Towards linking teaching, technology and textbooks

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The aim of the research project MAL (multimodal algebra learning) is to develop a digital algebra learning system (MAL-system) that provides an accessible approach for teaching and learning algebra. The subproject MAL textbook conducts an expert study with secondary school teachers who are also textbook authors to identify current teaching practices and needs. These are reconstructed by three iterations combining group discussions and questionnaires. The results are considered in the design of the MAL-system and lead to principles for integrating the MAL-system into textbooks. This paper shows how the surprisingly high commitment to the balance model is extracted from the data and its impact on the design of the MAL-system.

Keywords: Delphi technique, textbooks, digital tools, anthropological theory of the didactic.

Introduction

Although digital tools are available for everyday classroom practice, researchers as well as teachers are questioning how the use of digital tools can improve teaching and learning (Hillmayr, Reinhold, Ziernwald, & Reiss, 2017). Despite from the digital tool itself, two main factors are responsible for students' success: the teachers' integration of digital tools into classroom practice and a supplementary use of digital and traditional resources (Hillmayr et al., 2017). Teachers' integration requires change or innovation of current teaching practices. Research on innovation of teaching and learning shows that teachers are more likely to use digital tools if they address teachers' needs. Moreover, fruitful innovation in schools should rather be an incremental process building on current teaching practices than a revolutionary break (Bikner-Ahsbahs & Doff, 2019). These teaching practices are highly influenced by textbooks: on the one hand, textbooks specify the mathematical content; on the other hand, they shape the didactic style that teachers apply in their teaching (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Thus, textbooks represent teaching practices as well as traditional resources. Taking this into consideration, to provide the potential for integrating a digital tool for learning in classroom practice, already the development of the tool should follow a user oriented design that supports students' learning and furthermore takes into account the textbook and teachers' practices and needs.

In the research and development project Multimodal Algebra Learning (MAL), the overall goal is to develop a digital algebra learning system (MAL-system) that supports the students' transition from arithmetic to algebra, however, this paper is restricted to linear equations. With the design of the MAL-system we want to overcome the disadvantages of using either digital or physical manipulatives by integrating physical manipulatives in a digital environment (Reinschlüssel et al., 2018). The subproject MAL textbook focuses on (A) integrating teachers' perspectives into the

design process of the MAL-system and (B) providing strategies for using the MAL-system facilitated by textbooks. This will be achieved by answering the following research questions:

(1) What kind of teaching practices do teachers consider when confronted with the concept of the MAL-system?

(2) What kind of needs for teaching do teachers address for the design of the MAL-system?

(3) What kind of criteria for accepting or rejecting the MAL-system do teachers express?

(4) Which principles for integrating the MAL-system into textbooks can be extracted from (1)-(3)?

This paper focuses on the methodical design of the MAL textbook study based on a Delphi study. It will show that this choice is fruitful to inform research and development of digital tools with teachers' practices. These practices are conceptualized in the subsequent theoretical framework.

Theoretical Framework

In order to consider established teaching practices in the design of the MAL-system and, beyond that, to develop guidelines to include the MAL-system into established textbooks, the theoretical framework should address institutionalized forms of teaching and learning. This is possible by considering a theory that captures these teaching practices, the Anthropological Theory of the Didactic (ATD). The ATD (Bosch & Gascón, 2014) models human activities by the concept of *praxeology*. According to ATD, human activity splits up into a *practical* and a *theoretical component*. The former is broken down into a set of *types of tasks* and a *technique* that tells how these tasks are or might be carried out; the latter is broken down into *technology* and *theory*. The technology explains how and why a technique works. The theory comprises basic assumptions and views supporting and justifying the technology. In many cases, it is hard to reveal the theory because it is usually taken for granted. The practical component is often referred to as the know-how, whereas the theoretical component is the knowledge behind, in the sense of *raison d'être*. The set of all praxeologies of one person is called her *praxeological equipment*. In general a person's praxeological equipment emerges out of previous and current institutional settings including the people surrounding her.

