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Numerical and Experimental Analysis of Gas Flow Laminar to Turbulent Transition through Adiabatic Rectangular Microchannels

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Abstract. In this paper, laminar to turbulent flow transition in rectangular microchannels is studied using a modified $K - \omega$ SST transitional turbulence model available in ANSYS CFX. Originally proposed for external flows, model constants of SST $\gamma - Re_{\theta}$ transition turbulence model have been modified using values provided by Abraham et al. [1]. Transitional turbulence model is validated with experimental results obtained for two rectangular microchannels having hydraulic diameters of 295 μm and 215 μm with aspect ratios (height to width) of 0.7 and 0.24 respectively. Channels are micro milled in PMMA plastic and their surface roughness is in between 0.9 – 1.05 μm for both channels. Comparison of numerical and experimental results of average friction factor using Nitrogen as working fluid, yields that intermittency based transitional turbulence model can predict laminar to turbulent transition of gas microflows with a reasonable accuracy. After validating the model with experimental results, numerical analysis is extended further to study the effect of aspect ratio of rectangular microchannels on the laminar to turbulent transition. This was done by varying the aspect ratio of a rectangular microchannel from 0.1 to 1 for a constant hydraulic diameter of 300 μm . Reynolds number is varied from 400 to 20000 in order to capture laminar, transitional and fully turbulent regimes. Results showed that critical Reynolds number decreases from 4000 to 2100 with an increase in aspect ratio from 0.1 to 1 for the simulated microchannels. The trend of results is in coherence with numerous experimental studies available in the literature. This suggests that adapted transitional turbulence model is sensitive to geometric changes of microchannels and therefore can be used as first engineering approximations in extracting the microflow physics in transitional flow regime.

1. Introduction

Knowledge of turbulent transition in rectangular microchannels (MCs) is of prime importance as such geometries are being extensively employed in MC heat sinks and MC heat exchangers. Flow transition and consequent turbulent flow is of benefit for increased mixing and heat transfer in microsystems. After the pioneering work of Tuckerman and Pease [2], most of earlier experimental groups have reported an anticipated flow transition in MCs [3, 4, 5]. While a few groups [6, 7] also reported the conformance to the macro scale theory. Recent reviews [8, 9] however conclude that experimental data obtained in the recent decade establishes that single phase fluids in MCs follow conventional macro laws. A comprehensive review outlining possible reasons of deviations from macro theory in earlier reported experiments is due to Morini et al.

[10], according to which macro theory is applicable to MC flows if scaling effects are negligible. It also details various scaling effects that need to be ascertained before interpreting experimental results which may cause deviation from conventional correlations. In another work Morini [11] gathered previously published experimental data for laminar to turbulent flow transition in MCs and compared it with macro laws. Analysis showed that various published results for critical Reynolds number (Re_c) are in accordance with Obot-Jones model developed for conventional sized channels.

