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# Professional development through a web-based portal: The progress of mathematics teachers teaching algebra based on hypothetical learning trajectories

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*The aim of this paper is to present preliminary findings of a large-scale project for designing a web-based portal to contribute to the professional development of mathematics teachers for teaching algebra in lower secondary education. Eleven teachers volunteered to participate in this research that aimed to design hypothetical learning trajectories (HLTs) for sixth-grade algebra context over a three-week period. The teachers designed their own HLTs through a web portal by receiving feedback from mathematics teacher educators online. Their processes of designing and applying lesson plans during the first and third week were analysed according to the components of HLT and the theoretical perspective of the quartet of subject matter knowledge. Data analysis revealed that the teachers managed to define more coherent learning goals and hypothesis for their students' learning as well as making connections between the concepts.*

*Keywords: Mathematics teacher, Professional development, Hypothetical learning trajectories.*

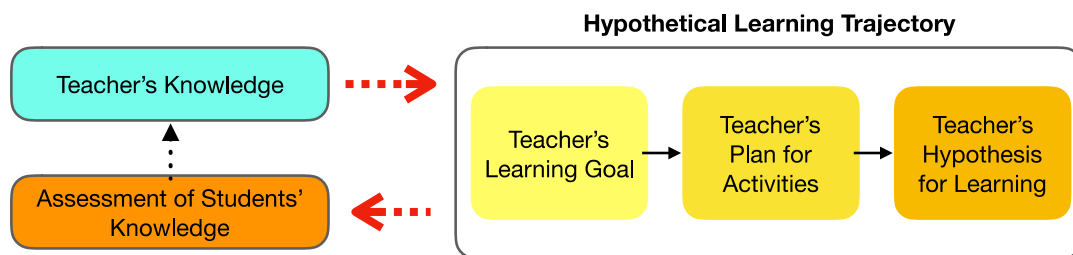
## Introduction

In Turkey, as in many countries, studies on the mathematics teacher knowledge received particular attention from researchers (Gökkurt & Soylu, 2016; Türnüklü, Akkaş & Gündoğdu Alaylı, 2015). Research on in-service teachers commonly indicates that a number of issues exist regarding teachers' subject matter knowledge of mathematics. Researchers have addressed major problems in the Turkish context: (i) a lack of certain professional development programs and their sustainability; and (ii) a gap between research and practice, which, in particular, means a lack of communication between mathematics teacher educators and in-service mathematics teachers (Yıldırım, 2013). The latter is of crucial importance since professional development activities are generally organised independently by each public school whereby teachers prepare termly plans without any collaboration with mathematics teacher educators. This research study elaborates the design and use of a web-based portal to contribute to the professional development of mathematics teachers, in which they design their own teaching-learning algebra activities within a perspective of hypothetical learning trajectories (HLT) (Simon, 1995) through close interaction with mathematics teacher educators.

We focus on the algebra context of Turkish (lower) secondary school curriculum and design a large-scale research project adopting a number of theoretical and conceptual elements, such as *mathematics teaching cycle* and HLT (Simon, 1995) and the mathematics *subject matter knowledge quartet*, as described by Rowland, Huckstep and Thwaites (2005).

## Theoretical Framework and Description of the Project

The notion of a *learning trajectory* corresponds to possible (anticipated) learning paths that students could follow in a proposed content designed under certain objectives. Since learning, in itself, is a complex process and depends on phenomenological experiences, Simon (1995) adds the word *hypothetical* to learning trajectory to emphasise a teacher's prediction about students learning and elaborates the notion of HLT under three components; 'the learning goal that defines the direction, the learning activities, and the hypothetical learning process' (p. 136). In fact, Simon (1995) considers the HLTs as a part of mathematics teaching cycle, which includes certain key components such as a teacher's knowledge and an assessment of student knowledge (Figure 1).



