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A Porous Media Model for a Double-Layered Gas-to-Gas Micro Heat Exchanger operating in Laminar Flow Regime

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Abstract. In micro heat exchangers, due to the presence of distributing and collecting manifolds as well as hundreds of parallel microchannels, a complete conjugate heat transfer analysis requires a large amount of computational power. Therefore in this study, a reduced order model based on porous medium approximation is developed for microchannels. With the help of this model, a detailed flow analysis through individual microchannels can be avoided by studying the device as a whole at considerably less computational cost. A cocurrent flow micro heat exchanger with 133 parallel microchannels (average hydraulic diameter of $200\mu m$) is employed for current study. Hot and cold streams are separated by a stainless-steel partition foil having thickness of $100\mu m$. Rectangular microchannels have a cross section of $200\mu m \times 200\mu m$ with wall thickness of $100\mu m$ in between. As a first step, a numerical study for conjugate heat transfer analysis of microchannels only, without distributing and collecting manifolds is performed. Mass flow inside hot and cold fluid domains is increased such that inlet Reynolds number for both domains remains close to the laminar regime. Inertial and viscous coefficients extracted from this study are then utilized to model pressure and temperature drops within porous medium. In order to cater for density dependence of inertial and viscous coefficients due to high pressure drop in microchannels, a modified formulation of Darcy-Forchheimer law is adopted. A complete model of a double layer micro heat exchanger with collecting and distributing manifolds where microchannels are modeled as porous medium is finally developed and used to estimate the overall heat exchanger effectiveness of the investigated micro heat exchanger. A comparison of computational results using current model with previously published experimental results of the same micro heat exchanger showed that adopted methodology can predict the heat exchanger effectiveness within the experimental uncertainty for the range of mass flows considered in the current study.

1. Introduction

Pressure drop (Δp) of a fluid through a porous medium of length L can be expressed using an extended Darcy-Forchheimer (here after referred to as simply Darcy) law as follows:

$$\frac{-\Delta p}{L} = \frac{\mu \dot{G}}{\alpha \rho} + \frac{\Gamma \dot{G}^2}{2\rho} \quad (1)$$

where $\frac{1}{\alpha}$ is viscous coefficient representing porous medium permeability and Γ is an inertial coefficient of the Darcy's law, ρ and μ denote density and dynamic viscosity of the fluid respectively, and \dot{G} denotes mass flow rate (\dot{m}) per unit area A ($\dot{G} = \dot{m}/A$). Calculation of viscous ($\frac{1}{\alpha}$) and inertial (Γ) coefficients, is usually done using experimental pressure drop results and therefore various empirical relations exist for different porous media geometries. Though originally proposed for estimating pressure drop of incompressible flow over bed of spheres, porous medium approximation can also be extended to multiple parallel microchannels (MCs) in heat sinks. An analytical model was first developed by Kim et al. [1, 2] where they modeled the MCs as porous media and compared modeling results with experimental results of Tuckerman & Pease [3] as well as with Knight et al. [4]. Results showed that developed model can be used for thermal performance and optimization of MC heat sinks. The same model has been studied by Liu and Garimella [5] and improved by Lim et al. [6]. Porous media based analytical models tend to solve a three equations model for fluid flow and heat transfer through MC heat sinks using simplified momentum and energy equations. A common trait of all the studies conducted for MC heat sinks is the use of an incompressible fluid. A porous medium approximation of a compact heat exchanger used in micro gas turbine application has recently been presented by Jojomon et al. [7]. Channel dimensions and operating pressure were such that gas was incompressible whereas operating temperature of the hot fluid was higher than 1000 K. However, a porous approximation model for a multi layered micro heat exchanger (μ Hx) where gas undergoes strong compressibility, has not been reported yet upto the best of authors knowledge. This serves as the main motivation to undertake current study.