Laminar to turbulent flow transition even in conventional sized channels is still an open field of investigation. Based on original experiments of Reynolds [12], transition in pipe flow occurs around Re_c of 2000 for a rough entrance. Underlying processes that trigger such flow transition with an increase in speed for pipe flows are still not clear, though there have been remarkable works in this regards reported recently [13, 14, 15]. Pipe flow experiments of Avila et al. [14] showed that when flow velocity is such that Re is lower than its critical value, turbulence was not sustained. Though turbulent puffs were generated in the flow field but were decayed swiftly by the surrounding laminar flow. This process is referred to as relaminarization. Above $Re_c = 2040$, splitting process of these turbulent puffs outweighed the decay and hence turbulence could sustain. Afterwards turbulent puffs increased their size and whole flow field suddenly exhibited a disordered motion. Later Barkley et al. [15] performed experiments on 10 mm pipe and 5 mm square duct. They introduced the perturbations in the flow field at a precise location along the length to initiate the turbulence. They also conducted DNS of the flow field and showed that even when the flow is fully turbulent, just by modifying the velocity profile to more plug shaped caused fully turbulent flow to break into turbulent puffs that were experimentally observed for $Re < 2300$. Such DNS inspired experiments were then conducted by Kühnen et al. [16] where experimentally it was shown that sustained turbulence in a pipe flow can be destabilized and flow relaminarized in the pipe. Starting with Osborne Reynolds [12] more than a century ago, turbulent transition within wall bounded flows has been investigated mainly through experimental observations. Similarly, flow visualization as well as pressure drop studies have been utilized to observe Re_c of microflows. For liquid flows, μ PIV studies were conducted by Li and Olsen [17]. They experimented four MCs with almost same D_h with aspect ratios (α), which is ratio of MC height and width, between 0.2 – 1. They found that Re_c increased from 1715 to 2315 by decreasing α from 1 to 0.2. Similar experimentation was also performed by Wibel and Ehrhard [18] where they reported that an increase in Re_c from 1600 to 2200 is observed for a decrease in α from 1 to 0.2. Both studies showed conformance with macro scale results. More recently Kim [19] presented experimental results of fluid flow and heat transfer for 10 chips containing 10 parallel MCs each. The critical Reynolds number increased from 1700 to 2400 with a decrease in the aspect ratio from 1.0 to 0.25. Unlike the previous μ PIV studies MC dimensions are changed such that α and D_h varied simultaneously. Due to relatively higher density of seeding particles, μ PIV cannot be applied to gas flows in MCs and therefore a pressure drop study (Moody chart) remains the only dispensable tool to observe turbulent transition. Such analysis for gas flows in 11 different microtubes with $D_h = 125 - 180 \mu m$ has been presented by Morini et al. [20]. By comparing their results with previously published gas flow pressure drop studies, it was shown that if friction factor is calculated by taking into account compressibility effects and minor losses, Re_c was in between 1800 – 2000, contrary to anticipated transitions reported by other groups [3, 21]. An analytical study based on energy gradient that deals with current subject is reported by Chang et al. [22]. Based on their analysis it was shown that in a rectangular channel the shorter side dictates the transition. Moreover a correlation was also provided to predict Re_c as a function of α for rectangular MCs.

As highlighted earlier, aside from experimental and analytical observations, DNS has been applied to understand the physics of flow transition [15, 23, 16]. However due to the lower computational cost, RANS based turbulence modeling is preferred in industrial environments

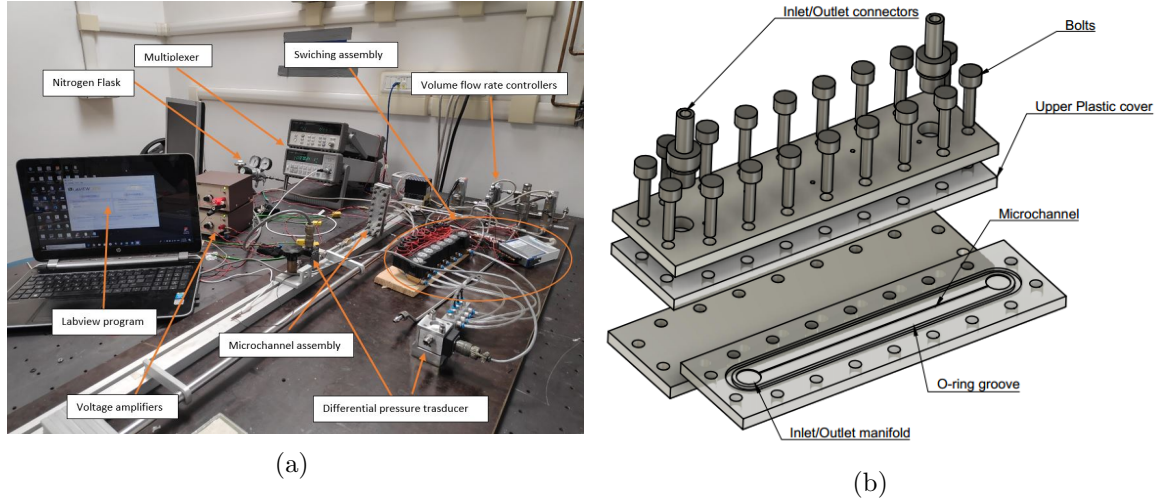


Figure 2: Experimental setup (a), and an exploded view of MC assembly (b).

for engineering approximations. A major breakthrough in modeling transitional flow using two equations turbulence models is due to Menter et al. [24]. They augmented original SST $k - \omega$ model [25] to incorporate flow transition based upon two additional intermittency transport equations. This model was later modified to be implemented into commercial CFD codes [26]. Although originally developed for external flows, model constants were modified by Abraham et al. [1] to predict transition in internal flows. It was shown for pipe flow that fine tuned model predicted laminar and fully turbulent friction factors similar to that of Poiseuille's law and Blasius law respectively. Therefore transitional behaviour was deemed to be representative of the reality. The same transitional turbulence model was later used by Minkowycz et al. [27] to investigate the effect of turbulence intensity (TI) on laminar to turbulent transition for straining pipe flow. For TI=10%, transition was delayed and abruptly reached fully turbulent state whereas for TI=5% transition initiated around 3000 and gradually developed to fully turbulent. The aim of the current work is to experimentally validate this transitional turbulence model for gas microflows, which has not been reported yet to the best of authors knowledge. After rigorous validation, model has been used to investigate the effect of α on laminar to turbulent transition in rectangular MCs where compressible gas is used as working fluid.

