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The co-design of a c-book by students and teachers as a process of meaning generation: The case of co-variation

Dimitris Diamantidis¹, Chronis Kynigos¹, Ioannis Papadopoulos²

¹University of Athens, Educational Technology Lab, dimitrd@ppp.uoa.gr ; kynigos@ppp.uoa.gr ;

²Aristotle University of Thessaloniki, ypapadop@eled.auth.gr

In this study we focus on the role of students as co-designers of digital resources for learning mathematics. Students with the use of c-book technology construct in collaboration with their teachers units of narrative blended with artifacts and constructions designed with digital tools for mathematics education, acting as designers. We see the design phase as a learning process for students, searching for new mathematical meanings that emerge around the concept of co-variation. For our analysis we use the lens of Documentational Approach as a means of a deeper understanding of the learning process.

Keywords: c-book, Documentational Approach, design of digital tools, co-variation

Introduction

Proposing answers on how to create didactic objects that may provoke reflective mathematical discourse in the classroom, fostering students to make significant mathematical advances and transforming grounded conceptual understandings about certain mathematical concepts constitute a generic aim of instructional design research (Thompson, 2002). The design of didactic objects entails making assumptions on students' and teachers' way of participating in this discourse. However, there is a productive tension between teachers' intentions and actions when they design tasks and the students' activity when they are involved in them (Johnson, Coles & Clarke, 2017), that could be studied in more detail. Research into the design and use of mathematical tasks accommodating students' intentions, actions and interpretations to the same extent as those of teachers' is fragmented. Focusing on the use of digital media in mathematics education, the students' agency during the learning process and its confluence on instructional design could become more visible; students along with mathematics teachers may share the role of co-designers of digital resources that can be used as didactic objects. In this report we build on previous research about students' activity with a 'c-book' technology, a new digital environment where storytelling is blended with mathematical tasks (Papadopoulos, Diamantidis & Kynigos, 2016). But, now we make a step further, using c-book technology to put students in the role of the co-designer (the 'narrator') in the sense of building on the notion of design as a learning process and as a means to intervene in educational practice (Kynigos, 2015a). We focus on the question: How does the students' role as co-designer affect the mathematical meanings they make through the design process? In our research, we employed the Documentational Approach (DA) a framework hitherto used to understand and support teachers as they iteratively design and use resources in the classroom (Gueudet & Trouche, 2009), in order to understand classroom practice and student meaning making.

Theoretical Framework

The focus of this research is on the design of a digital medium as a learning process; a group of teachers and students jointly design such a resource for learning mathematics through the pages of a digital narrative. Assuming that the element of storytelling may provoke students' initiative, the design process could be a field of dense interaction between students and teachers. It could yield a series of digital artefacts (i.e. narrative meshed with constructionist digital 'widgets') as foregoing versions of the end product, along with discussion among the members of the group that refers mostly to argumentation about them, reconstruction and exchange of different versions of artifacts. From the constructionist point of view, this communication between teacher and students results to a learning collective where meaning making process occurs naturally (Papert & Harel, 1991). In this sense, the digital medium to be designed is more than a product for the designers' group; it may be an object-to-think-with (Brennan, 2015; Kynigos, 2015b). Especially for students, the design process of this kind of medium could foster the generation of new meanings around mathematics.

To investigate this aspect in detail we have chosen the approach of DA (Gueudet & Trouche 2009; Guin & Trouche, 1999) that describes the creation of new resources. Taking into account that DA has been used to study teacher's role as a designer of digital resources, we adopted it to study the students as they co-design resources with the teacher being part of the mathematics course. According to Documentational Genesis (DG) (Gueudet & Trouche 2009; Guin & Trouche, 1999), a 'document' is the set of the existing resources the designer uses together with the scheme of use for these resources in a certain class of situation; i.e. when a mathematics teacher constructs a certain task for teaching functions with a digital medium (this is the class of situation), he/she attaches a scheme of use for a set of resources (the curriculum, another related task, a discussion with other teachers etc), shaping a document. In parallel, this task can be a new resource for him/her or others. In this study, we have students collaborating with teachers in the design of new resources, shaping new documents about mathematics blended with the element of narrative, which might affect the set of the existing resources. Teachers, at the same time have their own teaching agenda. Thus, in terms of instructional design, the documents that teacher and students jointly create can be seen as didactic objects, as well. Thus, we found DA useful to help us enhance our understanding of a process through which new mathematical meanings for students may emerge.

