

DE LA RECHERCHE À L'INDUSTRIE



# 15th Multiphase Flow Conference :

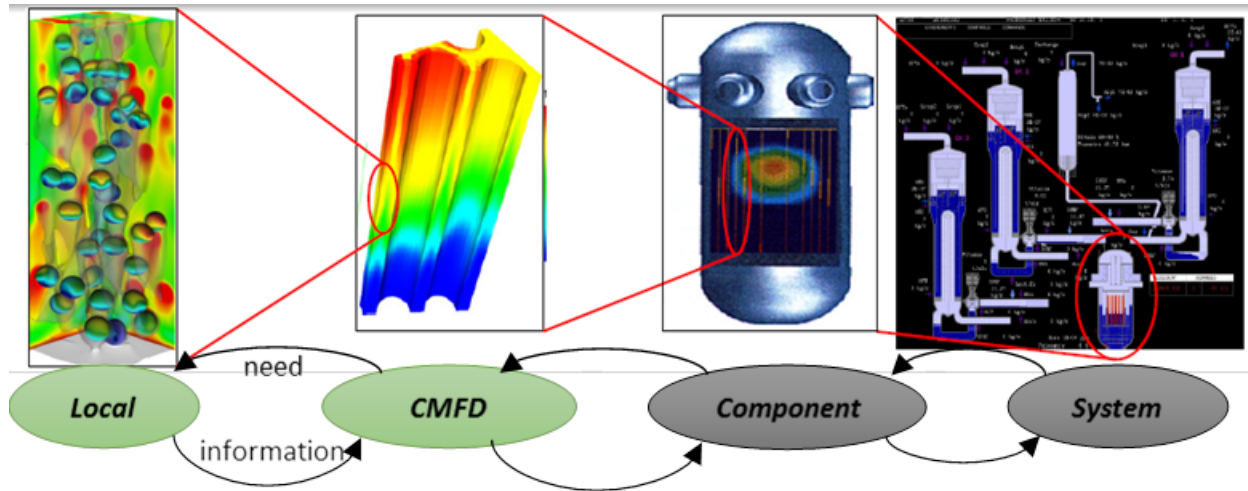
Surface tension energy in bubbly flows :  
physical impact and modelling in RANS  
Eulerian-Eulerian two-fluid model

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# Context : Solve complex two-phase bubbly flow ?



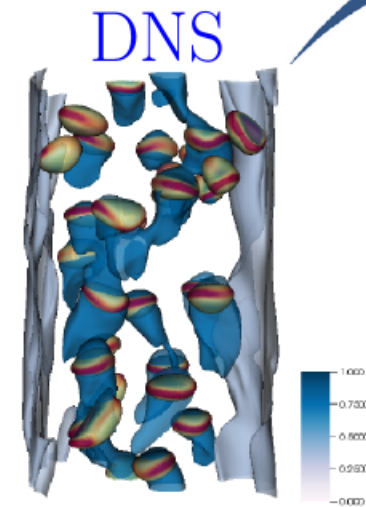
## 2-phase RANS simulation

- The only practical tool able to deal with industrial applications
- Need strong modelling and validation efforts
- Further progress is now limited by the lack of accurate knowledges of local phenomena : interfaces & turbulence interactions...

## Solution → Up-scaling

- To access to local information with experiments is limited by measurement techniques  
→ To use DNS as "numerical experiments"
- To analyze DNS results to improve modeling of 2-phase RANS CFD

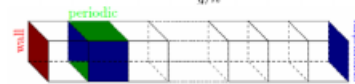
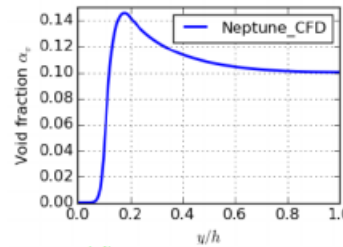
upscaling



**TRUST**

Local scale  
 $\chi_k, \mathbf{v}_k, p_k$

RANS



**NEPTUNE**

averaged scale

$$\alpha_k, \overline{\mathbf{v}_k}^k, \overline{p_k}^k$$

## upscaling $\neq$ calibration

- Access to all statistical quantities :
  - Interfacial forces
  - High-order turbulent statistics

- Assess the impact of fundamental RANS hypothesis :

→ **Interfacial forces :**

$$\mathbf{M}_I + \mathbf{M}_V = \mathbf{M}_\sigma$$

model for  $\mathbf{M}_I = \mathbf{M}^D + \mathbf{M}^L + \mathbf{M}^{TD} + \dots$

→ **One-pressure closure :**

$$\overline{p}_v^v = \overline{p}_l^l + \sigma \overline{\kappa}$$

→ **Turbulence closure :**

$$\overline{\mathcal{R}_v}^v = 0$$

$\overline{\mathcal{R}_l}^l$  modelling with  $R_{ij} - \epsilon$  or ...

$$\frac{D\alpha_k \rho_k \overline{\mathbf{u}_k}^k}{Dt} = -\nabla \cdot (\alpha_k \overline{\mathbf{P}_k}^k) + \alpha_k \rho_k \mathbf{g}$$

$$+ \nabla \cdot \alpha_k \overline{\mathbf{T}_k}^k + \nabla \cdot \alpha_k \overline{\mathcal{R}_k}^k + \mathbf{M}_k$$

## Introduction

Up-scaling

Physical setup

Numerical setup & physical description

RANS modelling

## One-pressure closure

Initial problem

New closure

Solution

Conclusion

## conclusion

## Local instantaneous description

- Navier-Stokes equations :

$$\frac{D\rho_k \mathbf{u}_k}{Dt} = -\nabla P_k + \rho_k \mathbf{g} + \underbrace{\nabla \cdot \mu (\nabla \mathbf{u}_k + \nabla^T \mathbf{u}_k)}_{\tau_k}$$

