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Portuguese and Spanish prospective teachers' functional thinking on geometric patterns

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Generalization of patterns and regularities to describe functional relationships is a way for students to develop functional thinking, a core expectation in mathematics curriculum orientations (Blanton, Levi, Crites, & Dougherty, 2011; NCTM, 2000). Thus, it is important to provide students with opportunities to interpret variables and relations between variables, and to use co-variation and correspondence approaches to generalization. The promotion of functional thinking in primary school requires a specific preparation of prospective teachers (PTs), which in turn claims to assess their knowledge on the subject at the beginning of the teacher education programs they are attending. Taking this into account, a study was developed to analyze how Portuguese and Spanish PTs identify and express generalization in a task involving geometric patterns.

The data was collected through a questionnaire which included 6 tasks to evaluate relevant aspects of algebraic thinking. The questionnaire was applied to 94 Spanish and 70 Portuguese PTs attending the 1st year of a degree in primary education teaching, respectively at a public university in north of Spain and at two public schools of education in the center of Portugal. The PTs were not familiar with this kind of tasks and had received no instruction in algebraic thinking before the test. One of the tasks, that is focus on this poster, is a problem of generalization of a geometric growing pattern (linear) intending to assess PTs' functional thinking. Based on the first four terms of a pictorial sequence, made up of squares and indicating the number of vertices of each figure, the PTs are required to provide two possible strategies to find the number of vertices of a distant term of the sequence and to identify a generalization of the functional relation between the number of vertices and the number of squares of the figures of the sequence.

For a descriptive and interpretative analysis of the PTs' responses to this task, the following categories of strategies for obtaining distant terms and general rule were established (adapted from Barbosa, 2010): (1) *difference-recursive*: continues the sequence using the pattern or the numerical difference between consecutive terms (e.g. "with each new square we have to increase the number of vertices by 3"); (2) *difference-covariation*: uses the difference between consecutive terms as a multiplicative factor (e.g. "we add three more vertices in each step, so the general term for the number of vertices is $4+3(n-1)$, with n the number of squares"); (3) *multiplicative reasoning-missing value*: uses the rule of three to find distant terms (e.g. "if a figure with two squares has 7 vertices, then a figure with 25 squares has $7 \times 25 / 2$ "); (4) *multiplicative reasoning-proportional reasoning*: uses multiplicative strategies, starting from one known term of the sequence to find distant terms or a general rule (e.g. "the number of vertices is x times the number of squares"); and (5) *correspondence*: expresses a relation between the two varying quantities, based on the numerical

or pictorial sequence (e.g. “ $v=3n+1$ where v is the number of vertices, and n the number of squares”).

In the poster, we will present a complete analysis of the PTs’ strategies for obtaining distant terms and a general rule, providing a table with the percentage of PTs’ answers displaying each of the described strategies, both for Spanish and Portuguese contexts, as well as examples of those answers. Those results, in summary, indicate that in 62% of the Portuguese PTs’ answers they used a correct strategy to find a distant term of the sequence. In most of these cases they used a recursive strategy but also privileged the difference between consecutives terms as a multiplicative factor to obtain distant terms and the correspondence strategy. As expected, the PTs revealed more difficulties to find a generalization of a functional relation, since 50% of them did not answer and only 31% of their answers were correct. In these answers, they used difference-covariation and correspondence strategies. Almost half (48%) of their generalizations of a functional relation were based on a recursive strategy. However, in this case it resulted on incorrect generalizations.

Spanish results show that only 43% of the PTs’ answers were correct for computing the distant term, most of them based on a correspondence strategy. The recursive strategy was also quite frequent for obtaining distant terms (25%), although unlike in the case of Portugal, it led in most cases to incorrect results. One third of the Spanish PTs did not provide any answer to the question about generalization. Among those who did, correspondence and difference-recursive strategies were predominant, with only 20% of correct responses.

Moreover, it was seen that PTs of both countries adopted both visual and numerical approaches in the generalization of patterns, although they have privileged the former, and that, in general, they took into account the strategies used to find a distant term in the far generalization. However, the results related to the geometric pattern show important gaps in the algebraic reasoning of the PTs evaluated in both countries. These results provide information about the reasoning of PTs in tasks involving functional thinking and reveal the main difficulties they faced. In view of these results, possible considerations could be taken into account, like the benefits of including this type of tasks when designing courses of mathematical contents to help PTs develop their functional thinking.

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