**Figure 1: Mathematics Teaching Cycle (adapted from Simon, 1995, p. 136)**

In Figure 1, particular emphasis is given to the teacher's knowledge as a starting point. Therefore, in this research, we consider and refer to a quartet of subject matter knowledge of mathematics (Rowland et al., 2005) that has four dimensions; *foundation*, *transformation*, *connection*, and *contingency*. The first component, foundation refers to a repertoire of the teacher's academic knowledge for teaching-learning mathematics including his/her beliefs regarding why mathematics is important and why it should be taught. Here, transformation refers to the transformation of theoretical knowledge into practice by designing and planning pedagogical tasks in terms of choosing appropriate examples and activities for the construction of mathematical meanings. The notion of connection refers to the coherence of designed parts of a lesson or series of lessons through deliberately chosen activities and domain-specific tasks. Such pedagogical task sequences enable students to make a connection between different concepts as well as to the interplay between different representations. The final component contingency refers to 'classroom events that are almost impossible to plan for' (Rowland et al., 2005, p. 263).

Related literature confirms that Simon's (1995) mathematics teaching cycle within HLTs could be a heuristic tool for teachers' professional development (e.g. Wilson, Mojika, & Confrey, 2013). Consequently, we exploit mathematics teaching cycles as twofold: as a perspective for designing interfaces of the web portal (which will be explained in the next subsection); and as an analysis tool to explore teachers' progress as a part of their professional development. In order to elaborate development of teachers' knowledge in depth, we also refer to the subject matter knowledge quartet as an additional analysis tool while looking deep into the teachers' lesson plans.

### The Project and Research Question

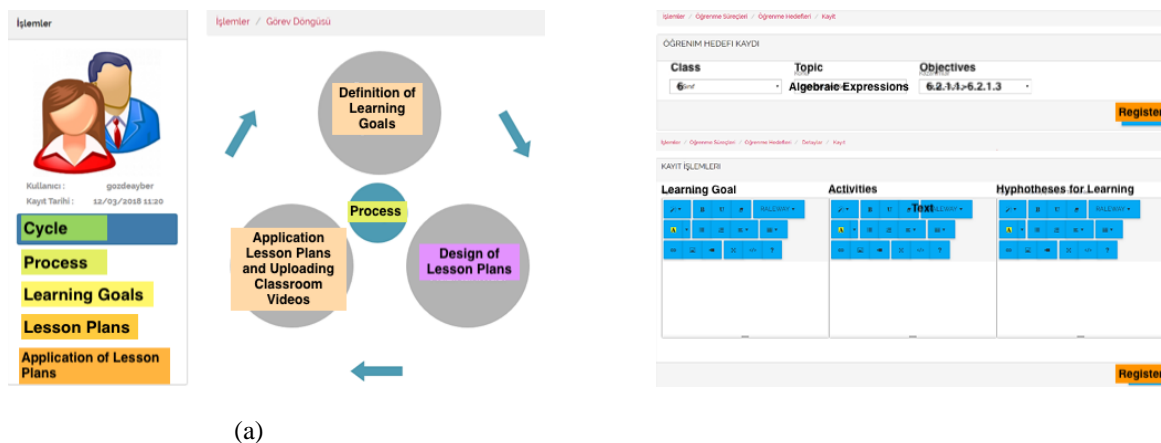
Since the Internet is now widely used in different platforms as well as in educational contexts, we designed a sustainable and specific web portal to ensure mathematics teachers' professional

development regarding algebra content of the (lower) secondary school mathematics curriculum. We specifically focused on algebra content as the researchers in the project team have background knowledge in epistemological issues and misconceptions regarding learning algebra. In order to create such a portal, at first, a domain <http://megedep.anadolu.edu.tr> was taken, where ‘megedep’ referred to a shortcut of the project’s title in Turkish. Figure 2 displays the (homepage) interface of the website.



**Figure 2: Megedep’s (Web portal) interface**

The interface includes some practical information in addition to theoretical foundations that are necessary for participating teachers in the project. More precisely, the interface includes a video describing the aim of the project, a manual for using the website, an introductory description for constructivism and the notion of HLT, as well as exemplary lesson designs under the notion of HLT. After registering to the website, the teachers could connect to a user interface including the components of the mathematics teaching cycle, which is sketched in Figure 3a.