- Interfacial jump conditions :

$$\mathbf{u}_l^i = \mathbf{u}_v^i, \quad \sum_k (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) = \sigma \kappa$$

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## Extension to full space

- Multiply by phase indicator function :  $\chi_k = 1$  in phase  $k$ , 0 otherwise.

$$\begin{aligned} \frac{D\chi_k \rho_k \mathbf{u}_k}{Dt} = & -\nabla (\chi_k P_k) + \chi_k \rho_k \mathbf{g} + \nabla \cdot \chi_k \tau_k \\ & - (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k \end{aligned}$$

## Local instantaneous description

- Navier-Stokes equations :

$$\frac{D\rho_k \mathbf{u}_k}{Dt} = -\nabla P_k + \rho_k \mathbf{g} + \underbrace{\nabla \cdot \mu (\nabla \mathbf{u}_k + \nabla^T \mathbf{u}_k)}_{\tau_k}$$

- Interfacial jump conditions :

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$$\frac{D\chi_k \rho_k \mathbf{u}_k}{Dt} = -\nabla (\chi_k P_k) + \chi_k \rho_k \mathbf{g} + \nabla \cdot \chi_k \tau_k - (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k$$

## One-fluid formulation

- 'one-fluid' variables :  $\phi = \sum_k \chi_k \phi_k$

$$\frac{D\rho \mathbf{u}}{Dt} = -\nabla P + \rho \mathbf{g} + \nabla \cdot \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u}) + \sigma \kappa \mathbf{n} \delta^i$$

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- Navier-Stokes equations :

$$\frac{D\rho_k \mathbf{u}_k}{Dt} = -\nabla P_k + \rho_k \mathbf{g} + \underbrace{\nabla \cdot \mu (\nabla \mathbf{u}_k + \nabla^T \mathbf{u}_k)}_{\tau_k}$$

- Interfacial jump conditions :

$$\mathbf{u}_1^i = \mathbf{u}_v^i, \quad \sum_k (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) = \sigma \kappa$$

## Extension to full space

- Multiply by phase indicator function :  $\chi_k = 1$  in phase  $k$ , 0 otherwise.

$$\frac{D\chi_k \rho_k \mathbf{u}_k}{Dt} = -\nabla (\chi_k P_k) + \chi_k \rho_k \mathbf{g} + \nabla \cdot \chi_k \tau_k - (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k$$

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## Definition of average operators

- Statistical average = temporal average :

$$\overline{\phi(x, y, z)} = \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} \phi(x, y, z, \tau) d\tau$$

- Phasic average :

$$\overline{\phi}^k = \frac{\overline{\chi_k \phi_k}}{\overline{\chi_k}}$$



# Local scale / Averaged scale

## Local instantaneous description

- Navier-Stokes equations :

$$\frac{D\rho_k \mathbf{u}_k}{Dt} = -\nabla P_k + \rho_k \mathbf{g} + \underbrace{\nabla \cdot \mu (\nabla \mathbf{u}_k + \nabla^T \mathbf{u}_k)}_{\tau_k}$$

- Interfacial jump conditions :

$$\mathbf{u}_I^i = \mathbf{u}_V^i, \quad \sum_k (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) = \sigma \kappa$$

## Extension to full space

- Multiply by phase indicator function :  $\chi_k = 1$  in phase  $k$ , 0 otherwise.

$$\frac{D\chi_k \rho_k \mathbf{u}_k}{Dt} = -\nabla (\chi_k P_k) + \chi_k \rho_k \mathbf{g} + \nabla \cdot \chi_k \tau_k - (\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k$$

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- 'one-fluid' variables :  $\phi = \sum_k \chi_k \phi_k$

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## Definition of average operators

- Statistical average = temporal average :

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- Phasic average :

$$\overline{\phi}^k = \frac{\overline{\chi_k \phi_k}}{\overline{\chi_k}}$$

## Two-fluid averaged formulation

- Note  $\alpha_k = \overline{\chi_k}$

$$\frac{D\alpha_k \rho_k \overline{\mathbf{u}_k}^k}{Dt} = -\nabla (\alpha_k \overline{P_k}^k) + \alpha_k \rho_k \mathbf{g} + \nabla \cdot \alpha_k \overline{\tau_k}^k + \nabla \cdot \alpha_k \overline{\mathfrak{R}_k}^k - \underbrace{(\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k}_{\mathbf{M}_k}$$

- $\mathbf{M}_k$  is the interfacial force exerted on phase  $k$

$$\mathbf{M}_I + \mathbf{M}_V = \overline{\sigma \kappa \mathbf{n} \delta^i}$$

## Physical setup

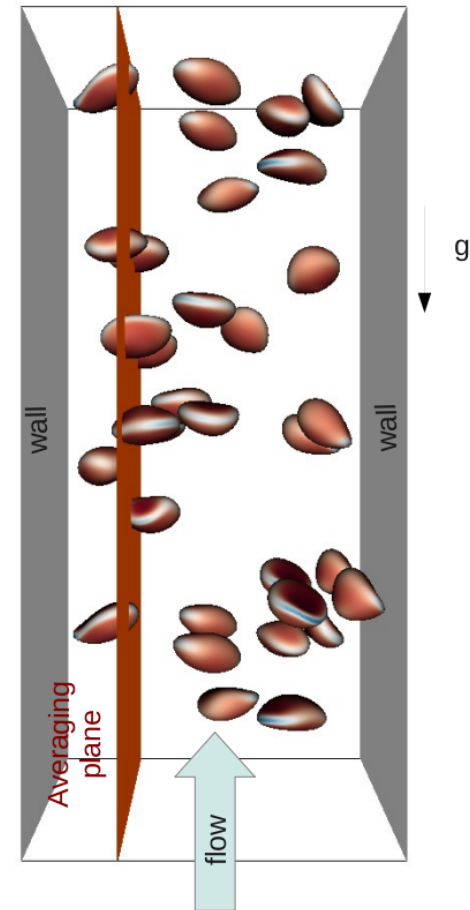
- One-fluid formulation

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla P' + (\rho - \langle \rho \rangle) \mathbf{g} \underbrace{-\beta \mathbf{e}_z}_{-\nabla \bar{P} + \langle \rho \rangle \mathbf{g}} + \nabla \cdot \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u}) + \sigma \kappa \mathbf{n}_v \delta^i$$

$$Re_\tau \text{ imposed by : } \beta = \frac{-\rho l}{h} \left( \frac{\nu_l Re_\tau}{h} \right)^2$$