In this work a two step methodology is proposed to conduct a performance evaluation study on μ Hx with acceptable computational power. As a first step, a 3D conjugate heat transfer (CHT) model of gas to gas μ Hx is developed without distributing and collecting manifolds. Resulting pressure, velocity and temperature fields are utilized to calculate inertial and viscous coefficients of the Darcy's porous law. A complete single layer of μ Hx with manifolds is then modeled with boundary conditions such that MCs are modeled as a porous medium with a low resolution mesh. This is achieved by modeling the required pressure drop as a momentum source term using inertial and viscous coefficients of porous medium. Similarly to incorporate heat transfer in MCs core, a source term derived by CHT analysis of MCs only, is also introduced. As seen by the literature review of the recent developments, though analytical studies to model heat sinks as porous media exist, no methodology to calculate porous medium coefficients for a μ Hx can be found in literature. Moreover as highlighted by many researchers, these coefficients may need to be updated with fluid velocity (mass flow) in order to correctly predict the pressure and temperature difference from inlet to the outlet of MCs. Furthermore, if coefficients are extracted directly by comparing numerical results of pressure drop with Darcy's law, they would not suffice due to the strong compressibility effects of gas flow through MCs. Therefore, a modified formulation of Darcy's law is adopted where inertial and viscous coefficients are obtained by integrating original Darcy's law along the length of the MC by considering density variations. This will give an integral average of these coefficients along the length of MC by incorporating density variations due to pressure and temperature drops.

2. Determination of Porous Media Coefficients

Thus integrating Equation 1 in the streamwise direction 'x' of the MC yields:

$$\int_{in}^{out} \frac{-\Delta p}{L} dx = \frac{\mu \dot{G}}{\alpha} \int_{in}^{out} \frac{dx}{\rho} + \frac{\Gamma \dot{G}^2}{2} \int_{in}^{out} \frac{dx}{\rho} \quad (2)$$

Combining gas law and a first order Taylor approximation of temperature change along the length of the MC, gas law can be rewritten in differential form as:

$$dp = RT \left[d\rho + \frac{\rho dx}{L} \right] \quad (3)$$

For a compressible fluid pressure and density can be used to define local speed of sound (a) as:

$$a = \sqrt{\frac{dp}{d\rho}} \quad (4)$$

Combining Equations 3 and 4 and utilizing ratio of specific heats (γ) yield following to relate dx with density of the flow:

$$dx = \frac{\gamma L}{\gamma - 1} RT \frac{d\rho}{\rho} \quad (5)$$

Substituting Equation 5 in Equation 2 and integrating between inlet ‘in’ and outlet ‘out’ of the MC results in:

$$\frac{-\Delta p}{L} = \mu \xi \dot{G} \left(\frac{1}{\alpha} \right) + \frac{\xi \dot{G}^2}{2} (\Gamma) \quad (6)$$

where $\xi = \frac{1}{\ln \left(\frac{\rho_{out}}{\rho_{in}} \right)} \left(\frac{1}{\rho_{in}} - \frac{1}{\rho_{out}} \right)$. Boundary conditions of CHT analysis are chosen such that

there are no heat losses to the surroundings, therefore all the heat loss by hot fluid must be shared between partition foil (solid wall) and cold fluid. Therefore porous medium coefficients can be extracted from either side of the CHT model. In this work, inertial and viscous coefficients are extracted from the channel with hot fluid. From a CHT analysis of MCs only, all parameters in the Equation 6 are available for a specific mass flow, with inertial and viscous coefficients as the only two unknowns. However, if Equation 6 is applied to two consecutive mass flows \dot{m}_i and \dot{m}_{i+1} , an average value of these coefficients between i and $i + 1$ can be found out by solving a system of two linear equations for two unknowns. This is repeated for range of mass flows being studied and a polynomial fit on these evaluated coefficients is used as an input for the porous model. Similarly a volumetric heat loss of hot fluid channel is calculated for all mass flows from CHT analysis using $q_v = \frac{\dot{m} C_p \Delta T}{AL}$, where C_p is the specific heat of fluid at constant pressure, A and L represent the cross sectional area and length of the MC respectively. A polynomial fit onto this volumetric heat loss is given as a source term in energy equation while solving for porous model.