2. Experimental Setup and Data Reduction

Experimental test bench and a schematic of MC assembly used in this work are shown in Figure 2. Pressurized Nitrogen gas is allowed to enter the MC assembly perpendicular to the axial direction of MC and leaves perpendicularly as well through outlet manifold. A detailed description of sensors used and associated uncertainties is documented in [28] and therefore is not repeated here. Voltages from employed sensors are fed to internal multiplexer board of Agilent 39470A and are read by means of a PC using a Labview[®] program. Two MCs are fabricated by milling a PMMA plastic sheet with a nominal thickness of 5 mm. CNC milling is performed using Roland[®] MDX40A with a 100 W spindle motor. Modeling and CNC toolpath are generated in Autodesk[®] Fusion360. Dimensions and inner surface roughness of resulting channels are measured using an optical profilometer. The average width (w), height (h) and surface roughness (ϵ) of realized MCs are reported in Table 1. Lower PMMA sheet containing milled MC is sealed by means of a top cover of same material and an O-ring as shown in Figure 2b. Both top and bottom plastics are sandwiched between two 5 mm thick Aluminum plates.

Table 1: Channels geometry used for experiments.

| Channel | h (μm) | w (μm) | α | D_h (μm) | ε (μm) |
|---------|-----------------------|-----------------------|----------|-------------------------|---------------------------------|
| MC1 | 250 | 360 | 0.694 | 295 | 1.05 |
| MC2 | 134 | 554 | 0.241 | 215 | 0.9 |

Whole assembly is then bolted to ensure leak tightness. Test section is checked for leakage by applying a pressure of almost 10 bars between the inlet and outlet connectors and closing the outlet manifold. Pressure inside the assembly is monitored for at least 5 h to spot any major leakage. Such a test is repeated before initiating experimental test campaign.

Considering one dimensional flow of ideal gas, average Fanning friction factor between inlet 'in' and outlet 'out' of a MC with length L can be defined by the following expression for a compressible flow [28]:

$$f_f = \frac{D_h}{L} \left[\frac{p_{in}^2 - p_{out}^2}{RT_{av}\dot{G}^2} - 2 \ln \left(\frac{p_{in}}{p_{out}} \right) + 2 \ln \left(\frac{T_{in}}{T_{out}} \right) \right] \quad (1)$$

where p and T denote cross sectional average pressure and temperature of gas, T_{av} is the average temperature of the gas between inlet and outlet of MC, and \dot{G} is mass flow per unit area ($\dot{G} = \frac{\dot{m}}{A}$). Hydraulic diameter of a rectangular MC is defined as:

$$D_h = \frac{4A}{Per} = \frac{2wh}{w+h} \quad (2)$$

Reynolds number at the inlet of MC can then be calculated using measured mass flow rate and calculated viscosity at inlet temperature with the following equation:

$$Re = \frac{\dot{m}D_h}{\mu A} \quad (3)$$

In addition, under the hypothesis of adiabatic compressible flow, the energy balance for one dimensional Fanno flow, between inlet 'in' and any other cross section at a distance 'x' from the inlet of the MC yields the following quadratic equation for the estimation of average cross-sectional temperature [29]:

$$\left(\frac{\rho_{in}^2 u_{in}^2 R^2}{2c_p p_x^2} \right) T_x^2 + T_x - \left(T_{in} + \frac{u_{in}^2}{2c_p} \right) = 0 \quad (4)$$

Finally, knowing the average pressure and temperature of a specific cross section, average density and velocity of compressible gas can be obtained using gas and continuity equations, respectively. An initial estimate of the gas velocity at MC inlet is made using the measured mass flow rate and density of the gas at the inlet of MC assembly. Whereas MC inlet properties are then calculated iteratively using the methodology outlined in [28, 30]. Once inlet conditions are determined, temperature at MC outlet can be easily evaluated using Equation 4 given the cross sectional pressure is known. In current work, perfect expansion is assumed which essentially assumes atmospheric pressure at the MC outlet ($p_{out} = p_{atm}$). Utilizing the outlet temperature evaluated by Equation 4 and keeping perfect expansion assumption, average Fanning friction factor of MC can finally be calculated using Equation 1.