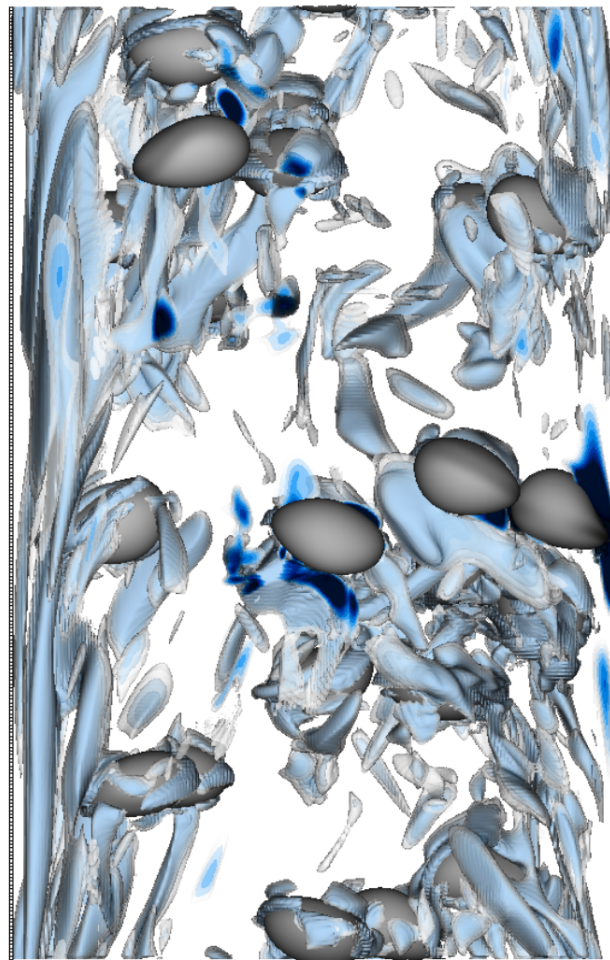
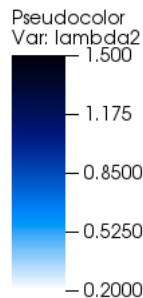
- Portion of a real channel bubbly upflow
- Isothermal / incompressible
- constant properties for each phase
- Non-slip boundary condition on wall
- Periodicity conditions in other boundaries
- Driving pressure gradient
- Deformable bubbles
- Ergodicity of statistical average :

$$\overline{\phi(\mathbf{y})} = \frac{1}{\Delta t L_x L_z} \int_{t-\Delta t/2}^{t+\Delta t/2} \int_0^{L_x} \int_0^{L_z} \phi(x, y, z, \tau) dx dz d\tau$$



# Numerical setup & physical description

$Eo$	4.5
res	$256 \times 192 \times 128$
$\alpha$	3%
$\sigma$	0.002
$d_b$	0.3
$N_b$	21
dom	$\pi \times 2 \times \pi/2$
$\beta$	0.0018
$Re_\tau$	127
$\rho_l$	1
$\rho_v$	0.1
$\mu_l/\nu$	0.0003
$g$	0.1



- Lift orientation is dependant of deformability  
→ Wall-picking for  $Eo < Eo^c$   
→ Core-picking for  $Eo > Eo^c$
- The boundary layer is free of bubbles  
→ Classical turbulent with elongated streaks
- Bubbles induced agitation in the core region  
→ Pseudoturbulence

**One-pressure closure**

# One-pressure closure : Interfacial force content

Two-fluid averaged formulation at statistical steady state :

$$0 = -\nabla \left( \alpha_k \overline{P_k^k} \right) + \alpha_k \rho_k \mathbf{g} + \nabla \cdot \alpha_k \overline{\tau_k^k} + \nabla \cdot \alpha_k \overline{\mathfrak{R}_k^k} - \underbrace{(\rho_k \mathbf{n}_k - \tau_k \cdot \mathbf{n}_k) \cdot \nabla \chi_k}_{\mathbf{M}_k} \quad (1)$$

Interfacial force equation :  $\alpha_I \times (1)_{k=v} - \alpha_V \times (1)_{k=l}$  :

$$\begin{aligned} \mathbf{M}_I &= \alpha_V \alpha_I (\rho_V - \rho_I) \mathbf{g}_i && = \text{Buoyancy force} \\ &+ \alpha_V \nabla \cdot \alpha_I \overline{\mathfrak{R}_I^I} - \alpha_I \nabla \cdot \alpha_V \overline{\mathfrak{R}_V^V} && + \text{Turbulent dispersion force} \\ &+ \alpha_V \nabla \cdot \alpha_I \overline{\tau_I^I} - \alpha_I \nabla \cdot \alpha_V \overline{\tau_V^V} && + \text{Lift \& Drag forces} \\ &- \nabla \left( \alpha_V \alpha_I (\overline{p_V^V} - \overline{p_I^I}) \right) - (\overline{p_I^I} - p_w) \nabla \alpha_V - \alpha_V (\overline{p_V^V} - \overline{p_I^I}) \nabla \alpha_V && + \text{pressure force ?} \\ &+ \alpha_I \mathbf{M}_\sigma && + \text{surface tension force ?} \end{aligned}$$

- Give the content of interfacial forces
- Drag, lift and turbulent dispersion forces are extensively studied.
- Almost nobody talks about surface tension & pressure forces

