



HAL
open science

Forming basic conceptions in dealing with quadratic equations

Alexander Wolff

► **To cite this version:**

Alexander Wolff. Forming basic conceptions in dealing with quadratic equations. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02416505

HAL Id: hal-02416505

<https://hal.science/hal-02416505>

Submitted on 17 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Forming basic conceptions in dealing with quadratic equations

Alexander Wolff¹

¹University of Hildesheim, Faculty of Mathematics and Applied Informatics, Hildesheim, Germany;
wolff@imai.uni-hildesheim.de

Keywords: quadratic equations, solving, transforming, understanding, basic conceptions.

Introduction

Transforming and solving quadratic equations, such as $-2x^2 = -8$, is part of the curriculum taught in school. To support the learning process of equations and their equivalent transformations, models such as the balance scale-, the box or the strip model, can be and are usually used in class (see Holzäpfel/ Barzel, 2010). The work and the support thereby reach its limits when dealing with negative coefficients in quadratic equations or by multiplying/dividing with negative numbers (see Vlassis, 2002). On the one hand its whole variety of formal expressions of quadratic equations and its equivalent transformations cannot be supported and visualized with the above mentioned models. On the other hand the work with the suggested models can lead to inadequate conceptions (misconceptions) of the mathematical concepts: E. g. facing problems to be able to interpret formal expressions such as $2x^2 + 2 = x^2$ or $2 = 3$ as equations as well (with an empty set of solutions). E.g. regarding the division of both sides of the equation by x or the extraction of the square root as an equivalent transformation. But working with models and forming content adequate conceptions for the learning process is indispensable. According to vom Hofe/ Blum (see 2016) the formation of basic conceptions (Grundvorstellungen) belongs to a competent and sensible dealing with mathematical concepts. But which basic conceptions, that students are about to form in the learning process, can be cited up? Two basic conceptions are cited normatively and visualized in a model further down. They allow to interpret a quadratic equation and its equivalent transformations illustratively in a graphical (or tabular) way and consequently grasp the core of understanding

Model

Basic conception 1:

A quadratic equation as an intersection of two functions, whereby at least one has to be quadratic. To determine the set of solutions the x-coordinate of the intersections S_1 and S_2 has to be considered.

Example:

$$-2x^2 = -8$$

$$f(x) = g(x)$$

$$\mathbb{L} = \{-2; 2\}$$

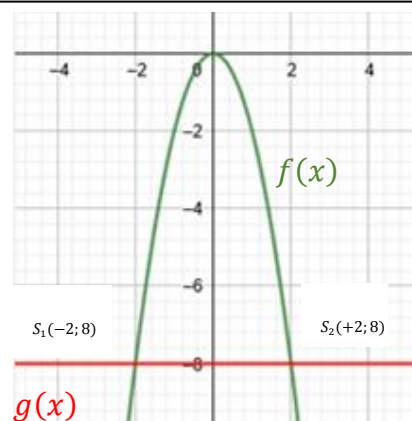


Figure 1: Graphical visualization of basic conception 1 (GV1)

Basic conception 2:

An equivalent transformation keeps the x-coordinate (the set of solutions) constant and simultaneously leads to a change of the functional value of the intersections.

Example:

$$-2x^2 = -8$$

$$f(x) = g(x)$$

$$\Leftrightarrow -x^2 = -4$$

$$\Leftrightarrow h(x) = i(x)$$

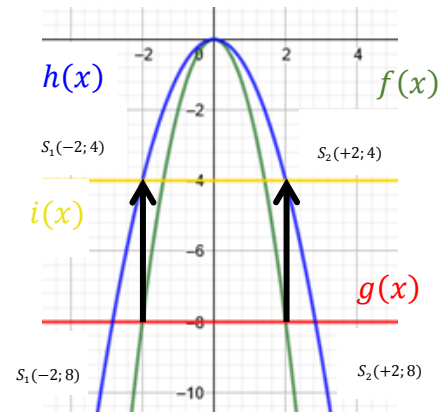


Figure 2: Graphical visualization of basic conception 2 (GV2)

Study design

To evaluate if these normatively cited basic conceptions can be formed by students and hence can be proved empirically, special learning assignments have to be designed. Aim of the presented study is thus to develop a learning environment and subsequently to investigate if those basic conceptions can be formed by students. For this purpose, a teaching sequence of 2x45 minutes was designed. Both sets of 45 minutes dealt with one basic conception. Each set consists of a specially designed common starter with a subsequent individual working phase of 4 learning assignments. The utilized exercises in the working phase were supplementary designed with regard to a high potential of diagnosis implying a written response in addition to the actual task.

Preliminary results and outlook

A first postprocessing of students' solutions of the learning assignments revealed to evaluate the collected data quantitatively through a deductive procedure.

This leads with regard to the formation process of the first basic conceptions to the following preliminary results: The solution quote over the first 4 learning assignments amounts up to 81%.

In summary, it could be said, that the dealing with the exercises for the formation of the first basic conception (GV1) worked out well. Missing percentages are due to a lack of time, inaccurate drawing of graphs and mistakes in reading off exactly the relevant coordinates and distinguishing both from each other.

The evaluation of the learning assignments for the formation of the second basic conception (GV2) showed up to be more difficult. The used tasks were created too open and required for its processing the correct and precise use of mathematical language. These circumstances made the students' solutions difficult to evaluate. The tasks are now going to be revised more closely. The following study is going to be carried out once again, under the revisions of the exercises, this year.

References

- Holzäpfel, L. & Barzel, B. (2011). Gleichungen verstehen. *Mathematik lehren*, 169, 2-7.
- Vlassis, J. (2002). The balance model: hindrance or support for the solving of linear equations with one unknown. *Educational Studies in Mathematics*, 49, 341-359.
- vom Hofe, R./Blum, W. (2016). „Grundvorstellungen“ as a Category of Subject-Matter Didactics. In: *Journal für Mathematikdidaktik* 37(1), p. 225-254.