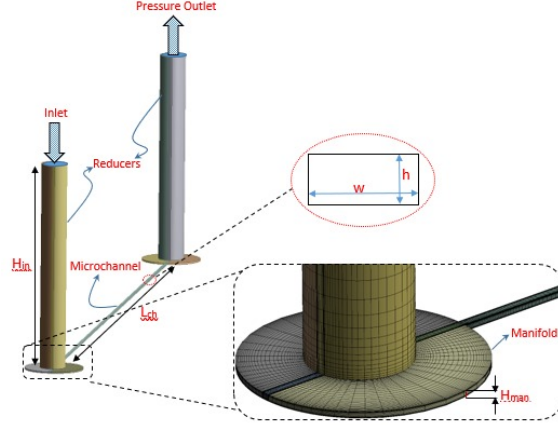


Figure 3: Details of numerical model and meshing.

3. Numerical Model

Geometry and meshing is done using Design Modeler and ANSYS Meshing software, respectively. After performing grid independent study, structured mesh locally refined at the walls of the channel and manifolds is employed as shown in Figure 3. The mesh expansion factor is kept as 1.1 and first node point is placed such that y^+ , which is non-dimensional distance between first mesh node and MC wall, is close to 1 in transitional regime Re for all simulated MCs. A commercial solver CFX based on finite volume methods is used for the flow simulations. Reducers and manifolds are also simulated along with the MCs. Height of the reducer (H_r , see Figure 3) is 30 mm with an internal diameter of 4 mm. Whereas diameter of the circular manifolds is 9 mm and height (H_{man}) is kept same as the height of the simulated MC ($H_{man} = h$). Dimensions of these parts are chosen based upon the experimentally tested MC assembly. Ideal nitrogen gas enters the entrance manifold that is orthogonal to MC and leaves again orthogonally through the exit manifold. For the MCs that are also experimentally tested (MC1 & MC2), measured mass flow rate is used while for the rest it is calculated from Equation (3). Steady state RANS simulations are performed for all turbulent cases. Laminar flow solver is used for the cases where $Re \leq 1000$ and for $Re > 1000$, SST $k-\omega$ transitional turbulence model is used. A modified formulation of $\gamma-Re_\theta$ transition turbulence model for internal flows is applied [1, 27]. High-resolution turbulence numerics are employed with a higher order advection scheme available in CFX. Pseudo time marching is done using a physical timestep of 0.1s. A convergence criteria of 10^{-6} for RMS residuals of governing equations is chosen while monitor points for pressure and velocity at the MC inlet and outlet are also observed during successive iterations. In case where residuals stayed higher than supplied criteria, the solution is deemed converged if monitor points did not show any variation for 200 consecutive iterations. Reference pressure of 101 kPa was used for the simulation and all the other pressures are defined with respect to this reference pressure. Energy equation was activated using total energy option available in CFX which adopts energy equation without any simplifications in governing equations solution. Kinematic viscosity dependence on gas temperature is defined using Sutherland's law. Further details of boundary conditions can be seen in Table 3. To estimate average friction factor, two planes defined at x/L of 0.0005 and 0.9995 are treated as the inlet and outlet of MC respectively. Analysis results from these planes are further post processed in MATLAB to deduce required flow quantities. Numerical friction factors are then evaluated simply by using Equation (1).

Table 2: Channels geometry used for simulations.

| Channel | h (μm) | w (μm) | α | D_h (μm) |
|---------|-----------------------|-----------------------|----------|-------------------------|
| MC3 | 165 | 1650 | 0.1 | 300 |
| MC4 | 187.5 | 750 | 0.25 | 300 |
| MC5 | 225 | 450 | 0.5 | 300 |
| MC6 | 300 | 300 | 1 | 300 |

Table 3: Boundary Conditions.

| Boundary | Value |
|----------|--|
| Inlet | - mass flow rate: experimental or from Equation (3) - Turbulence Intensity, TI = 5% - Temperature $T_{in} = 23$ °C |
| Walls | - No slip - Adiabatic |
| Outlet | Pressure outlet, Relative p = 0 Pa |