## Exact DNS equation

$$\begin{aligned}
 \mathbf{M}_I &= \alpha_v \alpha_I (\rho_v - \rho_I) \mathbf{g}_i \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\mathbf{R}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathbf{R}}_v^v \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\pi}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\pi}_v^v \\
 &- \nabla \left( \alpha_v \alpha_I (\overline{\rho}_v^v - \overline{\rho}_I^I) \right) - (\overline{\rho}_I^I - \rho_w) \nabla \alpha_v \\
 &- \alpha_v (\overline{\rho}_v^v - \overline{\rho}_I^I) \nabla \alpha_v + \overline{\alpha_I \sigma \kappa \mathbf{n}} \delta^i + \text{residu}
 \end{aligned}$$

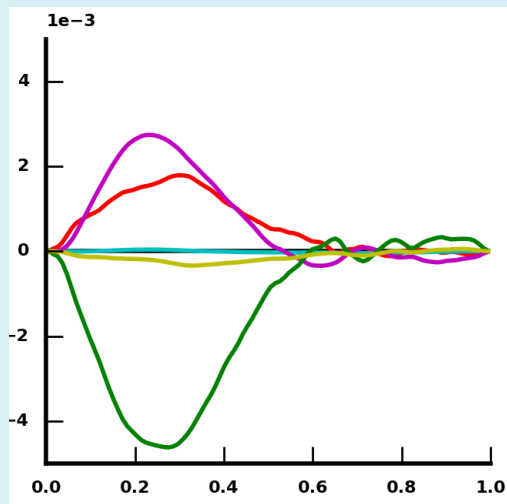


FIGURE – Local : wall-normal component of the  $\mathbf{u}_r$  equation

# One-pressure closure : Problem

## Exact DNS equation

$$\begin{aligned}
 \mathbf{M}_I &= \alpha_v \alpha_I (\rho_v - \rho_I) \mathbf{g}_i \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\mathcal{R}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathcal{R}}_v^v \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\mathcal{T}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathcal{T}}_v^v \\
 &- \nabla \left( \alpha_v \alpha_I (\overline{p}_v^v - \overline{p}_I^I) \right) - (\overline{p}_I^I - p_w) \nabla \alpha_v \\
 &- \alpha_v (\overline{p}_v^v - \overline{p}_I^I) \nabla \alpha_v + \overline{\alpha_I \sigma \kappa} \mathbf{n} \delta^i + \text{residu}
 \end{aligned}$$

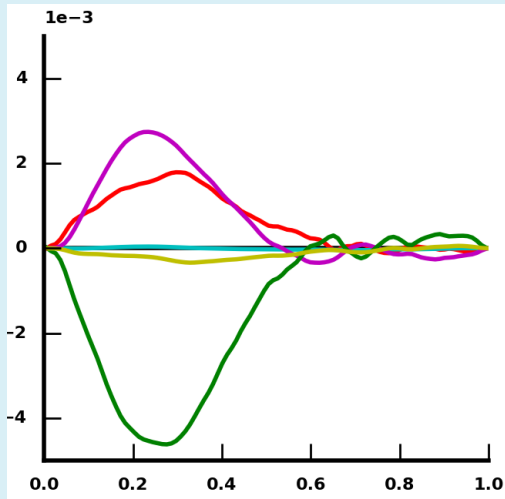


FIGURE – Local : wall-normal component of the  $\mathbf{u}_r$  equation

## Approximate RANS equation

■ Classical one-pressure closure

$$\overline{p}_v^v - \overline{p}_I^I \approx \sigma \kappa \quad \text{and} \quad \kappa = \frac{4}{d_0} = Cte$$

$$\begin{aligned}
 \mathbf{M}_I + (\overline{p}_I^I - p_w) \nabla \alpha_v &\approx \alpha_v \alpha_I (\rho_v - \rho_I) \mathbf{g}_i \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\mathcal{R}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathcal{R}}_v^v \\
 &+ \alpha_v \nabla \cdot \alpha_I \overline{\mathcal{T}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathcal{T}}_v^v + \text{residu}
 \end{aligned}$$

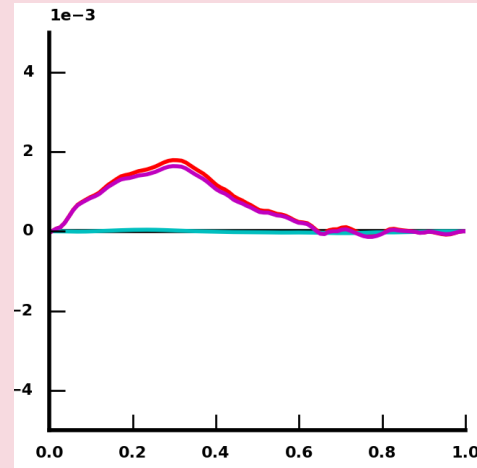
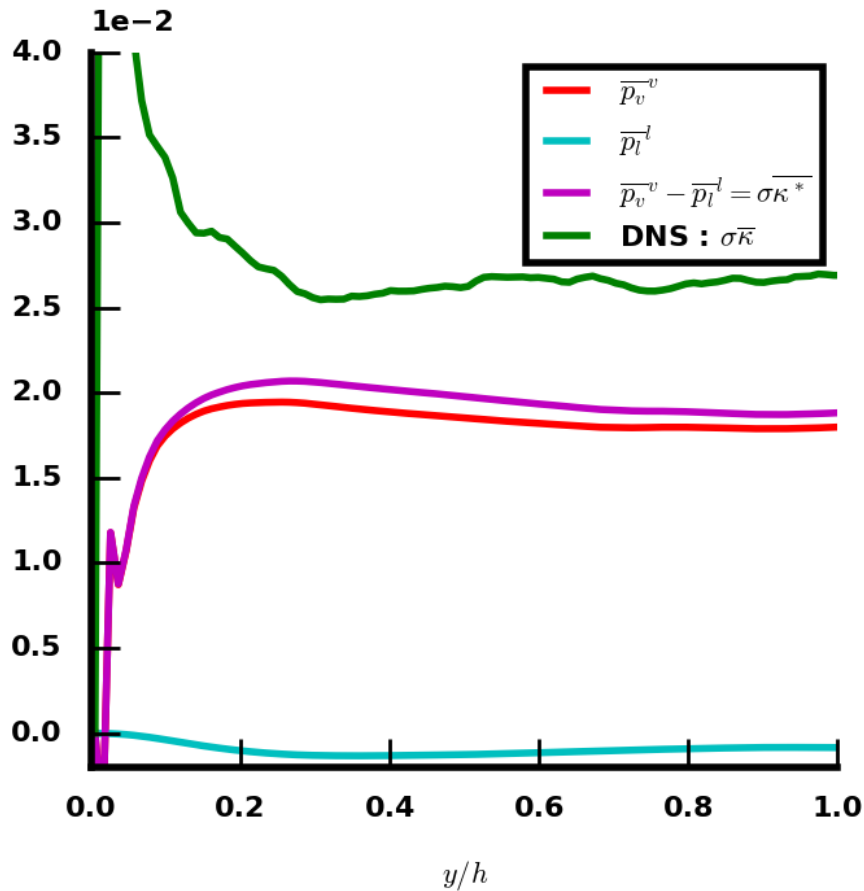


