



HAL
open science

Introduction to the papers of TWG15: Teaching Mathematics with Technology and Other Resources

Alison Clark-Wilson, Ornella Robutti, Melih Turgut, Iveta Kohanová

► To cite this version:

Alison Clark-Wilson, Ornella Robutti, Melih Turgut, Iveta Kohanová. Introduction to the papers of TWG15: Teaching Mathematics with Technology and Other Resources. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02417049

HAL Id: hal-02417049

<https://hal.science/hal-02417049>

Submitted on 18 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Introduction to the papers of TWG15: Teaching Mathematics with Technology and Other Resources

Alison Clark-Wilson¹, Ornella Robutti², Melih Turgut³ and Iveta Kohanová⁴

¹University College London, London, United Kingdom; a.clark-wilson@ucl.ac.uk

²University of Turin, Italy; ornella.robutti@unito.it

³Eskişehir Osmangazi University, Turkey; mturgut@ogu.edu.tr

⁴Norwegian University of Science and Technology, Norway; iveta.kohanova@ntnu.no

Keywords: Technology uses in education, teaching mathematics, teacher competencies, teacher behavior, teacher professional development.

Introduction

At CERME11 two groups addressed mathematics education research concerning technology. TWG15 focused on issues related to teaching, teacher education and professional development, whereas TWG16 focused on students' learning with technologies, alongside software and task design issues (see Introduction to TWG16 in this volume).

The TWG15 work was stimulated by contributions in the form of 12 research papers and 8 posters that had responded to the call that had highlighted the following themes:

- The specific knowledge, skills and attributes required for efficient/effective/equitable mathematics teaching with generic and mathematics-specific technologies and resources.
- The design and evaluation of initial teacher education and teacher professional development programmes (to include MOOCs) that embed the above knowledge, skills and attributes.
- Theoretical and methodological approaches to describe the identification/evolution of teachers' practices (and of 'best' practices) in the design and use of technology and resources.
- Theory and practice relating to the formative/summative assessment of mathematical knowledge in technological environments.

The work of TWG15 drew upon research from 13 countries: Austria, Brazil, Denmark, England, France, Iceland, Israel, Germany, Greece, Norway, Portugal, Sweden and Turkey.

Organisation of the TWG15 at the conference

During the conference the TWG15 convenors piloted a new format for the 30-minute paper presentations. Two weeks in advance of the conference, each lead paper author was paired with another participant in the group, their *partner*, and it was this partner who was invited to present the paper. She or he was asked to share their presentation slides, which were required to include some questions to the author. In reality, this led to email exchanges, online calls and face-to-face meetings, most of them before the conference, and some during the early days of the conference. The resulting presentations enabled the ideas in the submitted papers to be explored in much more depth than at previous CERMEs - often requiring the author(s) to show examples of tasks, resources and functionalities from within their selected technologies. Alongside this, the TWG15 poster

submissions were thematically aligned with the TWG session programme and the poster authors were invited to give a brief overview of their research. Following these paper and poster presentations, small group discussions were instigated within the TWG that focused on understanding the presented work in its cultural context. In particular, we probed the terminologies and contexts within the reported research that related to technological, pedagogical, mathematical, theoretical and methodological ideas. The TWG leaders supported the analytic and synthetic processes of the group by capturing the comments and questions made by participants as an ongoing process. We reflect on participants' perspectives of this approach later in the TWG15 introduction.

The emergence of TWG15 sub-themes

During the final two TWG sessions, the group spent time to discuss and agree the emergent sub-themes, which were: teachers' uses of students' (digital) productions; sorting and organising digital content such as: simulations, applets, Open Educational Resources (OERs); the teaching of computing/programming in, and through, mathematics; teachers' choices and beliefs concerning technology use; and the group's ongoing grappling with theory. The TWG sub-divided into smaller groups, each of which then contributed text to the summaries that follow.

Teachers' uses of students' productions

A number of papers and posters described research that featured teachers' practices when using students' work as a basis for discussion in the mathematics classroom. In these projects different technologies were employed, e.g. *c-book* units (Diamantidis, Kynigos & Papadopoulos), videos (Kristinsdóttir, Hreinsdóttir & Lavicza; Fidje & Erfjord), connected classroom technologies (Fahlgren & Brunström), and mini-whiteboards (Eidissen, Hreinsdóttir & Lavicza). These technologies served to both make students' thinking visible and to stimulate classroom discussions based on students' own work/solutions, which are known to be important formative assessment practices (Black & Wiliam 1996).

