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A study of structured uncertainties in wave characteristic of one-dimensional periodic structures

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Abstract

Periodic structures have properties of controlling mechanical wave. These are used in aircraft, trains, submarines, space structures and demand high level of robustness, which can be ensured with consideration of the presence of uncertainty in the numerical models. The uncertainties, in terms of material properties and geometrical parameters, are mostly introduced in both the manufacturing and assembly process. In order to predict the wave characteristics of the periodic structures under the uncertainty, the stochastic wave finite element method has been extended to periodic structures; wave finite element method (WFEM) added with an additional dimension to model the uncertain parameter using the perturbation approach. Numerical experiments are performed to test the method with one-dimensional periodic rod and periodic beam with parametric uncertainty. The performance of the developed method is compared in terms of computational cost which offers computational advantages over the Monte Carlo Simulation.

1 Introduction

The vibroacoustic performance and dynamics of the structure are important subjects in aeronautics, transport, energy, and space where the periodic media can be used for vibration reduction, acoustic blocking, acoustic channeling and acoustic cloaking. Application for these unusual phenomena is mainly correlated to the inherent band gaps or frequency regions in which the propagation of acoustic/elastic waves in the periodic media can be manipulated. Inherent material inhomogeneity, manufacturing errors (ex. variation of geometry) and environmental influences are unavoidable.

To address this unavoidable actuality, the effects of uncertainties need to be considered when analyzing band structures (pass and stop band) and dynamics of the periodic structures. Also, to meet the regulatory compliance and user requirement, the designer accounts for variation in the input parameter at the design phase. For example, in the space industry designers consider the uncertainty in the system parameter to ensure that the during launch and orbital operation the vibration level are in a range that is acceptable.

Three different frequency ranges of analysis can be defined for the typical dynamic response of weakly dissipative structures. At the low-frequency range, FE-like approaches are applicable because of the low order modes and low sensitivity to variability, contrary the high frequency with many high order modes involved, energy-based methods (e.g. Statistical Energy Analysis (SEA)) are well suited. The mid-frequency range affected by variability of the model parameter and the wave-based approaches have been developed by increasing computational efficiency. However, most of them assume that waveguide properties are homogeneous in the direction of the travelling wave, which is the limitation of such approaches for variety of application where uncertainty can be considered. Also the analytical solutions are available but limited to the particular cases (e.g. acoustic horns, ducts, rods and beams) [1]. Manohar et al. [2] considered the randomness in the wave propagation in waveguides using spectral element analysis. Ichchou et al.[3] proposed a numerical approach using the Wave Finite Element Method (WFEM) [4] considering spatially

homogeneous variability in waveguides using a first order perturbation and extended with second order perturbation is proposed [5].

When dealing with layered structures the material or geometrical uncertainty often exhibits spatial correlation and random fields theory can be used to model spatially distributed variability with probability measure [6]. In addition, a Karhunen–Loeve (KL) expansion provides characterization of the random field in terms of deterministic eigenfunctions weighted by uncorrelated random variables. Also the (after Wentzel, Kramers and Brillouin) WKB approximation are employed in the modelling the random media [7]. The limitation of the method is that it does not consider the internal reflections which occur due to any local changes in the material or geometrical properties. This assumption can be mitigated if one considers a waveguide with piecewise constant material variability, where internal reflections due to local changes in the impedance can be taken into account at the junctions of the sections and the piecewise constant separated into a finite number of discrete sections [8]. Also, this would be an approximate representation for the spatially varying system [9].

The literature reveals that the effects of uncertainty in the material properties, geometry, loading condition and model play significant role in altering wave states. The work reported so far in the literature with the best of authors knowledge, is limited to wave propagation (with structured and unstructured uncertainty) in elastic media and random media only. In the present work, a Stochastic Wave Finite Element Method (SWFEM) [10], [11] is employed to analyze the complex band gap in 1D periodic structures. The results of the stochastic formulation are compared those obtained with analytical and Monte Carlo Simulation (MCS) for the bar and beam.

2 Formulation of SWFEM

One dimensional periodic structures are obtained by formulating the unit cell and then repeating in the propagation direction. Then, the study of this structure can be converted into study of unit cell based on the Floquet-Bloch theorem [3]. The schematic representation is shown in Fig. 1.

To consider the effect of the uncertainty on the response of the structure, the SWFEM is formulated which is extension of the classical WFEM to accommodate the uncertainty. The workflow of the SWFEM is depicted in Fig. 2.