In this paper, we present instances of students as they generate meanings around the concept of co-variation. Following Thompson and Carlson (2017) approach of co-variational thinking we build on the hypothesis that co-variation emerges when students conceptualize a situation envisioning two quantities varying simultaneously and dynamically. Quantity refers to students' conceptualization of an object's attribute that can be measured. A quantity is varying in the sense that students envision the object having momentary states, thus its attribute has momentary values (Thompson, 2002).

The Digital Medium

The c-book technology

C-book technology affords the design and use of modules named c-book units. Each c-book unit consists of a narrative blended with diverse "widgets"; hyperlinks, videos and mostly instances of

digital tools like GeoGebra, DME (Digital Mathematics Environment) and MaLT2 (Kynigos, 2015b). DME provides also an authoring tool for the teacher to design new modules, while MaLT2 is a web-based Turtle Geometry environment which affords the design through Logo programming along with dynamic manipulation of 3D geometrical objects using sliders. GeoGebra and MaLT2 allow customization and personal construction of tools (Healy & Kynigos, 2010) in the form of microworlds, fostering the meaning making process. The end users (mostly students) of a c-book unit can explore the narrative, experiment with the “widgets between the lines” and be involved in mathematical tasks. C-book environment also includes the “Workspace”, a tool that affords asynchronous discussions between the designers or the users of a c-book unit organized in ‘trees’ (Fig.1).

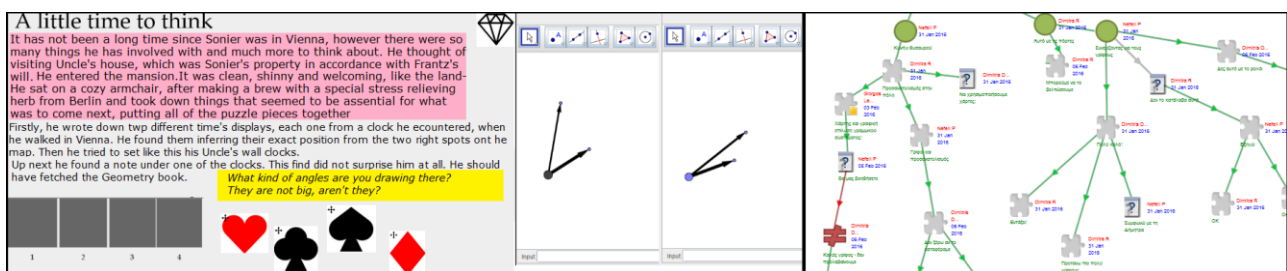


Figure 1: A c-book unit page on left hand side. The Workspace on right hand side.

The Design of the Study

Jewelry c-book unit was jointly designed from scratch by a group of students and teachers, for the purposes of this study. The storyline in this c-book unit is about a dinky merchant named Sonier back in 1820 who got an inheritance after his uncle’s death. However, the size of the uncle’s assets was not clear, so Sonier got in a quest to detect the inherited goods. His adventure - full of math related riddles and playing cards - involved a trip from Berlin to his uncle’s house in Vienna. His uncle’s passion for gambling -in contrast to his love for mathematics- was well-known. Widgets were used by the designers’ group to make up challenging tasks as parts of a riddle. A riddle was formed either by one task alone, or by a sequence of tasks, and the answer of each task was prerequisite for solving the next one and moving forward with the story. The plot was narrated through comic frames and text giving hints that might help the readers to solve the riddles (Fig. 1).