However, orchestrating class discussion based on students' contributions was reported to be challenging for teachers for which the selection, deliberate grouping and sequencing of student productions are useful strategies. When it comes to sequencing, the role of teacher preparation was considered important, however sometimes students' responses opened opportunities for discussion that could not have been realistically predicted by the teacher. Such opportunities can contribute to the classroom discussion, if the teacher has both the flexibility to take it up and it aligns with the teacher's goals for the lesson. As such, this relates to the existing framework of instrumental orchestration (Trouche, 2004) within the socio-mathematical norms where students' various personal meanings of concepts are valued and 'up for discussion' as a crucial factor to foster classroom discourse. The TWG group agreed that the aforementioned technologies might aid teachers in this endeavour.

The use of students' productions in education was exploited by Diamantidis et al., who focused on the role of students as co-designers - with teachers - of digital resources for learning mathematics. Their project design involves collaboration between students and teachers, who used digital tools for mathematics education for the design and production of *c-books*, a set of narrative units blended

with digital artefacts. The research, which was framed by the *documentational approach* (Gueudet & Trouche, 2010), aims to research the design phase as a learning process for the students as they search for new mathematical meanings that emerge around the concept of co-variation (in the sense of Thompson, 2002).

Kristindóttir et al. presented a poster on the use of silent videos to the whole class in Iceland, with the possibility for the students to view them as often as they want. The students, divided in pairs, were asked to plan and to record their voice-over for the video. The role of the teacher is of facilitator, encouraging students in many ways, for example by reminding them that their voice-overs might help their classroom peers to gain access to the mathematics shown in the video. The research is highlighting how the use of silent video in this way is particularly helpful to teachers as a form of formative assessment, because data about students' knowledge can be collected in an indirect way.

Fidje and Erfjord's research also focused on videos produced by students, this time in Norway, which are used to stimulate discussions that are coordinated by the teacher. Their project *Digital Interactive Mathematics Teaching*, explores the use of digital tools in three lower secondary mathematics classes. The student-produced videos are presented as a tool for the students to show strategies related to inquiry-based tasks given by the teacher (in this case, relating to similar triangles). The idea emerged from the teachers as they wanted to explore different uses of videos in teaching, both teacher- and student-produced. The three-part lesson sequence used in the analysed lesson was developed by both teachers and researchers in a workshop. The aim for both teachers and researchers was to develop an approach for eliciting student talk in full-class mathematics discussions. The results show that the teachers adopt different 'teacher moves' to steer the discussion towards both didactical and mathematical goals.

The final two research studies that relate to this sub-theme are those of Fahlgren and Brunström and Eidissen et al.. Both sets of authors presented posters of work in progress. Fahlgren and Brunström's study concerns research into the impact of *connected classroom technology*, a networked system of personal computers or handheld devices specifically designed to be used in a classroom for interactive teaching and learning, through a design-based study with Swedish mathematics teachers to establish design principles for its use for formative assessment purposes. The contribution by Eidissen et al., notable by its absence of digital technology, proved to be a significant one for the TWG as it challenged the group to consider how theories such as instrumental genesis and orchestration (Trouche 2004, and Drijvers' plenary paper in this volume) might be applied for classroom practices involving non-digital resources.

Sorting, organising, and increasing quality of mathematical digital content

The current diversity, complexity and potential of mathematical digital content demands research studies to support better understanding and communication of the main characteristics, constraints, didactical value, and quality. The abundance of digital resources for the teaching and learning of mathematics highlights the need to provide teachers with guidance and support to make discerning choices. In the previous CERME, a paper was addressed specifically on discerning quality issues of web materials useful for teaching mathematics with technologies (Kimeswenger, 2017). In this

CERME, TWG15 included several contributions that had adopted both empirical and theory-driven research approaches to address the challenges to sort, organise and make judgments about the quality of mathematical digital content.

