commented, and made decisions on her practices and students' learning in the lesson and reflected on the further changes for the lessons in the instructional sequence, while watching the episodes of video recording of the classes. In this paper, the data constituted 180 minutes lessons for two classes (90+90) and 90 minutes analyzing instruction session. The data was collaboratively analyzed considering the iterative approach of design research and the comparative analysis method of grounded theory (Glaser & Strauss, 2006). In the analysis, primarily, the researchers met to develop and evaluate the instructional sequence, the progress of the students in classes, and the observations/comments about students' thinking about slope of an object and their use of GeoGebra materials and other representations. The researchers wrote the analytic memos documenting their emergent understandings of the teacher's mathematical practices and observations about students' learning. Secondly, the researchers started with the microanalysis of the transcripts of the videotapes through line-by-line analysis. While doing systematic coding in the analysis, the teachers' acts, events and activities were conceptualized and classified when the actions of the teacher occurred in a pattern during the classroom teaching. Moreover, theoretical comparisons were made to identify the derived properties and dimensions of the concepts from the incidents in the literature. Additional evidences were used

from the interviews with the teacher to confirm and disconfirm the actions and the teacher's reasoning about those actions. In this paper, we present two results of the analysis of the teacher's mathematical practices that involves interrelations among each other: trimming slope computation and its relation to a bridging practice and decompressing components of slope and its relation to trimming slope computation.

## **Findings**

### **Trimming slope computation and its relation to a bridging practice**

In the PoB activity, as a trimming practice, the teacher trimmed the computation of slope of the batten using the GeoGebra material by showing/checking change in the slope. In detail, this practice involved actions of showing and checking the correctness of the computations of slopes of the battens through the use of slope checkbox (that opens dynamic slope computation text) with slider (that changes horizontal length of the batten), and horizontal and vertical lengths checkboxes (that shows their images and values). As evidence, for example, after the groups of students made measurements on their battens and the teacher showed/checked the horizontal and vertical lengths of their battens using GeoGebra material, in the conversation with students the teacher said:

Yes. You take the vertical length of the batten as 57 cm (*The teacher checks the value of the vertical length on the computation after moved slider to make students' vertical length of the batten*). Let's look. Is the value of your slope like this? (*The teacher shows the value of the slope on the slope computation text in the GeoGebra graphics view*).

That is, the teacher moved the slider to create the students' battens and then showed and checked those values on the GeoGebra material. Thus, she both constrained the slope computation as ratio of vertical length to horizontal length and provided various slope computations with different values.

In addition, that trimming practice took place in the background of bridging practice of connecting mathematical situation/concept and GeoGebra material. That is, as a bridging practice, the teacher used the slider tool in connection with the line segment (i.e. the batten) in the GeoGebra graphics view while posing the questions about the position of batten. Therefore, when relating such trimming and bridging practices, the teacher put forward her intention of using the line segment to create a model of a batten at the beginning of the students' learning process and then a model for reasoning about the slope as steepness at the end of their learning experience.

### **Decompressing components of slope and its relation to trimming slope computation**

In all activities, as a decompressing practice, the teacher interpreted the components of the slope of an object using GeoGebra materials in the following of the practice of trimming computation of slope using GeoGebra materials. In detail, in the PoB activity, the teacher interpreted the factors of slope (i.e. attributes that affect/do not affect the slope of the line segment representing the object of batten) using the GeoGebra material (i.e. using the slider tool and the graphics view) and interpreted logic in computation of slope using GeoGebra material after she trimmed the slope computation. That is, after the teacher showed/checked horizontal and vertical lengths of the batten (i.e. line segment object in the GeoGebra graphics view) using the slider of the horizontal length and checkboxes of the lengths and showed/checked the slope of the batten using the slope checkbox

(that opens dynamic slope computation text) with the slider and the checkboxes of the lengths, the teacher and students discussed on how the slope changes in accordance with the changes of the horizontal and vertical lengths while representing slopes of various positions of the batten with their horizontal and vertical lengths values. In the continuum of the discussion, she also used ‘trace’ tool on the line segment (i.e. batten) that shows the trace of the line segment as the slider of horizontal length moves from 1 to 60 with the dynamic slope computation. As evidence, the following conversation occurred:

- Teacher: Ok then, I want you to see how the slope value changes when I change the [position of] batten. Even I clicked trace on command in GeoGebra. Look at both the batten [line segment] and the slope. The value of the computation of the slope! What happens to the slope? *(The teacher moves the slider to the lower part, which increases the horizontal length of the batten, and eventually that moves the batten)*
- Student: Decrease!
- Teacher: It decreases, doesn't it? Look it is 1.17 and then 1.12. What happens gradually?
- Students: Lessens. Becomes smaller.
- Teacher: The slope decreases more and more. What happens when it moves to the other part? *(The teacher moves the slider.)*
- Students: Increases!
- ... *(The students discuss it in their groups)*
- Student Our batten has the longest vertical length and the least horizontal length with the biggest slope.

As seen from the conversation, the teacher paid attention to the complexity of change of the slope that depends on the change in the horizontal length and the vertical length. Therefore, as evidence of decompressing, she unpacked the idea of slope in the physical situations context that involves proportional approaches.