**Figure 3: a. The user interface (translated), b. The user interface to enter components of HLT**

When a teacher selects ‘Process’, Figure 3b is shown, where the teacher enters the class level, the topic and the related learning objectives determined by the Turkish Ministry of National Education (MoNE, 2018) of Turkey. In the portal, we followed Khan (2005) for designing an e-learning platform where there exists an effective *feedback* system. When teachers enter the learning goal, possible activities and their hypotheses for students learning, the mathematics (teacher) educators

(MTEs) (in this paper, the authors) of the project receive alerts by email. Through entering their own interface, MTEs review the teachers' input and give feedback to the teachers to improve the anticipated teaching and learning processes. Thereafter, the teachers make revisions, and if they complete necessary improvements, then they could proceed to design a more fine-grain teaching plan. After that, the MTEs again review their plans and provide feedback. The progress continues in a cycle like this. However, it ends when the teachers record their own teaching episode and upload videos to the web portal after having written their responses to some reflective questions (described in the methods section) regarding the process. Thereafter, the MTEs watch the teaching episodes and review the teachers' responses to reflective questions by taking notes and then communicate their ideas to the teachers through the portal for the teachers to prepare and develop the next episodes.

In this paper we provide a brief analysis of the teachers' initial lesson designs and the final (third) lesson designs created within the web portal to explore their professional development progress. We address the research question: How does mathematics teachers' professional development progress when they use megedep portal to design HLTs and lesson plans for teaching algebra?

## **Methods**

Theoretical insights of the mathematics teaching cycle (Simon, 1995) and quartet of subject matter knowledge (Rowland et al., 2005) refer to a qualitative paradigm. In this paper, in order to explore progress of teachers' professional development in depth, we consider the same perspective and adopt a case study design (Bogdan & Biklen, 1998). Eleven (all six grade teachers) secondary school mathematics teachers (T1–T11) with different teaching experiences (ranging 1–15 years) volunteered to participate in the project. All of the participants work in the same city located in central Turkey and have experience of the objectives (algebra) of the national curriculum, having all graduated from faculties of education from different universities. In addition, while T1, T5, T6, T8 and T10 have only a bachelor degree, T7 is studying for a master's degree and also T2, T3, T4 and T11 are studying for doctorates. One participant, T9, already holds a master's degree. All of the participants learnt of the notion of HLT for the first time as they become involved in the project. During the first project meeting (lasting around three hours), all of the participants were introduced to the philosophy behind constructivism, the use of the megedep web portal, and the notion HLT with a number of exemplary cases.

## **The Data and Analysis**

The data comes from two sources: the first is based on the MTEs' evaluations of the teachers' designs of HLTs. In other words, the teachers designed their initial HLTs and MTEs wrote feedback to improve each part. After the teachers' improvements on the HLTs, the designs were reviewed once more. The first part of the data comes from the MTE final reviews of the HLTs. The second part of the data comes from the teachers' responses to reflective questions that were proposed after they completed teaching episodes. Three reflective questions were asked before the teachers applied their HLTs whereas four questions being proposed after they applied their lesson plans.

Table 1 shows the reflective questions that were proposed to the teachers through the megedep interface.

Before application of the lesson plans	After application of the lesson plans
<p><i>-What were the key elements in your lesson plan?</i></p> <p><i>-How and why did you determine the activities in your plan?</i></p> <p><i>-How did you determine your hypotheses for students learning?</i></p>	<p><i>-Which learning goals were obtained and which were not? Explain in detail.</i></p> <p><i>-Which parts of your plan worked well, or which did not? Explain in detail.</i></p> <p><i>-Which hypotheses of student learning appeared in the classroom? Did all? Were there any contingency cases? If so, what did you do?</i></p> <p><i>-In the teaching process, which concepts were conceptualised and which were not? Explain in detail.</i></p>

**Table 1: Reflective questions proposed to the teachers through the megedep interface**

The teachers prepared three teaching episodes corresponding to three weeks. In other words, they prepared an HLT for each week considering the objectives of the curriculum. MTEs evaluated the first and third week's HLTs, in order to explore the teachers' progression. Similarly, the MTEs also collected all of the responses given to the reflective questions. A thematic analysis (Miles & Huberman, 1994) was employed to elaborate emerging main themes within a perspective of components of HLT and the quartet of subject matter knowledge.