The group of designers consisted of three Grade-9 students in a public Experimental School in Athens (where teachers follow the same curriculum with other schools, adopting innovative teaching methods and/or conducting teaching experiments in collaboration with the University) and two teachers of mathematics. One of the teachers was the school teacher of the students and he additionally shared the role of the “participant observer”. The communication between students and teachers while making the c-book unit, took place exclusively in a shared digital Workspace. The whole process lasted almost two months. Students and teachers were exchanging ideas and widget drafts in the Workspace in order to discuss and decide their inclusion to the c-book unit. Whilst the students were akin to the use of digital media, they did not have the competence to make changes in an already designed widget, so when they wanted to suggest a modification of an existing widget, they used to “draw a picture” to inform their teachers. The 68 posts uploaded in the Workspace and the produced c-book unit constituted our data. The analysis of the collected data took place on the

level of identifying and analyzing critical episodes defined as selected segments of (discourse) activity with a single theme as a focus in the discussion among the design group members in the Workspace (Kynigos & Kolovou, 2018). The focus in these episodes lies on instances that show how the storytelling affected the emergence of new mathematical meanings around co-variation.

Results

The theme of “Jewelry” was used by the teachers as a sparker for the content of the c-book unit from the very first moment. As a result, one student posted in the Workspace a short story that triggered the whole group to make a decision about the main idea of the plot: The heir of a probably great but mysterious fortune. From the beginning of the discussion among students and teachers, the question “what kind of task should be constructed to be included in the c-book unit?” became the epicentre. Teachers (T1 & T2) proposed a couple of tasks, and posted them on the Workspace. The main issue for the reader was to fix a buggy-by-design diamond constructed in MaLT2 (Figure 2). The teachers’ aim was to underpin learning through tinkering. Students (S1 & S2) expressed a discontent, saying that this could be boring. Instead, they made a suggestion:

- 1 S1: I think this could be a little boring. It seems better if we had riddles,
2 not pure math problems; like an orientation game we recently played.
3 S2: Yes! And the riddles could be connected to each other; you must solve one
4 riddle before moving to the next, and go on with the story.

This was the designers’ first goal, to construct a ‘chain of riddles’ connected to each other in a way that the narrative would be garbled if one or more riddles remained unsolved. The teachers posted suggestions on possible twists of the narrative, including clues about riddles, as an effort to find out what a proper riddle might be for students, while students made their own suggestions. Soon, it was obvious that riddles’ selection would leave its mark on the narrative or vice versa:

- 5 S1: The uncle could have hidden a different diamond in each room at his house.
6 Sonier should replicate the correct diamond using MaLT2. The challenging
7 thing is that Sonier should find the correct room number, by solving an
8 equation, or by calculating a value of a variable x (Figure 2).
9 T2: Why is it challenging?
10 S1: It is more difficult to make these calculations, than having a number
11 ready to be used.

In the previous extract, S1 uses the word “variable” referring to a numeric expression (Fig. 2), maybe just because the value of x is not obvious or easy to calculate. Students made a suggestion about a riddle, where the challenge was to unlock a room, by calculating x ’s value which was the door’s ‘key’. In the next couple of days students reflected on their design, inferred with frustration that if someone solves the door’ riddle once, then he/she had no interest in reading the c-book unit again, since he/she knows the answer:

- 12 S2: The value of x must be different every time someone opens the c-book unit.
13 T1: We could use a slider in GeoGebra.
14 S1: T1, in which way? S2’s idea reminds me of a book I read. The story

15 had many different endings. We could also do the same; the flow of
 16 the narrative can vary in relation to the value of the variable x.