3. Numerical Strategy

As described earlier, two different numerical setups are used in the current work. First a 3D CHT of MCs only, without distributing and collecting manifolds is performed. And using the results from CHT model, performance of complete heat exchanger with manifolds is analyzed subsequently. Reynolds number at the inlet of MC is defined by:

$$Re = \frac{\dot{m} D_h}{\mu A} \quad (7)$$

where hydraulic diameter (D_h) of a rectangular MC with width (w) and height (h) is defined as:

$$D_h = \frac{2wh}{w + h} \quad (8)$$

From CHT analysis, heat transfer rate (\dot{Q}) on hot (h) and cold (c) side can be defined by using respective flow quantities as:

$$\dot{Q} = \dot{m}C_p\Delta T \quad (9)$$

For a cocurrent (cc) configuration, logarithmic mean temperature difference (LMTD) can be defined as:

$$\Delta T_{LMTD,cc} = \frac{(T_{h,in} - T_{c,in}) - (T_{h,out} - T_{c,out})}{\ln\left(\frac{T_{h,in} - T_{c,in}}{T_{h,out} - T_{c,out}}\right)} \quad (10)$$

Overall heat transfer coefficient is calculated by:

$$U = \frac{\dot{Q}_{av}}{A\Delta T} \quad (11)$$

where $\dot{Q}_{av} = \frac{\dot{Q}_c + \dot{Q}_h}{2}$. Finally heat exchanger effectiveness, defined as the ratio of actual heat transfer rate and maximum potential heat transfer rate available, can be calculated using:

$$\varepsilon = \frac{Q_{av}}{(\dot{m}C_p)_{min}(T_{h,in} - T_{c,in})} \quad (12)$$

For porous model of μ Hx, heat exchanger effectiveness to be compared with experimental results, is calculated for the MC core only. This essentially means that temperature difference from the inlet of MC core to outlet of MC core is used to calculate ε . Moreover, as only one layer is computationally modeled in porous model, resulting effectiveness is calculated as follows:

$$\varepsilon_p = \frac{Q_p}{(\dot{m}C_p)_p(T_{h,in} - T_{c,in})} = \frac{\bar{T}_{MC,in} - \bar{T}_{MC,out}}{T_{h,in} - T_{c,in}} \quad (13)$$

where subscript 'p' denotes the porous model. $\bar{T}_{MC,in}$ & $\bar{T}_{MC,out}$ denote the mass flow weighted averages of static temperatures, at the inlet and outlet of N number of MCs, respectively.

In order to extract inertial and viscous coefficients for the porous medium using methodology outlined earlier, a 3D CHT model is setup where only MCs are modeled without real design of collecting and distributing manifolds. However, to allow any possible underexpansion at the outlet of hot and cold MCs, computational domain is extended $15D_h$ in streamwise and $32D_h$ lateral direction as shown in Figure 1. A meshed model for the complete μ Hx is also shown in Figure 3. Geometry and meshing for both models is done using Design Modeler and ANSYS Meshing software respectively. A mesh of $40 \times 40 \times 100$ is used in the MCs for CHT analysis whereas a coarse mesh of $3 \times 3 \times 40$ is used for MCs in case of porous model. Mesh is refined near the walls to capture any flow vortices present in the model. A commercial solver CFX based on finite volume methods is used for the flow simulations. Ideal Nitrogen gas is used as working fluid for both models. Simulation relevant parameters used in analyses are tabulated in Table 1.

Laminar flow solver is used for CHT model whereas a transient turbulence model $\gamma - Re\theta$ [8, 9] is utilized for porous model. Higher order advection scheme available in CFX is utilized and pseudo time marching is done using a physical timestep of 0.01s. A convergence criteria of 10^{-6} for RMS residuals of governing equations is chosen while monitor points for pressure and velocity at the MC inlet and outlet are also observed during successive iterations. In case where residuals stayed higher than supplied criteria, the solution is deemed converged if monitor points did not show any variation for 200 consecutive iterations. Reference pressure of $101kPa$ was used for the simulation and all the other pressures are defined with respect to this reference pressure. Energy equation was activated using Total energy option available in CFX which adopts energy equation without any simplifications in governing equations solution. Kinematic