Wörler focused on finding central features of digital simulations for learning mathematics, and articulate the underlying traits that can be derived within the context of mathematical modelling. The resulting research question concerned how these features and traits might enable an a priori classifications of mathematical simulations. This question, and the results of the study give some order within the field of modelling and simulation for teaching mathematics and enable comparisons between the products used in different countries and contexts. Considering the importance of variation in the field of simulation, the author introduced a classification based on the kind of variation, number and possibilities of varying elements in a simulation model, and kind of representation used. In this way, different simulation applications are classified with a quantitative scale based on different levels of manipulation, and the classification is useful to both researchers and teachers.

Alongside simulations, applets are also widely (and increasingly) available for the teaching and learning of mathematics and it is widely evidenced that teachers find it challenging to select which ones to use, highlighting a need for some related quality criteria (Nakash-Stern & Cohen). The authors reported research that concluded teachers' choices, which were classified by the intended mathematical learning goals, the role of the applet in the planned lesson and the nature of its planned use. Nakash-Stern and Cohen's research aims to support teachers to cope with the propensity of such OER by offering a methodological tool (a meta-data map, as shown in Figure 1) to link a teacher's pedagogical-content considerations with the didactical aims.

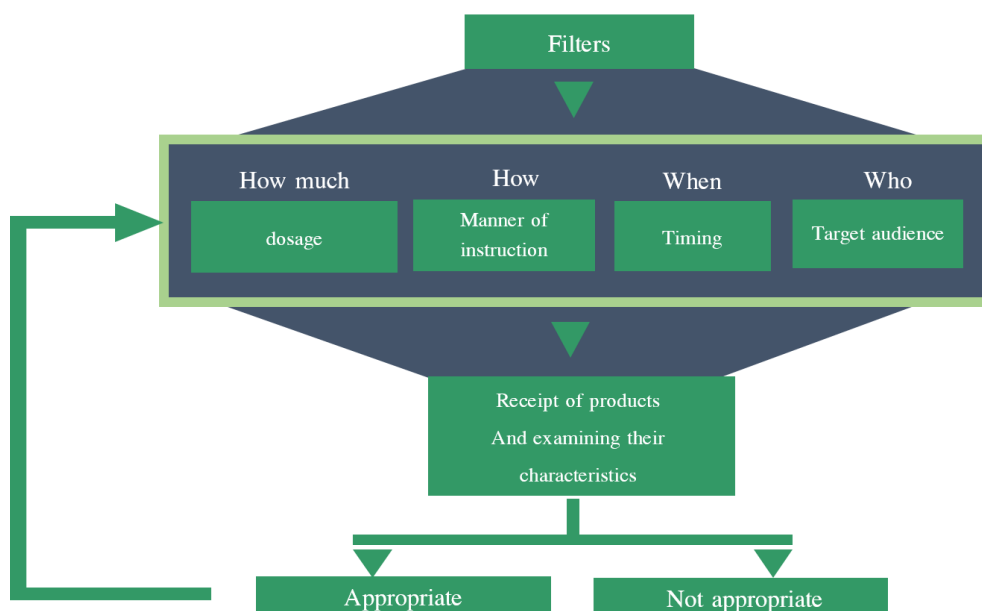


Figure 1. A Map for applets integration in teaching sequence (Nakash-Stern & Cohen)

The specific case of gamification within mathematics is addressed by Russo, whose research has reviewed the features of existing gamification platforms for mathematical learning to develop a

framework for the evaluation of their motivational aspects. This is to be used to inform the development of a gamification editing platform that will be explicitly designed for teachers' uses.

Braukmüller, Bikner-Ahsbabs and Wenderoth offer a publisher's perspective to the same challenge in their consideration of teachers' needs whilst developing a digital tool for learning algebra in the German *multimodal algebra learning* (MAL) project. They argue that, as textbooks are the dominant media in mathematics teaching, research with textbook authors is necessary to derive principles for integrating the MAL digital tool within textbook authoring.

These different approaches all address the aim to facilitate teachers' choices, uses and evaluation of all manner of digital tools in classroom practice. The collaborative work of the TWG resulted in new ideas for future research focusing on some aspects that are considered useful when evaluating a tool, for instance motivational features, authoring systems, types of use and levels of interaction/activity. Basically, two different approaches arose: platforms that suggest some tools according to teacher needs (using filters) and the provision of frameworks to evaluate a specific tool for a special use case.

Teaching computing/programming in and through mathematics

Discussions in the group revealed that several of the represented countries are now implementing computing curriculum in schools (England) and/or mandating compulsory technology usage (Denmark, Sweden). The legacy of Seymour Papert (1980) and his seminal work that resulted in the computer programming language LOGO stimulated the TWG discussions on this sub-theme.