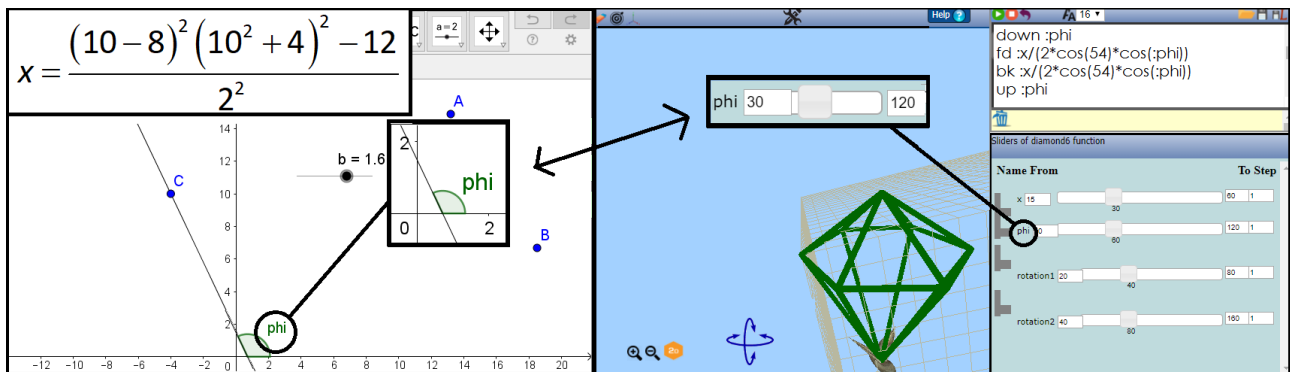


Figure 2: The GeoGebra widget-riddle on left hand side. The ‘diamond’-riddle in MaLT2 on right hand side. Designers used these artifacts in the c-book unit to form a narrative with alternate endings.

On top left hand side: the ‘x’ of line 8.

This might be considered the ultimate aim of the designers; A c-book unit that would be interesting to be read repeatedly. Soon, they realized that it was not feasible to have a different answer for the riddle every time someone opens the c-book unit. Hence, there was a shift in the group’s effort. They turned to the connection between the readers’ choices and the alternate versions of the narrative. Teachers made up a microworld that afforded the variation of a line’s position on the Cartesian plane, by the use of two sliders in GeoGebra, as a sparker for students to make a riddle. Therefore, the use of the slider came up again, as an option (Workspace):

- 17 T1: See the graph in GeoGebra. What changes when you move the sliders?
- 18 S1: The position of the line.
- 19 S2: We can relate the diamond’s shape that Sonier asks for, with the line.
- 20 T1: To what characteristic of the line?
- 21 S2: The sliders change the angle between the line and x-axis. We could imply
- 22 through the narrative that this angle should be equal to diamond’s angle.

It seems that the designers came up with the idea of using the notion of angle as an element that would give a potential relation between two riddles. Thus, students made up a story in GeoGebra as the first riddle. There was a map of Vienna on a Cartesian plane with a line on it. The position of the line was determined by two sliders. The reader should put the line in a position of his/her choice, in order to make a path. Then, using the size of the angle between the line and x axis in degrees, as a hint, the reader should fix the buggy diamond in MaLT2 (Fig. 2) that had been proposed by the teachers at the beginning; this was the second riddle. The twist of the narrative was depended on the diamond that was constructed, since its shape was crucial for the storyline. It seems that the designers used the answer of the first riddle as an output magnitude necessary for solving the second one. When the content (microworlds and narrative) was ready to be used, students suggested making a trial on how it works focusing on the use of constructions:

- 23 S1: We should play a couple of times, maybe more, to see how it works.
- 24 S2: Yes, we should draw a line that changes smoothly and normally in order to

So, the students tested the two microworlds many times, to observe if the variation of the line's position in GeoGebra had an elegant effect on the diamond's shape in MaLT2, through the common angle measure. Finally, they decided to use the same name 'phi' as the name of the variable for both the diamond's angle in MaLT2 and the angle in GeoGebra, too (the angles mentioned in line 25).