Algebra tiles and the MAL-system

The MAL-System is developed based on algebra tiles (Dietiker, Kysh, Sallee, & Hoey, 2010). Algebra tiles can be used for modeling linear and quadratic expressions and equations with integers. A small square represents the number one, a big square represents x^2, and a rectangle with the side lengths of the two squares represents the variable x. The three types of tiles usually have three different colors on one side and are all red on the other side. The red side indicates a "negative sign" (Figure 1 a). For modeling a linear equation a so-called equation mat with two distinct sides, left and right, is needed. By laying out the appropriate tiles, an equation is modeled (Figure 1 b). The equation can be solved by taking away or adding the same tiles on both sides and dividing both sides into the same number of equal sets. A subtraction zone – an area where all the tiles within are supposed to be subtracted – can be introduced; either to mark the difference of sign and arithmetic operator or to express subtraction without negative Tiles (Figure 1 c).

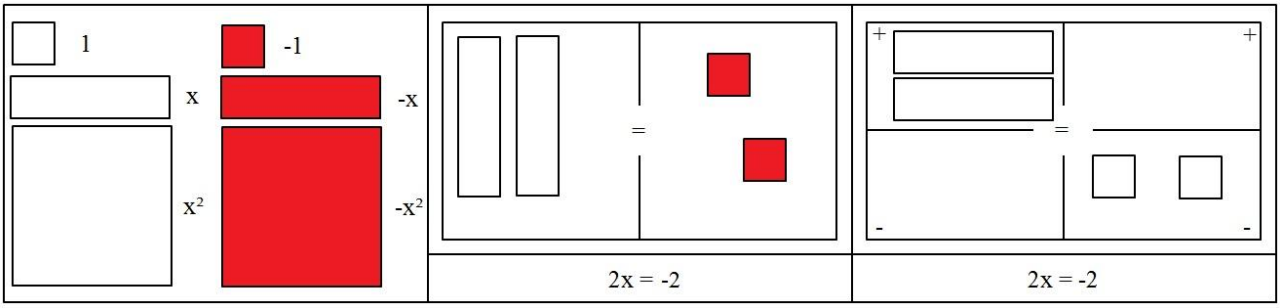

Figure 1 a: algebra tiles; b: equation with negative tiles; c: equation with subtraction zone

As the focus in this paper is on linear equations only, x^2-tiles are not addressed in this paper.

Methodology and Method

In Germany, textbook authors are primarily in-service teachers. Therefore, they are familiar with established teaching practices in their schools and many of the teachers' needs for teaching. As these teachers author a certain textbook series in a team, they are also influenced by the teaching practices provided by this textbook series. Hence, their praxeological equipments are affected by their experience as teachers as well as textbook authors. This awareness of relevant praxeologies makes these teachers experts for the MAL textbook study to inform the design of the MAL-system through established teaching practices and relevant needs for teaching algebra. In order to gain main directions of consensus for adapting the MAL-system to these practices and needs, an expert study in the style of a Delphi study is conducted.

According to various definitions, any Delphi study comprises an expert group communication with several iterations and guided feedback to achieve consensus on a given question (Linstone & Turoff, 1975). The MAL textbook study conducts three iterations combining group discussions and questionnaires building on one another (Figure 2). Three groups of authors, working on books for three different performance levels are involved (*Math Alpha*: comprehensive school (covers and combines lower secondary education, secondary education and high school education); *Math Beta*: special school; *Math Gamma*: high school). These groups are selected because they cover a wide range of school types at different performance levels in Germany. The first iteration includes group discussions of the teams Math Alpha and Math Beta. Due to organisational reasons the team of Math Gamma did not participate. The topics are algebra learning, digital learning, gamified learning, and the MAL-system, introduced by a concept video. The second iteration includes all experts to answer an online questionnaire readdressing the topics from the first iteration by open and closed items. For example, the high relevance of the balance model to the experts came up in both discussions. As this was unexpected, the questionnaire asked for advantages and disadvantages of it within the whole expert group. In the third iteration, comprised statements based on the answers in the second iteration are evaluated by the experts. According to the example with the balance model, this iteration reveals tendencies on the central advantages and shows the willingness to make compromises concerning disadvantages.