FIGURE – Averaged : wall-normal component of the  $\mathbf{u}_r$  equation

# One-pressure closure : closure of the vapor pressure



- The Laplace law is effective at the interface :

$$p_v^i - p_l^i = \sigma \kappa$$

- statistical quantities show :

$$\bar{p}_v^v - \bar{p}_l^l \neq \sigma \bar{\kappa}$$

- A new free parameter is introduced called 'mean effective curvature' such as :

$$\bar{\kappa}^* = \frac{\bar{p}_v^v - \bar{p}_l^l}{\sigma}$$

- $\bar{p}_l^l \ll \bar{p}_v^v$  because of the small void fraction ( $\alpha = 3\%$ )

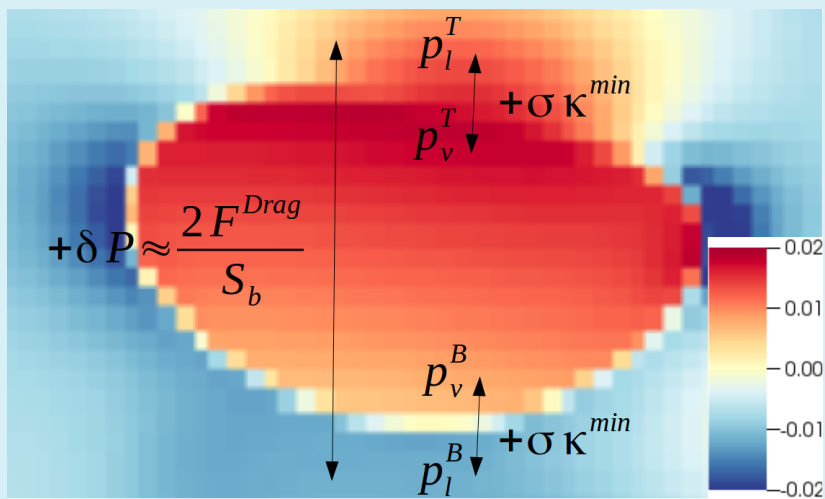
$$\bar{\kappa}^* \approx \frac{\bar{p}_v^v}{\sigma}$$



# One-pressure closure : closure of curvature

physical closure of  $\overline{\kappa^*}$  based on :

- DNS observation (ex : a constant pressure gradient in the bubble / an approximated ellipsoidal shape )
- Physical interpretation ( ex : the surrounding pressure at the top and at the bottom is led by drag force )



Closure equations of the vapor pressure

$$\overline{\kappa^*} = \frac{F^{Drag}}{S_b \sigma} (2C - 1) + \kappa^{min} + \frac{P_0}{\sigma}$$