Discussion

For the analysis of the Jewelry's design, under the lens of DA we have distinguished four phases. In the first phase, although they shared the same class of situation, i.e. to design a c-book-unit, students and teachers seemed to have their own separate agendas. Teachers suggested an instance with a buggy-by-design figure of diamond in MaLT2 which mostly promoted the added value of learning mathematics through tinkering, while students overrated the amusement of the reader. Furthermore, students at first almost rejected the microworld as 'pure mathematics' (line 2), suggesting to use riddles 'like in a game they recently played' instead of mathematical problems. This reaction may be in line with the hypothesis that students' set of resources was mostly related to storytelling, than to the design of mathematical tasks. This became more evident in the second phase, where the designers tried to make up riddles linked to each other, which probably is a good description of the new class of situation for the group. Despite the consensus on that, there was no common understanding of what kind of riddles should be used. Students made up an intriguing story about hidden diamonds in locked rooms, trying in the same time to involve a variable x as the 'key number' to unlock the door. We assume that they made an effort to use resources with mathematical content, such as a narrative based on the use of a variable, representing it with x (line 8). However, this effort was fruitless, since they did not manage to operate the variation of x . In the sense of co-variational thinking, the dysfunctional use of x seems to be congruent to Thompson's view (2002); although a person operates successfully on algebraic representations, it is hard to talk about the represented relationship. In the third phase, the aim of the design was more apparent. The riddles should be constructed in a way to provoke the reader to read the c-book-unit many times, which as a class of situation was more specific. To construct this kind of riddles, teachers proposed an instance in GeoGebra where a line changed position through the use of sliders (line 13). The core idea was to relate each time the 'key number' of the locked door to a different position of the line; Unfortunately, this did not work. It seems that the relation between sliders' manipulation and the line's changing position was not a usable case of co-variation for the students, since the varying attributes of the line might not be clear in the momentary states of its variation (Thompson, 2002). Using this kind of instance as a design unit might not be feasible for the students according to their scheme of use for their set of resources. However, students made a contribution which determined the final product, the c-book-unit as a 'document'. They proposed that the c-book-unit should have alternate endings, in order to be provoking. This idea, came up from a resource related with storytelling, (i.e. a book read by S1, line 14), and was soon adapted by the rest of the designers as the main goal of their efforts, during the fourth phase. Therefore, the role of instances-riddles was to give to the narrative the characteristic of different twists of the plot. To achieve this goal students and teachers used the instance in GeoGebra, along with buggy diamond in MaLT2, trying to 'link' them. Thus, it seems that they conceptualized this linking as a situation of two objects (the diamond

and the line) varying simultaneously and dynamically. This became clearer, when students tried to coordinate the manipulation of the line in GeoGebra with the diamond in MaLT2 (lines 23-25), so that a move of the sliders in GeoGebra should correspond to an ‘elegant’ and ‘normal’ change of shape of the diamond. This action seems to be in line with Thompson’s (2002) view of co-variation that points out the importance of coordinating actions among two attributes with an ‘operative image’ of their co-variation so that one can act out how the two are related. In phase four, students’ goal was to conceptualize and define a ‘co-variational relationship’ between two attributes; the position of the line in GeoGebra and the diamond’s shape in MaLT2. It seems that students, by making use of the two sliders that represented angles’ measures (one related to the line and another related to the diamond), they envisioned these two attributes’ momentary states varying simultaneously (lines 17-22). Thus, they decided to use a common variable ‘phi’ related with both angles as an operative image of the co-variation between line’s position and diamond’s shape, linking the two riddles. According to Johnson, McClintock, Hornbein, Gardner and Grieser (2017) students are facing fewer difficulties to discern co-variation among quantities (in the sense of Thompson) that can be measured with linear units. Building on this position, we assume that, in our case, students employed such a quantity ‘phi’ (that represented an angle’s measure) to define and use a ‘co-variational relationship’ among two objects/attributes (the position of the line and the shape of the diamond) that could not directly be measured.

Conclusions

In this study, the focus was on the role of students as co-designers of a digital medium, a c-book-unit, i.e. a digital narrative blended with tasks and constructions with mathematical content. We used the DA theory to understand in detail the way new meaning around co-variation during the design process emerged. During the four phases of the design process the students’ set of resources and their scheme of use seemed to be in flux. In the beginning, the use of resources had mostly to do with storytelling. This changed progressively, and in the last phase a combination of narrative with mathematical resources was used. It was in this phase when, according to our analysis, students made meanings around the concept of co-variation, while ‘building’ a relationship for the needs of the storyline. It is remarkable that, after building this relationship, students tried to attune the mathematical content to the narrative, referring to the line in GeoGebra, as a path on a map. It seemed that initially, the differentiation between students’ and teachers’ resources and their schemes of use, worked as a barrier to the design process. However, from the analysis of the ‘document’ is evident that when resources were related to both narration and available mathematical tools, this led to the emergence of new meanings. From this point of view, it seems that when students act as designers using this kind of media, they take the role of both the storyteller and learner. Given the lack of research in this field, we believe that such enquiries should come up, assuming that these two roles -of storyteller and learner- should not be seen as separated, since the interaction between different kinds of resources was crucial for the design as a learning process.

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