As another example of interpreting logic in computation of slope using GeoGebra material, in the FT activity, the teacher had students interpreted the change in computation of the slopes of fire truck ladder (that stops away 5-unit constant distance from a burning building) using vertical length slider tool after clicked the slope checkbox (that shows dynamic slope computation text) in GeoGebra graphics view and interpreted the change of slopes of the ladder when the horizontal length of the ladder was constant using the slider tool. For instance, while students using the GeoGebra material of the activity in a similar conversation given in below, she said “*We have two variables. One is the vertical and the other is the horizontal length (The teacher shows them on the GeoGebra graphics view). In this question, which one is constant?*” After the students explicitly mentioned the horizontal length was constant in that situation. The teacher added, “... *Thus, we can see how the vertical [length] affects the slope. Could we see clearly the effect of one variable on the slope when the other one is constant?*”. Therefore, the teacher again paid attention to the complexity of change of slope that depends on the vertical length when the horizontal length is constant. In this regard, she unpacked the idea of the slope that involves direct relationship with the vertical length when the horizontal length does not change in a physical situation. Similar

decompressing practice emerged in the Tt activity that the teacher unpacked the idea of slope involving indirect relationship with the horizontal length when the vertical length is constant.

## **Discussion and Conclusions**

This study began with the development of an instructional sequence involving GeoGebra integrated tasks and activity sheets to develop students' conceptualizations of the slope and interpret the slope computation in physical situations. The design of that sequence provided opportunities for students to experience slope with concrete objects and GeoGebra materials, to decide the factors that have an effect and do not effect on the slope of an object, to give meanings to various representations from concrete objects to dynamic visuals and mathematical notations, and to reason about the slope concept.

In accordance with the findings about teachers' mathematical practices, when teachers are engaged in such GeoGebra integrated activities after students experience the slope with the concrete objects, they need to have understanding of what is the meaning of the mathematical idea of the slope as a concept of steepness and as a process of measurement and computation in physical situations, and how this meaning of slope is in connection with measurement, ratio, rate, and division. Therefore, during the decompressing practices in those activities, teachers are likely to enhance students' proportional reasoning underlying the slope computation for an object (i.e. the slope and the vertical length directly proportional when the horizontal length is constant, slope and horizontal length indirectly proportional when the vertical length is constant). Current studies also emphasized this understanding of slope as steepness through its relationship with proportional reasoning (Cheng et al., 2013; Lobato & Thanheiser, 2002). In this regard, such kind of mathematical practices require teachers to have the understanding of the relationship between the concrete and visual (i.e. as having knowledge of the goals and subgoals of steps in using intertwined multiple representations for the computation and the environment in which the computation is used) and the way of giving the aforementioned relationship considering the students' experiences and reasoning. In addition, teachers need to have a mathematical understanding of how to give visual representations in a connected way not also for translating the selected concrete examples but also for generating all possible situations with the concrete object using the dynamic properties of the software (i.e. the constitution of relationship between deep knowledge of computational skill and knowledge of the dynamic representations of mathematical ideas) since this understanding is likely to enhance the background of removing the complexity of slope computation (i.e. trimming). In this regard, while using dynamic software to represent such situations, teachers' knowledge of content and technology come to the forefront. Therefore, while the KAT framework (McCrary et al., 2012) and other studies (Wasserman, 2015) did not mentioned a knowledge dimension related with the technology, it is considered that teachers knowledge of teaching mathematics with technology should be considered as a source for fostering the bridging and trimming practices in classroom teaching. In sum, interrelations among those practices are likely to vary depending on the mathematical idea, teachers' understanding of this mathematical content and their knowledge of students' understanding in this content. As Clark-Wilson (2013) suggested, this paper elaborated the situated nature of use of mathematical knowledge in technology-enhanced classrooms. In this regard, that elaboration illustrate content-specific mathematical practices in teaching with technology that could



be involved in various kinds of activity formats as Bozkurt and Ruthven (2017) exemplified and explored through Drijvers et al.'s (2010) orchestrations. In further research, such kind of mathematical practices could be investigated in teachers' various ways of orchestrations for elaborating their mathematical work as didactical performance.

## References

- Bozkurt, G., & Ruthven, K. (2017). Expert and novice teachers classroom practices in a technological environment. In K. Krainer & N. Vondrová (Eds.) (2015). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. 2319–2325). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.
- Cheng, D., Star, J. R., & Chapin, S. (2013). Middle school students' steepness and proportional reasoning. *New Waves-Educational Research & Development*, 16(1), 22–45.
- Clark-Wilson, A. (2013). How teachers learn to use complex new technologies in secondary mathematics classrooms - the notion of the hiccup. In B. Ubuz, Ç. Haser, M. A. Mariotti. *Proceedings of the Eight Congress of the European Society for Research in Mathematics Education (CERME 8, February 6-10, 2013)* (pp. 2544–2553). Ankara, Turkey: Middle East Technical University and ERME.
- Doerr, H. M. (2004). Teachers' knowledge and the teaching of algebra. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The Future of the Teaching and Learning of Algebra: The 12th ICMI Study* (pp. 265–290). Dordrecht: Springer Netherlands.
- Glaser, B. G., & Strauss, A. L. (2006). *The discovery of grounded theory: Strategies for qualitative research*. New Brunswick: Transaction Publishers. (Original work published in 1967).
- Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213–234.
- Lesh, R. A., & Kelly, A. E. (2000). Multitiered teaching experiments. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of research design in mathematics and science education* (pp. 197–230). Mahwah, N.J: Lawrence Erlbaum Associates.
- McCrary, R., Floden, R., Ferrini-Mundy, J., Reckase, M. D., & Senk, S. L. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education*, 43(5), 584–615.
- Wasserman, N. H. (2015). Unpacking teachers' moves in the classroom: navigating micro-and macro-levels of mathematical complexity. *Educational Studies in Mathematics*, 90(1), 75–93.
- Zawojewski, J., Chamberlin, M., Hjalmanson, M. A., & Lewis, C. (2008). Developing design studies in mathematics education professional development: Studying teachers' interpretive systems. In A.E. Kelly, R. Lesh, & J. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching*. (pp. 216–245). New York: Routledge.