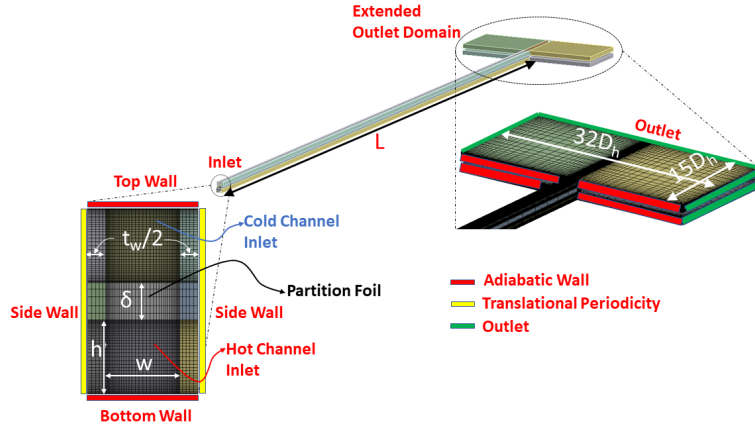


Figure 1: Mesh and geometric details for co-current CHT analysis

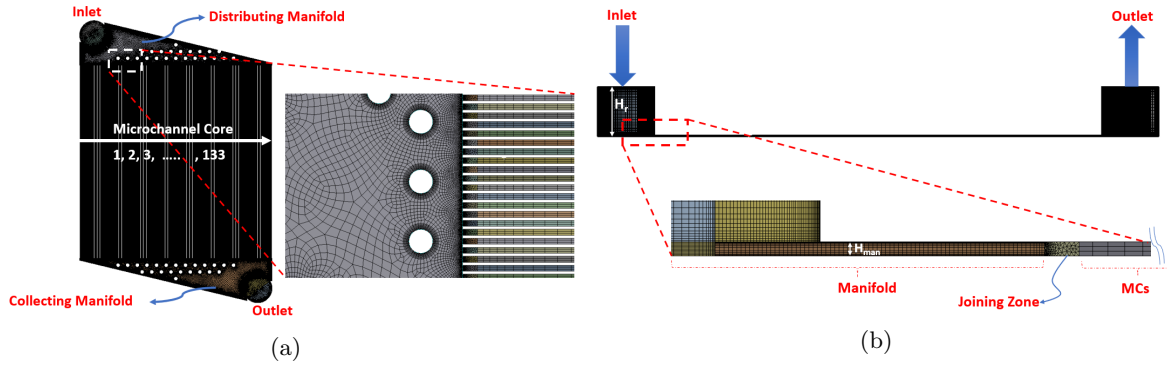


Figure 3: Geometry and mesh details of porous model: top view (a), and side view (b)

Table 1: μ Hx geometry used for simulations.

Parameter	Symbol (units)	Value
MC width	w (μm)	200
MC height	h (μm)	200
MC Length	L (μm)	40
Hydraulic Diameter	D_h (μm)	200
Wall Thickness	t_w (μm)	100
MC housing (PMMA) conductivity	k_{MC} ($W/m/K$)	0.25
Partition Foil (Stainless Steel) thickness	δ (μm)	100
Partition Foil conductivity	k_{PF} ($W/m/K$)	15

Table 2: Boundary conditions used in the CHT Analysis.

Boundary	Value	
	Hot Side	Cold Side
Inlet	- \dot{m} evaluated using Equation 7 for cold side - $T_{h,in} = 90\text{ }^\circ\text{C}$	- $T_{c,in} = 20\text{ }^\circ\text{C}$
Side Walls	Translational Periodicity	
Top & Bottom Walls	Adiabatic	
Outlet	Pressure outlet, Relative p = 0 Pa	

Table 3: Boundary conditions used in the porous model for μHx .

Boundary	Value
Inlet	- \dot{m} from experimental testing - $T_{h,in} = 90^\circ\text{C}$
MCs walls	Free slip
Inertial and visocus coefficients	Determined from CHT analysis
Energy source term	Determined from CHT analysis
Manifolds walls	Adiabatic/ No slip
Outlet	Pressure outlet, Relative p = 0 Pa

viscosity dependence on gas temperature is defined using Sutherland's law. Further details of boundary conditions used in CHT and Porous model can be found in Tables 2 & 3 respectively.

Using CHT model, global as well as local evolution of flow variables with inlet mass flow is evaluated at six different cross-sectional planes defined at x/L of 0.005, 0.1, 0.5, 0.9, 0.95 and 0.995 respectively. In addition, two planes defined at x/L of 0.0001 and 0.9995 are treated as the inlet and outlet of MC, respectively. Results from these planes for both hot and cold fluid sides are further post processed in MATLAB to deduce required flow quantities. Thermal effectiveness is then simply evaluated using Equation 12. Once the porous medium coefficients namely inertial (Γ) and viscous ($\frac{1}{\alpha}$) are determined using CHT model of a double layer μHx , next step is to setup a complete single layer porous model with inlet and outlet manifolds. A gas to gas double layer μHx that has been experimentally investigated previously by Yang et al. [10, 11] and Gerken et al. [12], is used for validation of proposed methodology.