The group discussions are audio taped, transcribed and analysed in a sequential way (Przyborski, 2004) based on ATD concepts to reconstruct the textbooks' and teachers' praxeologies, identify

teachers' needs and principles for textbook integration. The data set of the second iteration is analysed by theory-driven content analysis (Mayring, 2015) to capture individual praxeologies and needs based on the authors' experiences in school. The final data set is analysed by descriptive statistics to identify quantified relevancies of the praxeological elements from the iterations before.

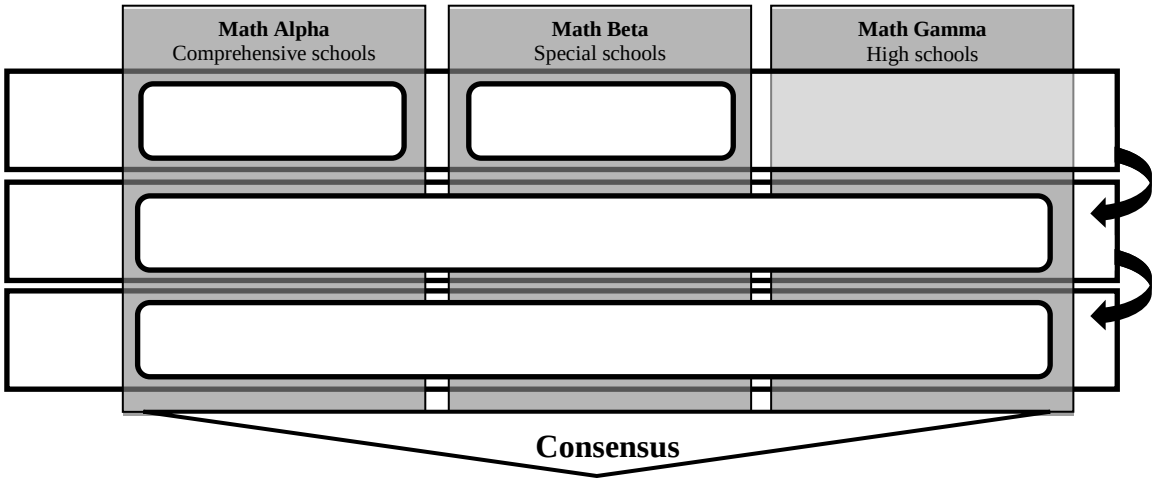

Figure 2: Procedure of the MAL textbook study

Results

This section shows the three steps of data analysis applied to reconstruct the praxeological equipment of the expert authors. Presenting the results, we focus specifically on the balance model since the teachers have shown a strong commitment to it throughout the three iterations. Praxeological elements that came up while comparing algebra tiles and the balance model in the first iteration are presented. As the experts could hardly detach from the balance model while judging algebra tiles, the second iteration asked separately for advantages and disadvantages of the two models. The categorized range of experts' opinions extracted from data analysis is then presented. From the third iteration, the final evaluation of selected (dis-) advantages is presented.

The first iteration

In the first iteration, the teams Math Alpha (12 participants) and Math Beta (5 participants) took part in a group discussion each. The researcher gave an introduction on algebra tiles and the experts were invited to work with them. Afterwards, the experts were asked if they can imagine using algebra tiles in class. Both groups immediately refused to do so, giving the same *technological*[1] arguments. Similar to expert A10[2]: "[the balance model] is just easier for the kids to understand" (#236-237)[3], teacher expert B2 said: "the balance model is much easier" (#76-77) and narrowed to "more illustrative" (#78) by B3. Another *technological* argument that came up in both groups in the beginning is that algebra tiles are "too abstract" (A10, #238) or rather "very abstract" (B3, #82).

[1] The reconstructed praxeological elements are highlighted in italics.

[2] The experts from Math Alpha are named AXX and those from Math Beta BXX.

[3] (#236-237) refers to the lines in the original transcript that is not shown here due to the limited space.

In the discussion of the Math Alpha team, A1 then mentioned that "the problem of the balance model is clearly that I cannot express negative numbers with it" (#239). This was a turning point in the discussion towards algebra tiles, validated by three more experts. A1 continued to point at a *technique* based on algebra tiles: "one can slide the […] *xs* that cancel out each other together" (#242-243). This was justified by the *technology* that "this […] also always helps" (#243). At this point, the Math Alpha group stopped comparing algebra tiles and the balance model.