4. Results and Discussion

4.1. Numerical Model Validation

Laminar to turbulent flow transition is established using the average friction factor curve. Re_{cr} is considered where friction factor attains its first minimum and then starts to increase. This point is individuated during MATLAB post processing of experimental and numerical results. A comparison between numerical and experimental friction factor is made in Figure 5. There exists an excellent agreement between the current numerical results and experimental results in the laminar flow regime where f_f follows the Shah & London correlation (S&L):

$$f_{f_{SL}} = \frac{96}{Re} \left(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5 \right) \quad (5)$$

In the turbulent flow regime, both experimental and numerical results are slightly above the Blasius law. However even in the turbulent regime numerical f_f is within uncertainty of experimental results. It is evident from Figure 5 that the model is able to predict the gentle change in slope of the f_f due to the transition from laminar to turbulent flow regime. In fact, the trend of numerical simulation is similar to what has been observed in experiments. In transition regime the values of f_f smoothly connect the limiting cases of laminar and turbulent flow regimes. Contrary to what occurs in external flow where there is a sharp transition to turbulence once disturbances break laminar flow stability, in internal flows a region of fully developed intermittent flow exists [1]. This interval could be identified in Figure 5a between $Re = 1800$ and $Re = 3000$ for MC1. In this range the conformity between simulated and experimental values is not as good as in the other regimes, but the relative error between the two values remains under the 15%. Another intriguing aspect is the ability of the $\gamma - Re_\theta$ transition turbulence model to predict critical Reynolds (Re_c) number with sufficient accuracy, in fact the difference between experimental and numerical Re_c is of about 200 Re . Experiments highlighted a Re_c equal to 2030, whereas numerical one is ~ 2200 . The correct trend is confirmed by MC2 for which f_f is reported in Figure 5b. In this case experimental Re_c is calculated to be 2536 whereas model predicts it to be ~ 2800 .

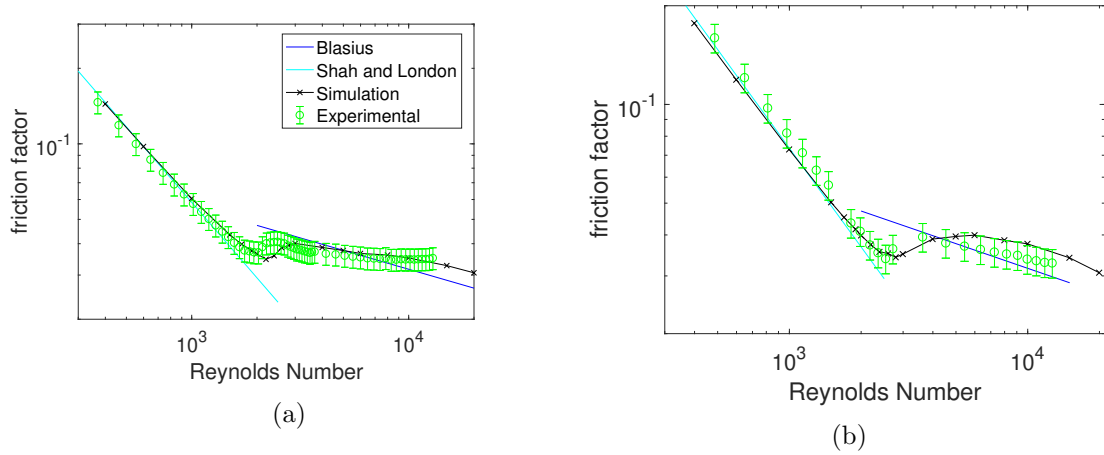


Figure 5: Friction factor for MC1 (a), and MC2 (b).