4. Results and Discussion

CHT model with linear periodicity at side walls represents an ideal situation where there exist no maldistribution for parallel MCs. CHT results for the heat transfer rate and heat exchanger effectiveness are shown in Figure 5. For an incompressible fluid, heat transfer rate using numerical model on both sides should be equal. But for gases, as the gas flow experiences additional acceleration due to compressibility, heat gain on the cold side tends to differs than heat loss on the hot side. An interesting fact is that heat transfer rate on the hot side keeps on increasing with increasing mass flow (see Figure 5a). On the contrary, it keeps on decreasing on the cold side. Therefore even though there are no losses modeled to the surroundings, due to compressibility gas flows still exhibit difference in heat transfer rate between hot and cold sides. Similar behavior can also be seen in effectiveness where it increases with mass flow for

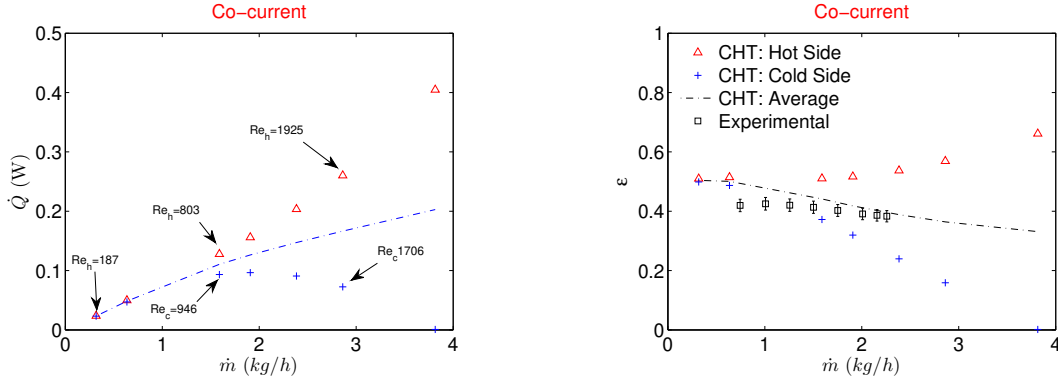


Figure 5: CHT Model: Heat transfer rate for CHT analysis (a), heat exchanger effectiveness (b)

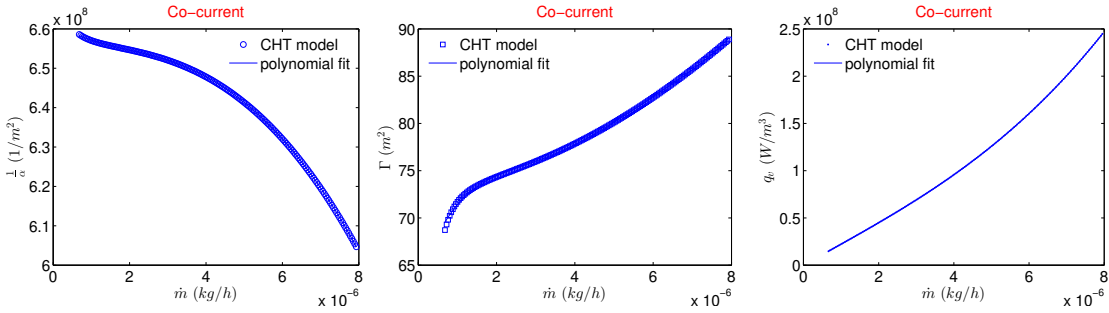


Figure 7: Porous medium coefficients: viscous (a), inertial (b), and volumetric source (c)