The process in the Math Beta group was slightly different. B6 presented a "counter *technology*" concerning two different *techniques*. One was working with the balance model on the iconic level; the other was working with algebra tiles on the enactive level. Based on the *technological* argument that the enactive helps in real life and the iconic does not, B6 saw an advantage of algebra tiles versus the balance model. Based on that, B5 suggested a didactic technique combining the two models. She started with the core *technology*: "the equal sign […] means the balance" (#125-126) that was substantiated by the *technology* that the pupils should interpret the equal sign as a relational and not as an arithmetical sign. Based on that, her *didactic technique* related to the balance model began with using it, "this [equal sign] could maybe be introduced with the balance" (#126-127). But then B5 suggested to use algebra tiles "in the beginning really only using additive elements" (#128-129), before involving the subtraction zone or negative tiles. This was justified by the *didactic technologies* that the concrete action of laying out tiles is helpful and the subtraction zone "is a demanding conception" (#132).

This analysis shows that the experts' praxeological equipments on teaching linear equations is highly bound to the use of the balance model. The experts could hardly think of *didactic techniques* involving algebra tiles that fit to their *theoretical arguments* and moreover they could hardly detach their *theoretical arguments* from the balance model. Even the positive statements on algebra tiles were mostly connected to weaknesses of the balance model. To reveal *theoretical arguments* detached from comparing the two models we asked for advantages and disadvantages of the two models separately in the second iteration.

Second iteration

In the second iteration a questionnaire was sent to 37 experts. 20 of them answered (5 Math Alpha, 4 Math Beta, 11 Math Gamma) the questionnaire. The answers on the advantages and disadvantages of the balance model are summed up in categories in Table 1.

	Advantages	Disadvantages
Balance model	• illustrative • intuitively accessible • enactive and iconic working • builds on students' knowledge • prepares new knowledge • illustrates equivalence (transformations)	• no negative numbers or quadratic equations • hard to illustrate fractions • more enactive working needed • transition to symbolic level problematic • students do not know the balance
Algebra tiles	• illustrative • negative numbers and quadratic equations	• abstract • linking of area and term is difficult

	• explain more than the balance model • link algebra and geometry • combinable with balance model • enactive and iconic working	• explanation takes much time • do not illustrate equivalence (transformations) • not intuitive for students

Table 1: Categorized results of (dis-) advantages of balance model and algebra tiles

We focus on the complementary findings in this iteration. As expected, the experts mentioned the core *technology* from the first iteration as an advantage: the balance model illustrates equivalence very well. In contrast, this is a disadvantage of algebra tiles, e. g. one expert from Math Alpha stated "[the] equivalence is taken for granted […]". This *technology* is connected to the *technological argument*: the balance model is intuitive for the students whereas algebra tiles are not.

The other way round, an advantage of algebra tiles is that negative numbers and quadratic equations can be illustrated. In particular, the lack of expressing negative numbers was mentioned as a disadvantage of the balance model several times. This also already occurred in the discussion of Math Alpha in the first iteration as a *technique* to favor algebra tiles.

The third iteration offers further clarity about the experts' preferences by rating the findings above.

Third iteration

In the third iteration, the 37 experts were again asked to answer a questionnaire. We received 19 answers (6 Math Alpha, 4 Math Beta, and 9 Math Gamma). Figure 3 shows the evaluation of the four complementary statements: (1) The balance model enriches my teaching because it illustrates the equivalence very well. (2) The algebra tiles are insufficient for my teaching because they do not illustrate the equivalence. (3) The use of algebra tiles would enrich my teaching because it can represent negative numbers. (4) The balance model is insufficient for my teaching because it cannot represent negative numbers.