$$a = r_0 (1 + 0.163 Eo^{0.757})^{1/3}$$

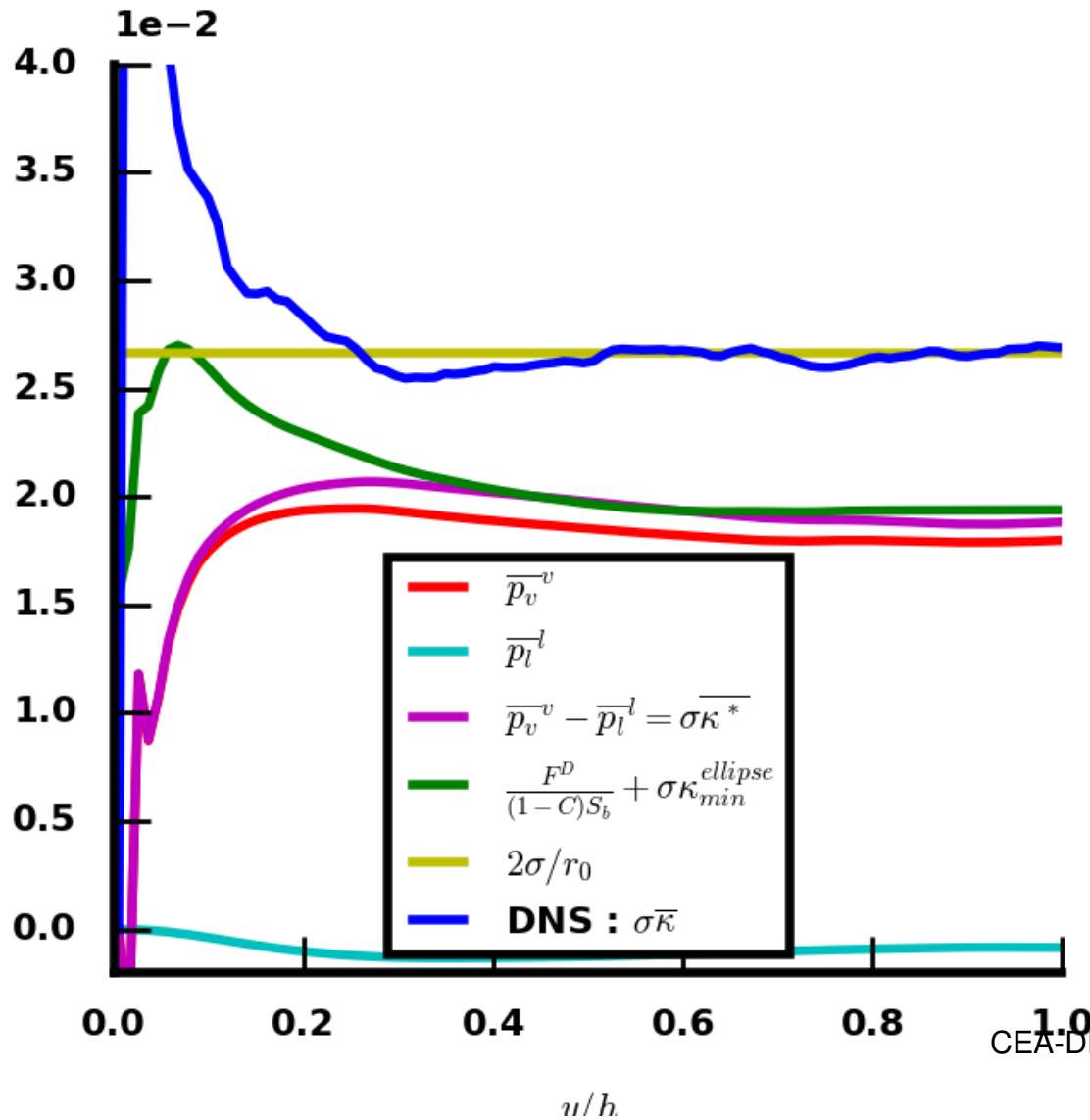
$$S_b = 2\pi a^2 + \frac{\pi b^2}{e} \ln \left( \frac{1+e}{1-e} \right)$$

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$\kappa^{min} = \frac{2b}{a^2}$$

$$b = \frac{r_0^3}{a^2}$$

# One-pressure closure : results of closure



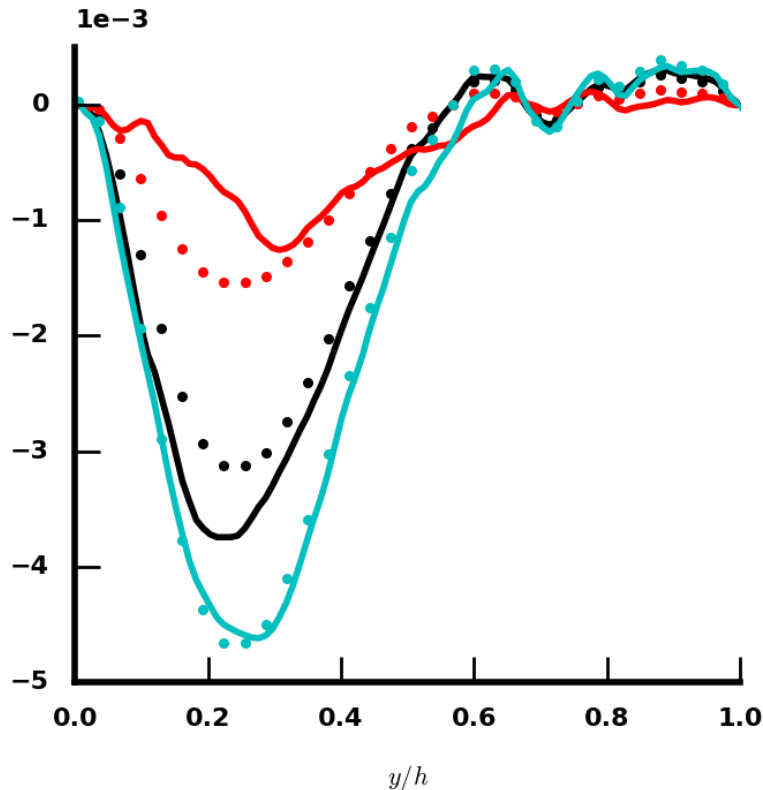
- $\overline{\sigma\kappa}$  fits with  $\approx \frac{2}{r_0}$
- $\overline{\sigma\kappa^*}$  fits with the new correlation for  $C = 0.9$  with :

$$F^{Drag} = \frac{1}{2} C_d \rho_l \mathbf{v}_r |\mathbf{v}_r| \pi a^2$$

$$C_d = \frac{4}{3} a \sqrt{\frac{g(\rho_l - \rho_v)}{\sigma}}$$

- $C = 0.9$  is relevant with DNS results
- Good behavior of the correlations in the bulk but not at the wall

# One-pressure closure : closure of the surface tension



Solid line : DNS  
Dot marker : model

- The splitting of the surface tension term with :  
 $\kappa = \bar{\kappa} + \kappa'$  and  $\sigma = cte$

$$\alpha_I \overline{\sigma \kappa \mathbf{n}_v \delta^i} = \alpha_I \sigma \bar{\kappa} \overline{\mathbf{n}_v \delta^i} + \alpha_I \sigma \underbrace{\frac{\overline{\kappa' \mathbf{n}_v \delta^i}}{\overline{\mathbf{n}_v \delta^i}}}_{\kappa^{**}} \overline{\mathbf{n}_v \delta^i}$$

- $\bar{\kappa}$  is the averaged curvature

$$\bar{\kappa} \approx 4/d_0$$

- $\kappa^{**}$  corresponds to the correlation between the fluctuating curvature and the normal at the interface. It fits well with the maximal fluctuation of curvature for an ellipsoidal bubble.