hot fluid and decreases for the cold fluid as shown in Figure 5b. This is simply because gas flow accelerates at the expense of kinetic energy, therefore a part of total energy is utilized for acceleration than to have a conjugate heat transfer [13, 14, 15, 16]. Further step is to evaluate the inertial and viscous coefficients of the modified Darcy's law using flow quantities evaluated in CHT analysis. Resulting viscous and inertial coefficients, alongwith the volumetric heat source are shown in Figure 7. Polynomial fits to these coefficients are used as input to porous model in CFX environment. Experimental pressure drop of the double layer μHx being considered in this study for different flow configurations has been reported by Gerken et al. [12]. Pressure drop showed dependence on the flow configuration as well as the material and thickness of the partition foil employed during the experimental tests. A possible reason of this deviation between different foil materials and thicknesses was associated with possible bending of the thin partition foils inside manifolds although a strong layer of circular pillars were realized underneath (in manifolds) to protect against such undesired deflection of partition foil. Computational results of pressure drop from porous model are compared with the experimental results reported for the same μHx from two different studies [11, 12] in Figure 9. As results differ from one separation foil material to the other, only results with stainless steel foil with thickness of $100 \mu\text{m}$ (as utilized in current study) are compared and are shown in Figure 9a. It can be seen that total pressure drop of the device shows a good agreement between the average of two experimental investigations on the same μHx . Results are more compliant to the results of Yang et al. [11] for smaller mass flows while they match better with Gerken et al. [12] for higher mass flow rates. Pressure drops in the distributing manifold and MC core are also shown in

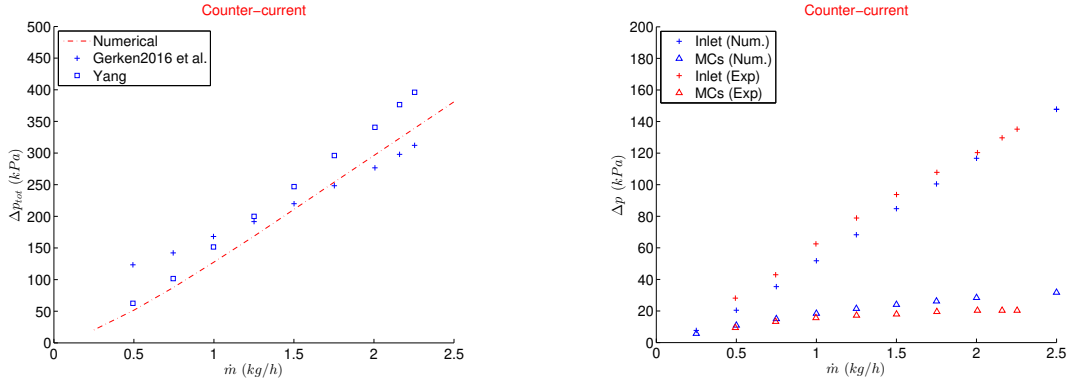


Figure 9: Comparison between experimental and numerical total pressure drop of μ Hx (a), and in the inlet and MCs only (b)

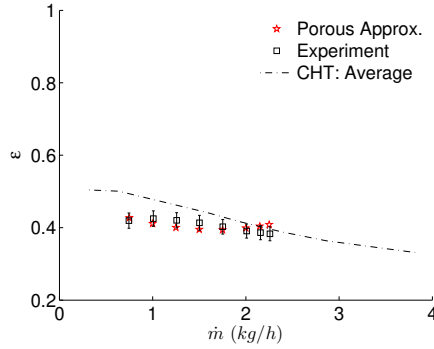


Figure 10: μ Hx effectiveness for cocurrent flow configuration

Figure 9b where there exists a very good match between current porous model and experimental results of Gerken et al. [12]. However, pressure drop of MC core is slightly overestimated at higher mass flow rates with modeled porous medium coefficients with incorporated source term in energy equation. Heat exchanger effectiveness evaluated using Equation 13 is shown in Figure 10 where porous model predictions match to the experimental results of Yang et al. [10] within the experimental uncertainty. Whereas, average ε using CHT model is overestimated compared to experimental results, especially for smaller mass flows.

5. Conclusions

- (i) Porous medium coefficients for parallel channel μ Hx can be extracted for compressible fluids by modifying the existing Darcy-Forchheimer law to incorporate for the strong density variations with increasing mass flow rates in MCs.
- (ii) Pressure drop of porous model is much higher compared to the CHT due to presence of collecting and dividing manifolds. Pressure drop estimation using porous model is in good agreement with the experimental results of the same μ Hx.
- (iii) For the cocurrent flow configuration, heat exchanger effectiveness of porous model agrees well with the experimental results reported earlier.

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