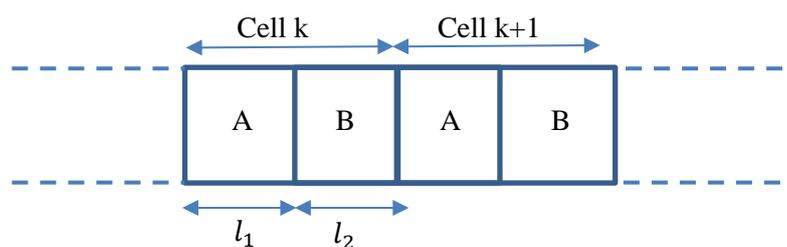


Figure 1: Schematic representation of periodic structure

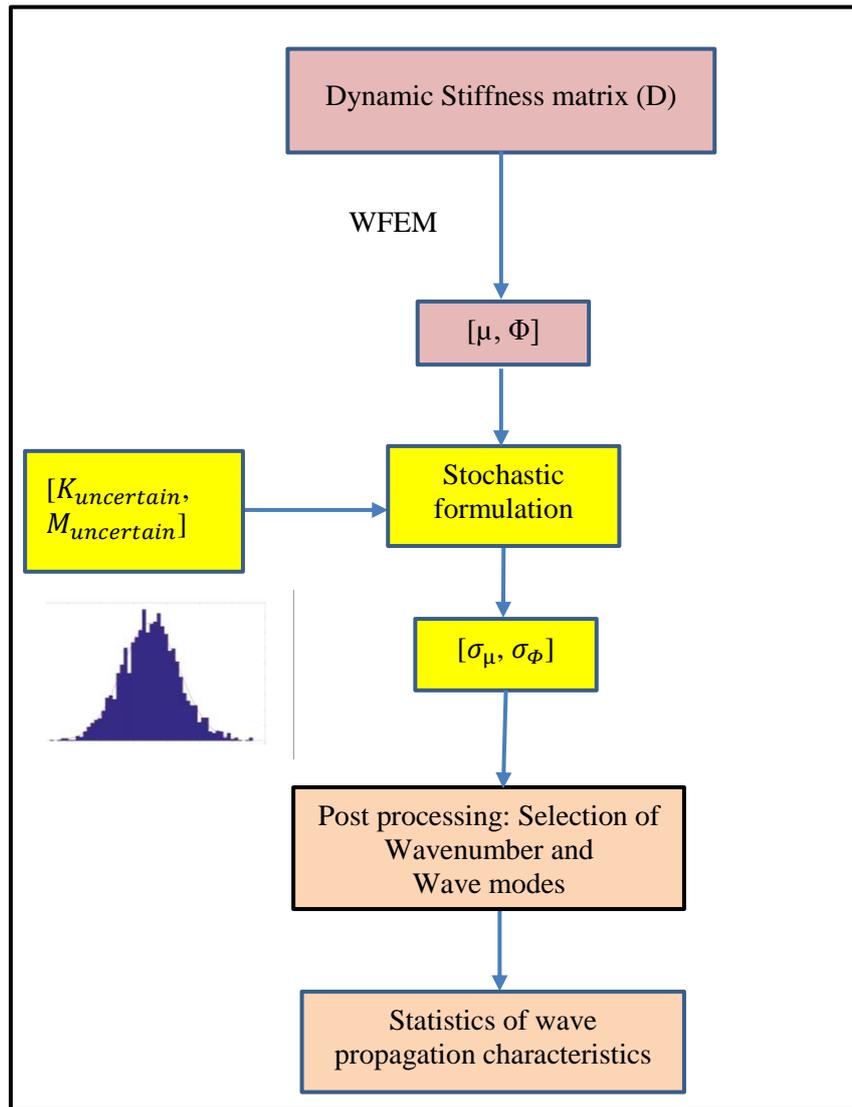


Figure 2: Flow diagram for SWFEM

To make the reader easily understand, the step in the formulation of SWFEM [3] is explained here.