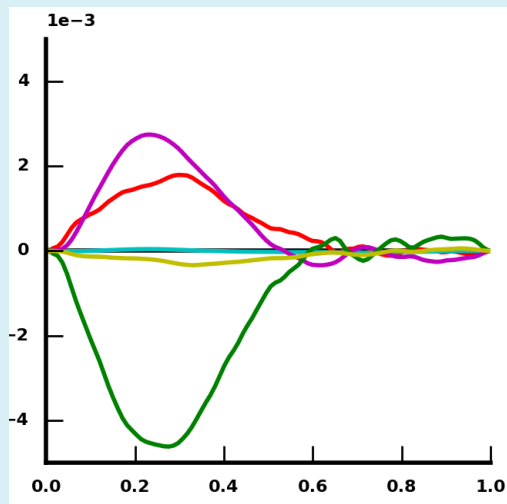
$$\kappa^{**} \approx 4/d_0 - \kappa^{min}$$

- Finally :

$$\alpha_I \overline{\sigma \kappa \mathbf{n}_v \delta^i} = \alpha_I \sigma (8/d_0 - \kappa^{min}) \overline{\mathbf{n}_v \delta^i}$$

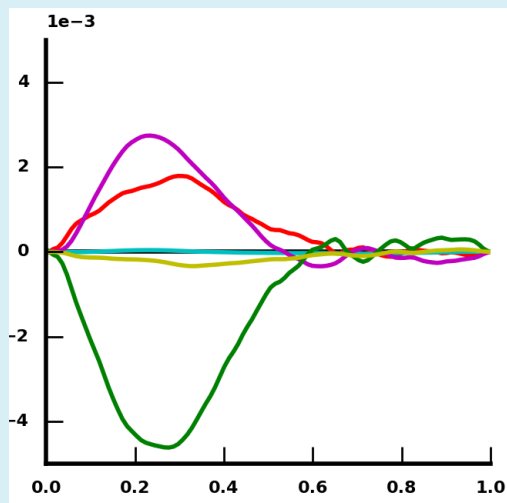
## Exact DNS equation

$$\begin{aligned}
 \mathbf{M}_I = & \alpha_v \alpha_I (\rho_v - \rho_I) \mathbf{g}_i \\
 & + \alpha_v \nabla \cdot \alpha_I \overline{\mathcal{R}}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\mathcal{R}}_v^v \\
 & + \alpha_v \nabla \cdot \alpha_I \overline{\tau}_I^I - \alpha_I \nabla \cdot \alpha_v \overline{\tau}_v^v \\
 & - \nabla \left( \alpha_v \alpha_I (\overline{\rho}_v^v - \overline{\rho}_I^I) \right) - (\overline{\rho}_I^I - \rho_w) \nabla \alpha_v \\
 & - \alpha_v (\overline{\rho}_v^v - \overline{\rho}_I^I) \nabla \alpha_v + \overline{\alpha_I \sigma \kappa \mathbf{n}} \delta^i + \text{residu}
 \end{aligned}$$

FIGURE – wall-normal component of the  $\mathbf{u}_r$  equation

## Exact DNS equation

$$\begin{aligned}
 \mathbf{M}_I &= \alpha_v \alpha_l (\rho_v - \rho_l) \mathbf{g}_i \\
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 &+ \alpha_v \nabla \cdot \alpha_l \overline{\mathcal{T}}_l^I - \alpha_l \nabla \cdot \alpha_v \overline{\mathcal{T}}_v^V \\
 &- \nabla \left( \alpha_v \alpha_l (\overline{p}_v^V - \overline{p}_l^I) \right) - (\overline{p}_l^I - p_w) \nabla \alpha_v \\
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 \end{aligned}$$

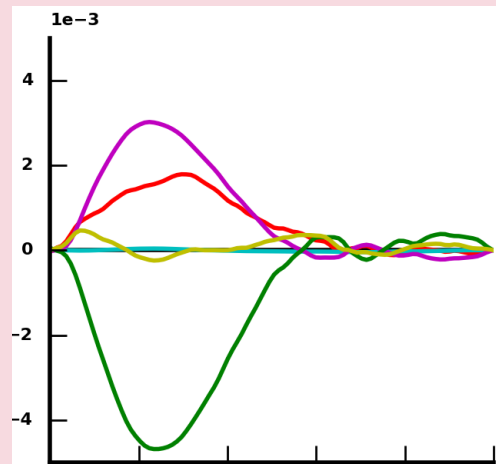
FIGURE – wall-normal component of the  $\mathbf{u}_r$  equation

## Approximate new RANS equation

## ■ One-pressure new closure

$$\begin{aligned}
 \overline{\alpha_l \sigma \kappa \mathbf{n} \delta^i} &\approx \alpha_l (\overline{\kappa}^{**} + \overline{\kappa}) \nabla \alpha_v \\
 \overline{p}_v^V - \overline{p}_l^I &\approx \sigma \overline{\kappa}^*
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{M}_I + (\overline{p}_l^I - p_w) \nabla \alpha_v &\approx \alpha_v \alpha_l (\rho_v - \rho_l) \mathbf{g}_i \\
 &+ \alpha_v \nabla \cdot \alpha_l \overline{\mathcal{R}}_l^I - \alpha_l \nabla \cdot \alpha_v \overline{\mathcal{R}}_v^V \\
 &+ \alpha_v \nabla \cdot \alpha_l \overline{\mathcal{T}}_l^I - \alpha_l \nabla \cdot \alpha_v \overline{\mathcal{T}}_v^V \\
 &- \alpha_l \sigma \overline{\kappa}^* \nabla \alpha_v + \alpha_l \sigma (\overline{\kappa}^{**} + \overline{\kappa}) \nabla \alpha_v + \text{residu}
 \end{aligned}$$