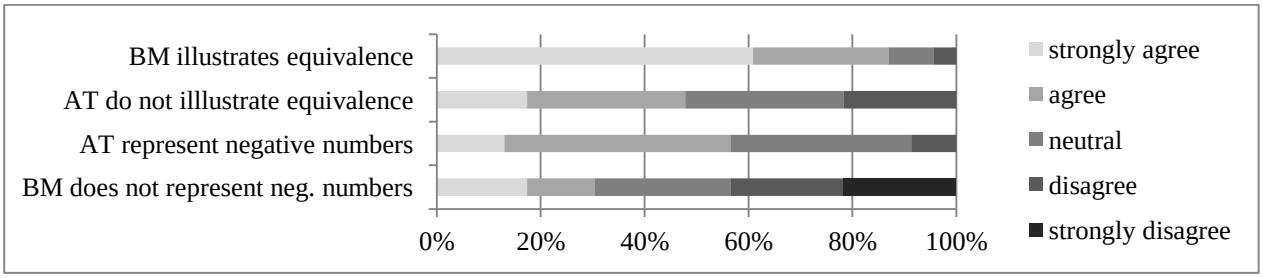

Figure 3: Ratings on complementary statements (BM: balance model; AT: algebra tiles)

These results emphasize the importance of the core *technology* substantiating the use of the balance model. More than 80% of the experts agreed that the balance model enriches teaching because it illustrates the equivalence very well. In contrast to that, about 50% of the experts rated algebra tiles as insufficient for their teaching because the intuitive access to equivalence is missing. Only 20% did not take this as a barrier for classroom teaching.

The strongest advantage of the algebra tiles over the balance model is the possibility to represent negative numbers. More than 50% of the experts evaluated this feature as enriching for their

teaching, and only 10% disagreed on that. However, only one third of the experts evaluated the balance model as insufficient due the lack of negative numbers and as one half of them disagreed, the balance model was still rated positive on average.

All in all, the results show that the balance model is indispensable in current teaching practices, however, the lacking representation of negative numbers gives space for the algebra tiles to enter classroom practice.

Discussion

The *didactic technique* - teaching linear equations with the balance model - reconstructed in the previous section reveals insight into current teaching practices. Independent of the type of school, the balance model turned out to be the most relevant model for teachers when teaching linear equations. However, the experts were aware of the problems and limitations of the model, e. g. no negative numbers, special kind of balance is unknown to kids, limited possibilities to explore equations concretely, aspects also discussed in the literature (e.g. Vlassis, 2002). Especially the missing representation of negative numbers points to a need in current teaching. The results from the third iteration suggest a complementary use of the two models allowing to address strengths of both models in a complementary way. We suggest introducing equations with the balance model by working enactively with the balance in class to make the students experience the equal sign as a relational sign of keeping two collections of weights in balance. The balance may then be readdressed in the MAL-system as a feedback symbol for equality when the students begin to work with algebra tiles. Starting only with positive tiles and additive equations, a balance in the middle, that is balanced or not (depending on the correctness of the actions carried out) instead of the equal or unequal sign is used.

As the authors *theoretical and technological arguments* on teaching linear equations are highly influenced using the balance model, it should not be replaced by the MAL-system. The balance model seems to be very fruitful for teaching in all the three types of schools in Germany since it provides intuitive access to the equal sign of equations and the equivalence relation. Addressing a current need like the representation of negative numbers which is missing in the balance model turns out to be a criterion for accepting the MAL-system. In addition, a careful integration of the MAL-system into textbooks could compensate the teachers' judgement of algebra tiles as being abstract. This could be done by creating a context that makes algebra tiles more accessible to students, for example by gamification or as an expanded digital environment for exploring problems. An important issue for increasing acceptance of the MAL-system is its correspondence to textbook praxeologies, as well.

Looking back to the methodology, the Delphi study with the three iterations has proven to be fruitful for investigating teachers' needs for teaching, their praxeologies and textbook praxeologies. The discussions have led to three important insights: the teachers' strong commitment to the balance model; contrasting the balance model with Algebra Tiles has made the teachers sensitive to disadvantages of a given technique that is normally not questioned; it has shown that a negotiation process is needed to disclose hidden needs for teaching underlying the praxeologies. The second iteration has provided the possibility to readdress aspects not considered before like the balance

model. The final questionnaire supplies a quantitative overview, showing possible differences according to the types of schools. In conclusion, the three steps of the Delphi study exactly fit the purpose of the MAL textbook study. As they are not tied to the topic of algebra, they unlock potential for further research on the role of textbooks in reform processes with a wider scope.

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