## Findings

Since our focus is on designing HLTs and lesson plans as components of professional development, we will present our findings in two subsections; an analysis of the teachers' designs of HLTs and an analysis of the teachers' designs and applications of lesson plans.

### Analysis of the Teachers' Design of HLTs

The analysis of the teachers' first HLT designs revealed: four themes appeared for determining learning goals; three themes for determining activities; and four themes for determining the hypotheses for students learning. Table 2 describes these themes and their respective frequencies.

Determining Learning Goals	Determining Activities	Determining Hypotheses
<p><i>-Defining learning goals coherent with curriculum objectives (T1)</i></p> <p><i>-Not sequencing learning goals (T2, T3, T4, T5, T6, T7, T8)</i></p> <p><i>-Referring to objectives as learning goals (T9, T10, T11)</i></p> <p><i>-Defining learning goals, not in relation to objectives (T8, T7, T6, T2)</i></p>	<p><i>-Determination of coherent activities (T7, T8, T6, T11, T9, T3, T2)</i></p> <p><i>-Activities not based on the curriculum objectives (T8, T6, T7, T10)</i></p> <p><i>-Writing only the title of activities (not based on any objective) (T5, T1, T4)</i></p>	<p><i>-Writing hypotheses based on activities (T2, T3, T11)</i></p> <p><i>-Formulating a partial hypothesis not focusing on entire student learning (T4, T9, T1, T5, T10, T8)</i></p> <p><i>-Hypotheses not based on the activities (T7)</i></p> <p><i>-Not a clear formulation (T6)</i></p>

**Table 2: Themes regarding the teachers' design of the first HLT**

Table 1 reveals that only one teacher defined learning goals according to the objectives of the curriculum, whereas seven teachers could not define sequencing learning goals, i.e. they were not able to express learning goals in order. Three teachers directly referred to the curriculum objectives

as learning goals, while four of them defined the learning goals independent of the curriculum objectives. With respect to a determination of the activities, seven teachers managed to define coherent activities, i.e. activities that are coherent with curriculum objectives and also with the learning goals that they defined. Four teachers expressed activities not based on the objectives and three teachers only wrote the title of activities without giving any details. Three teachers expressed their hypotheses that are coherent with the learning objectives and activities that had been written before. However, it should be noted that these three teachers thought that students would follow the same learning trajectory. Six teachers formulated their hypotheses partially, without focusing on the students' entire learning of the proposed concepts, while one teacher proposed a hypothesis independent of the activities and learning goals. Moreover, one teacher expressed his possible teaching steps, including no trace of the hypotheses of students learning.

After all the teachers had used the megedep portal for three weeks, we analysed their resulting HLTs. Tables 3 shows that there were two emerging themes for determination of the learning goals; one theme for the determination of the activities and two themes for the determination of the hypotheses.

Determination of Learning Goals	Determining Activities	Determining Hypotheses
<p><i>-Defining learning goals based on the curriculum objectives (T6, T1, T3, T4, T7, T11, T5, T2)</i></p> <p><i>-Defining learning goals part by part, not based on all curriculum objectives (T10, T8, T9)</i></p>	<p><i>- Determination of coherent activities (T6, T1, T3, T10, T4, T7, T11, T5, T2, T8, T9)</i></p>	<p><i>- Writing hypotheses based on activities (T3, T11, T5, T2)</i></p> <p><i>- Formulating a partial hypothesis not focusing on entire student learning (T6, T1, T10, T4, T7, T8, T9)</i></p>

**Table 3: Themes regarding the teachers' design of the third HLT**

Eight teachers managed to define learning goals based on the curriculum objectives. However, there were still three teachers who defined learning goals part-by-part, not based on the curriculum objectives. All of the teachers were able to define activities coherent with the learning goals and curriculum objectives and also explained them in detail. With respect to a determination of the hypothesis, four teachers expressed the hypotheses of students' learning that were coherent with the learning goals and activities, while seven teachers formulated hypotheses part-by-part, not focusing on the conceptual learning.