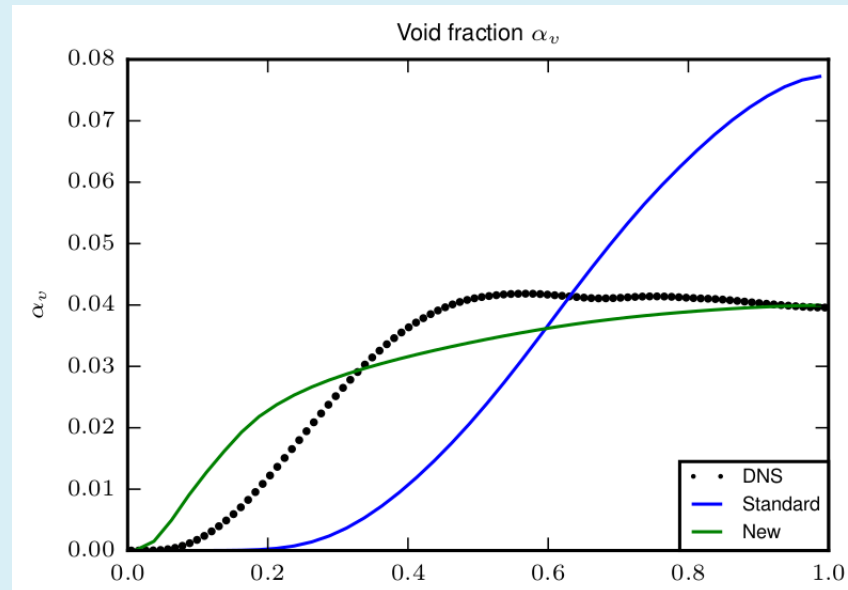
# One-pressure closure : Conclusion

## New model for dispersion force

- Bubble dispersion have two contributions.
  - Classical turbulent dispersion force
  - New dispersion force linked to bubble deformability and surface tension

$$\mathbf{M}^{\text{Dis}} = \left[ \underbrace{C^{TD} \rho_l K_l}_{\text{turbulent dispersion}} + \underbrace{\alpha_l \sigma (\bar{\kappa} + \bar{\kappa}^{**} - \bar{\kappa}^*)}_{\text{dispersion}} \right] \nabla \alpha_v$$

- A posteriori test : Results are much better in Neptune\_CFD with the addition of the new dispersion force.



## Summary

- Surface tension acts as a dispersion force
- It is an important parameter for the void fraction prediction
- The modelling of the surface tension is linked to the one-pressure closure
- A new physical closure has been found
- Validation on DNS and RANS simulations

## Prospects

- High Reynolds number → Next calculation
- Validation of the new closure on other cases
- Generalization for polydispersed flows

**Thank you for your attention !**

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# One-pressure closure : closure of curvature

- H1 : Linear pressure gradient inside bubbles

$$\bar{p}_v^v = 1/2 (p_v^T + p_v^B)$$

- H2 :  $p_l^T$  and  $p_l^B$  lead by the drag force

$$F^{Drag} = - \oint_{S_b} p_l \mathbf{n}_v dS \rightarrow p_l^T - p_l^B \approx \frac{2F^{Drag}}{S_b}$$

- H3 : Top and bottom pressure deviate from the reference pressure assymmetrically :

$$P_l^T \approx C \frac{2F^{Drag}}{S} + P_0$$

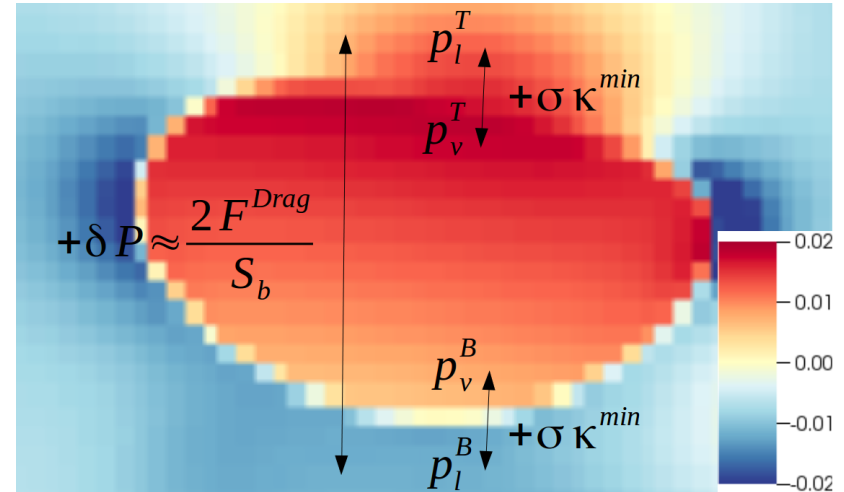
$$P_l^B \approx (C - 1) \frac{2F^{Drag}}{S} + P_0$$

- H4 : at  $p_l^T$  and  $p_l^B$ , the local curvature is minimized

$$p_v^T - p_l^T = \sigma \kappa^{min}$$

$$p_v^B - p_l^B = \sigma \kappa^{min}$$

- H5 : Bubbles have ellipsoidal shape governed by Wellek correlation



## Closure equations of the vapor pressure

$$\bar{\kappa}^* = \frac{F^{Drag}}{S_b \sigma} (2C - 1) + \kappa^{min} + \frac{P_0}{\sigma}$$

$$S_b = 2\pi a^2 + \frac{\pi b^2}{e} \ln \left( \frac{1+e}{1-e} \right)$$

$$e = \frac{\sqrt{a^2 - b^2}}{a} ; \quad \kappa^{min} = \frac{2b}{a^2}$$

$$a = r_0 (1 + 0.163 Eo^{0.757})^{1/3} ; \quad b = \frac{r_0^3}{a^2}$$