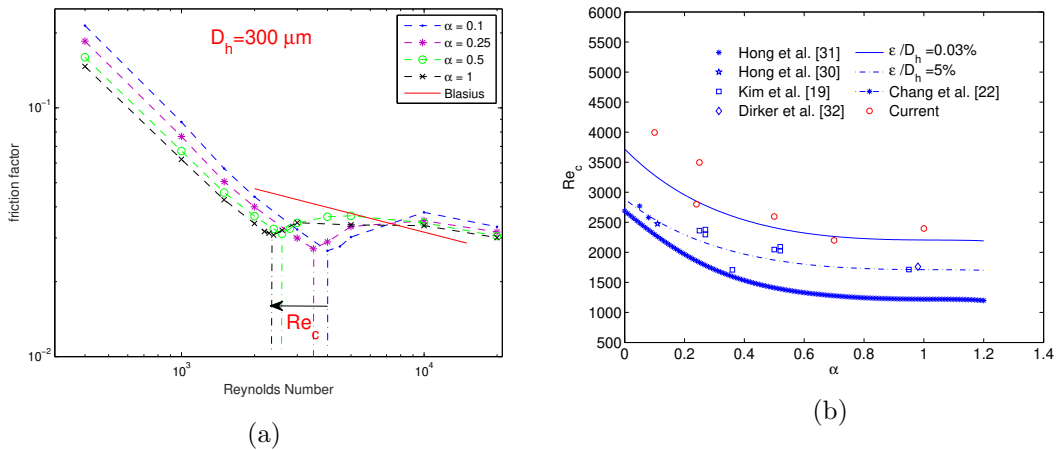


Figure 7: Moody diagram for simulated MCs (a), comparison with published data (b).

4.2. Effect of aspect ratio on transition

The comparison between numerical and experimental Re_c conducted earlier, highlighted that adopted transitional turbulence model is sensible to geometric changes and is capable of predicting Re_c with sufficient accuracy. Therefore this model has been further utilized to see the effect of aspect ratio of rectangular microchannels on laminar to turbulent flow transition. Four different aspect ratios are simulated for a constant D_h of $300 \mu m$. Results are shown in Figure 7a where Re_c increases with a decrease in aspect ratio. It decreases from ~ 4000 to ~ 2400 for aspect ratios of 0.1 and 1 respectively. Re_c for all the simulated cases are shown in Table 4.

A set of recent published experimental data [19, 30, 31, 32] is gathered and compared with current results in Figure 7b where a clear trend of an increase in Re_c with the decrease of α can be seen. Various researchers have associated such early transition to surface roughness e.g. [19], therefore experimental data points are also compared with a macroscale Obot-Jones model. According to this model Re_c of a channel with an arbitrary cross section ‘ g ’ is a function of

Table 4: Re_c for all simulated microchannels.

| Channel | D_h (μm) | α | Re_c |
|---------|-------------------------|----------|--------|
| MC1 | 295 | 0.7 | 2200 |
| MC2 | 213 | 0.24 | 2800 |
| MC3 | 300 | 0.1 | 3995 |
| MC4 | 300 | 0.25 | 3496 |
| MC5 | 300 | 0.5 | 2597 |
| MC6 | 300 | 1 | 2397 |

average surface roughness (ε) to D_h ratio as well as α of the channel as follows:

$$Re_{cg} = \frac{Re_{cc}}{\sigma_g} = \frac{1160 \left(\frac{\varepsilon}{D_h} \right)^{-0.11}}{\sigma_g} \quad (6)$$

where $\sigma_g = \frac{(f_f Re)_g}{(f_f Re)_c}$ is the ratio of Poisuille's numbers of rectangular geometry in investigation and of equivalent D_h circular tube. An approximate validity onto microchannels along with theoretical details of such a macroscopic model can be found in [11]. Results of rectangular microchannels from current study and literature seem to obey the model qualitatively well where a decrease in critical Re is observed with an increase in α . All experimental data falls in between current results and predictions of Obot-Jones model for MCs with relative surface roughness of 5%. However surface roughness reported by all the authors is well below this limit ($< 1\%$). Therefore it can be concluded that compared to experimental results, Obot-Jones model though qualitatively exhibit the similar behaviour but overpredicts the Re_c . Analytical correlation of Chang et al. [22] on the other hand, severely underpredicts the transition point especially for higher aspect ratios.

5. Conclusions

Laminar to turbulent transtion for gas microflows can be estimated with acceptable accuracy using $\gamma - Re_\theta$ transitional turbulence model. Validation of the model with experimental results showed that intermittency model predicted Re_c with a maximum error of 10% for the two channels with different D_h and AR . For a MC with $D_h = 300 \mu\text{m}$, Re_c decreased from 3995 to 2397 by increasing α from 0.1 to 1. A comparison with recent published experimental results of Re_c for rectangular MCs shows that current numerical results are relatively higher, but follow the same trend. Moreover, current and published results do not exhibit any anomalous transition as predicted by correlation proposed by Chang et al. [22] for rectangular MCs.

Acknowledgments

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