### **Analysis of the Teachers' Designs and Applications of the Lesson Plans**

The teachers' responses to the reflective questions posed after the first and the third weeks' lesson plans were analysed according to the quartet of subject matter knowledge. When the authors compared the personal analyses on the reflective questions, it was agreed to consider the same themes for the first and the third week's results. As a result, four themes emerged for the foundation component, three themes emerged for transformation component, five themes emerged for connection component, and three themes emerged for the contingency component. Table 4 summarises the emerging themes with the arrows indicating some themes that have specific meanings. For instance,  $a \rightarrow b$  means that  $a$  is the number of teachers in the first week, while  $b$  is the number of teachers in the third week.

Foundation	Transformation	Connection	Contingency
<i>-Recognition of students' misconceptions (7→11)</i> <i>-Proposing affective questions (3→5)</i> <i>-Focusing on computational applications (7→2)</i> <i>-Referring to multiple ways assessing student knowledge (3→8)</i>	<i>-Using multiple representations (11→11)</i> <i>-Selecting and/or preparing carefully-designed problems (2→9)</i> <i>-Explaining mathematical concepts in several ways (3→11)</i>	<i>-The interplay between mathematical concepts and notions (3→11)</i> <i>-The interplay between mathematical problem-solving techniques (4→11)</i>	<i>-Thinking all the goals were achieved by students: no contingency action (2→0)</i> <i>-Explaining which parts of the plans worked and why (6→11)</i> <i>-Explaining how and why certain activities did not work (2→11)</i>

**Table 4: Overview of the teachers' design and application of the lesson plans**

Table 4 indicates that there is an increase in the number of the teachers, who recognised students' misconceptions, proposed effective questions and referred to multiple ways of assessing student knowledge by the end of the third week's application. In parallel with this finding, the teachers' focus on computational applications decreased. No change occurred in the use of multiple representations while selecting and/or preparing carefully-designed tasks, and the teachers' emphasis on explaining mathematical concepts increased. Similarly, at the end of the third week, the teachers' interplay between mathematical concepts and interplay between mathematical problem-solving techniques increased. With regard to the contingency component, after the first week's application, two of the teachers believed that all of the goals had been achieved by the students. However, at the end of the third week, there is an indication that most of the teachers' beliefs were changed. Moreover, at the end of the third week, the teachers managed to explain how and why certain parts of the plans and proposed activities did not work.

## Conclusions and Further Steps

In this paper, we present preliminary findings of an on-going large-scale project aiming to contribute to the professional development of mathematics teachers through a web-based portal. The findings of the study suggested that at the end of three weeks' usage of the megedep portal, the participant teachers managed to define more coherent learning goals, classroom activities and hypotheses for students learning regarding (lower) secondary school algebra. Most progress is observed in the teachers' interplay between the mathematical concepts by referring to different mathematical problem-solving techniques. These conclusions were possible for the teachers' own readings of the existing literature on learning algebra and the MTEs feedback that was given at each step for designing HLTs through the megedep portal. However, expressing hypotheses for students learning as well as determining learning goals based on curriculum objectives are not trivial tasks for the teachers. We aim to acknowledge this fact by showing or preparing more examples from the literature (e.g. Simon, 1995; Simon & Tzur, 2004) in order to provide teachers with a detailed understanding to write hypotheses for their own classroom context. In summary, the main contribution of this paper is to show how a web-based portal could mediate the professional development of mathematics teachers, where the key component in such progress is peers' review



and feedback in lower algebra context provided by the megedep portal. The use of acknowledged digital technologies here constructs a link between a *collaborative design* for teaching-learning algebra and *classroom practice*, which can be considered as novel progress for their professional development. The main ease here is the collaboration, which is independent of time and place.

The next steps of the project can be summarised by conducting the same process with seventh grade teachers, but over seven weeks and this time focusing on the students learning and the use of pre-test and post-test tasks. The results of these steps will be the focus of future research papers.

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