The ScratchMaths project (Clark-Wilson, Noss, Hoyles, Saunders & Benton), a large-scale evaluation project in English primary schools focused on making the mathematics explicit by focusing on an explicit pedagogical framework, the *5 Es* (Explore, Explain, Envisage, Exchange, bridgeE). The research reports the important methodological considerations in relation to the teachers' adoption and adaptation of the pedagogical ideas as a means to assess the fidelity of classroom implementations of the computing curriculum.

In Norway, Munthe is undertaking design-based research with upper secondary students on an elected science and mathematics pathway in which they are presented with mathematical problems to investigate how the use of programming can facilitate their learning of mathematics in the classroom. His poster is a good example of how the mathematics explored was explicit, within the problem of solving quadratic functions with no real roots.

This idea is researched from the teachers' perspective in the paper by Misfeldt, Szabo and Helenius, in Sweden, where teachers' perceptions of the relationship between mathematics and programming is explored within the context where teachers are expected to integrate compulsory programming within the mathematics curriculum, without deep thought about its value.

There is a diversity in the perspectives of the different papers in relation to both the definition and positioning of computational thinking within the school mathematics curriculum, which lead to differing approaches to curriculum and teacher development. Critical to this were the questions, *where is the mathematics?* and *how is the mathematics transformed (in terms of syntax.*

representation and meaning) through the use of different programming tools? - and the implications of these approaches on teachers' roles and actions.

Teachers' choices, beliefs and practices with technology

A number of TWG15 contributions addressed issues related to mathematics teachers' attitudes to, and uptake of, digital technological tools. The paper by Thurm and Barzel adopts a quantitative approach to explore the link between teachers' self-efficacy and their use of *mathematical analysis software* (MAS). They adopt Bandura's definition of *self-efficacy*, 'a judgement of one's ability to organize and execute the courses of action required to produce given attainments' (1997, p. 3), to develop and validate a scale that concludes that teachers with high self-efficacy tend to use MAS more frequently. Furthermore, high teacher self-efficacy is less well-correlated with the teachers' level of experience with technology. A different methodological approach to explore teachers' perceptions of their technology use is evident in the work of Bang et al., who use a stimulated interview task to research how Danish teachers are integrating computer algebra software (CAS) into their teaching.

In his presentation of ongoing doctoral work, Dreyøe declares the integration of digital tools by teachers of mathematics to be a *wicked problem*, a phrase first coined by Rittel and Webber for problems with many interdependent factors making them seem impossible to solve (Rittel & Webber, 1973). Dreyøe's study explores the impact of participatory design with mathematics teachers as a means to create shared understanding of their declared intentions and practices with digital tools.

Focusing more on mathematics teachers' particular practices with digital tools, research by Rocha and Yemen-Karpuzcu & Isiksal-Bostan adopt a micro approach with respect to particular mathematical content. Rocha's study explores Portuguese teachers' representational fluency when teaching functions at high school level and the impact of their choices of representations within their practice, concluding hierarchies in algebraic, graphical and tabular forms. Yemen-Karpuzcu & Isiksal-Bostan research Turkish teachers' mathematical practices as they taught a sequence of lessons addressing the concept of slope that used both concrete and technology-based resources. Their findings highlight the nuances of teachers' practices, which vary by: the mathematical construct, their own understanding of it; and their knowledge of their students' understandings of the same construct.

The use and application of theories within TWG15

Over the years, and especially this year, the TWG15 papers have shown a strong appreciation of theory, which is used to inform hypotheses, research questions and research methodologies. During TWG15, this work continued as we discussed theories in relation to: individual papers; by making links with other papers in the group; the CERME plenaries; or other papers in the literature. One theory that continues to be present has been *instrumental genesis*, with its origins in French cognitive ergonomics (Verillon & Rabardel, 1995) and its applications in mathematics education (Artigue, 2002; Guin & Trouche, 2002). One discussion point was the complexity of two key components: *instrumentation* and *instrumentalisation*. In particular: how (and whether) to interpret these from a user perspective or an instrument perspective; their meaning in digital and non-digital

contexts; and whether to consider each separately or in a more dialectic relationship. However, as addressed by Paul Drijvers in his plenary, instrumental genesis or, in other words, the emergence of utilisation schemes, emerges from the intertwining of mathematical concepts and the employed techniques for the proposed task. In this process the *tools* have a central role but such tools ‘come with [their own] affordances and constraints, with opportunities and obstacles’. Our discussion, and resulting consensus was that instrumental genesis is a *long* and *continuing* process (as exemplified by Paul Drijvers’ piano recital in the Dom church), in which the user continually and spontaneously shapes his/her techniques and associated cognition. Consequently, the separation of instrumentation and instrumentalization as distinct processes is not an easy task and may not be observable. The paper by Bozkurt et al. prompted rich debate on the process of instrumentalization as, in their research, they had particularly ‘focused on a stage of discovery’ of a specific tool and how this related to the instrumentation of other tools.

Moreover, to describe the design of material in a technological context, Diamantidis et al. used the *documentational approach* (derived from the instrumental approach, Gueudet & Trouche, 2010). The interesting and new perspective introduced in this study is the involvement of students alongside with teachers in the phases of design.

An emergent phenomenon in the TWG15 contributions is the increasing number of studies that report the involvement of teachers in mathematical, technological contexts as active protagonists, not only as consumers of professional development made by researchers. The engagement of teachers in working, not only learning, is the novelty coming to the fore of the theme in the last years, over the world: teachers as designers, with researchers or with students, teachers involved in the research, teachers aware of the materials produced, of their role, of their choices.

And, for the future, it seems increasingly important to research situations where teachers are working more collaboratively with each other and more knowledgeable others for the purpose of developing knowledge of, and practice with, digital mathematical tools. For that reason, theoretical approaches that take care of praxeologies of the teacher both in the activity of teaching (i.e. didactical), as well as in the activity of working and learning (i.e. meta-didactical), can be used to describe possible evolution and improvement of teachers in their learning and working trajectories. The recently announced 25th ICMI Study: *Teachers of mathematics working and learning in collaborative groups* will be an opportunity to explore the role of technology in this respect.

Areas for further research

Following the conference, the group created a ResearchGate¹ project page to enable the community to share, and comment upon research projects, articles and conference presentations that align with the TWG15 focus.

The group identified the following emergent research themes that might be included in future TWG calls for proposals:

¹ <https://www.researchgate.net/project/CERME10-TWG15-Teaching-Mathematics-with-Technology-and-Resources>

- Teachers' decision-making for the selection and use of digital tools for teaching and learning mathematics.
- Quality criteria for digital tools that enable the teaching and learning of valuable and meaningful mathematics.
- Developing collaborative groups to scale digital mathematical implementation – How to implement different technologies in schools? What are the challenges and barriers? What makes a successful implementation?
- The implications of the emergence of 'big data' in mathematics education and its impact on how mathematics is assessed.
- Teachers' appropriations of emerging technologies, for example virtual and augmented reality, artificially intelligent digital tools etc.

References

- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274.
- Bandura, A. (1997). *Self-Efficacy. The exercise of control*. New York: Prentice-Hall.
- Black, P., & Wiliam, D. (1996). *Inside the black box*. London: King's College.
- Gueudet, G., & Trouche, L. (2010). Teaching resources and teachers' professional development: towards a documentational approach of didactics. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 1359–1368). Lyon, France: Institut National de Recherche Pédagogique and ERME.
- Guin, D., & Trouche, L. (2002). Mastering by the teacher of the instrumental genesis in CAS environments: necessity of instrumental orchestrations. *ZDM Mathematics Education*, 34(5), 204–211.
- Kimeswenger, B. (2017). Identifying and assessing quality criteria for dynamic mathematics materials on platforms. Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10). Dublin, Ireland.
- Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books.
- Rittel, H. W., & Webber, M. M. (1973). "Dilemmas in a General Theory of Planning." *Policy Sciences*, 4(2), 155–169.
- Thompson, P. W. (2002). Didactic objects and didactic models in radical constructivism. In K. Gravemeijer, R. Lehrer, B. v. Oers, & L. Verschaffel (Eds.), *Symbolizing and Modeling in Mathematics Education* (pp. 197–220). Dordrecht, The Netherlands: Kluwer.
- Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9, pp. 281–307

Verillon, P., & Rabardel, P. (1995). Cognition and artefacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education*, 10(1), 77–102.