The WFEM is used to study the behavior of periodic structures. This is extended to apply the WFEM for the uncertainty study. This starts with consideration of the random field as a supplementary dimension through the spatial discretization using the finite element steps. Which start from discretization of one subelement of length (d). This discretization leads to stochastic dynamic equilibrium of any substructure in the following manner.

$$(\tilde{D}) \begin{pmatrix} \tilde{q}_L^k \\ \tilde{q}_R^k \end{pmatrix} = \begin{pmatrix} \tilde{F}_L^k \\ \tilde{F}_R^k \end{pmatrix} \tag{1}$$

Here (\tilde{D}) is the stochastic complex dynamic stiffness matrix of the substructure, condensed on left and right boundaries degree of freedom at the pulsation ω :

$$(\tilde{D}) = -\omega^2 \tilde{M} + \tilde{K}(1 + i\eta) \tag{2}$$

Here \tilde{M} and \tilde{K} are stochastic mass and stiffness matrix respectively.

In the probabilistic tools, the parametric approach allows to consider the uncertain parameters (material, geometrical properties etc.) as random quantities. This is modeled using the stochastic finite element discretization approaches as expressed in [12].

The random variables are modeled using the first order perturbation, as Gaussian variables, such that the dynamical equilibrium is expressed as:

$$[-\omega^2(\bar{M} + \sigma_M \varepsilon) + (\bar{K} + \sigma_K \varepsilon)(1 + i\eta)] \begin{pmatrix} \bar{q}_L^{(k)} + \sigma_{q_L}^{(k)} \varepsilon \\ \bar{q}_R^{(k)} + \sigma_{q_R}^{(k)} \varepsilon \end{pmatrix} = \begin{pmatrix} \bar{F}_L^{(k)} + \sigma_{F_L}^{(k)} \varepsilon \\ \bar{F}_R^{(k)} + \sigma_{F_R}^{(k)} \varepsilon \end{pmatrix} \quad (3)$$

The overbar $\bar{\cdot}$ symbol denotes the mean of the random variable, σ ; is standard deviation and ε is a Gaussian centered variable [13]. In the expression \bar{M} , \bar{K} , \bar{q} , \bar{F} are the mean quantities of mass matrix, stiffness matrix, displacement vector and load. σ_M , σ_K , σ_q , σ_F are their standard deviation.

As in the classical case, the stochastic problem in equation (1) can be partitioned in the following way:

$$\begin{pmatrix} \bar{D}_{LL} & \bar{D}_{LR} \\ \bar{D}_{RL} & \bar{D}_{RR} \end{pmatrix} \begin{pmatrix} \tilde{q}_L^k \\ \tilde{q}_R^k \end{pmatrix} = \begin{pmatrix} \tilde{F}_L^k \\ \tilde{F}_R^k \end{pmatrix} \quad (4)$$

It is to be noted that equation (4) is valid and can accommodate the stochastic behavior of the stiffness and mass matrix.

The stochastic kinematic variable, \tilde{q} and \tilde{F} are represented through stochastic state vector and related by the stochastic transfer matrix \tilde{S} .

$$\tilde{u}_R^k = \tilde{S} \cdot \tilde{u}_L^k \quad (5)$$

Where $\tilde{S} = \bar{S} + \sigma_s \varepsilon$

$$\bar{S} = \begin{pmatrix} -\bar{D}_{LR}^{-1} \bar{D}_{LL} & \bar{D}_{LR}^{-1} \\ \bar{D}_{RL} - \bar{D}_{RL} \bar{D}_{LR}^{-1} \bar{D}_{LL} & -\bar{D}_{RL} \bar{D}_{LR}^{-1} \end{pmatrix}$$

$$\sigma_s = \begin{pmatrix} -\bar{D}_{LR} & 0 \\ -\bar{D}_{RR} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{D_{LL}} & \sigma_{D_{LR}} \\ \sigma_{D_{RL}} & \sigma_{D_{RR}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\bar{D}_{LR}^{-1} \bar{D}_{LL} & \bar{D}_{LR}^{-1} \end{pmatrix}$$

Following the steps of the deterministic development, a stochastic eigenvalue problem formulated as follows:

$$\begin{aligned} \tilde{S} \tilde{\phi}_i &= \bar{\mu}_i \cdot \tilde{\phi}_i \\ |\tilde{S} - \bar{\mu}_i I_{2n}| &= 0 \end{aligned} \quad (6)$$

The polynomial chaos projection of the eigenvalue problem leads to find their mean and standard deviation.

The standard deviation of the propagation constant μ_i (σ_{μ_i})

$$\begin{aligned} \sigma_{\mu_i} &= [(\bar{\phi}_i)^T \sigma_s^T (\bar{S}^T - \bar{\mu}_i I_{2n})^{-1} J_n (\bar{S} - (\bar{\mu}_i)^{-1} I_{2n}) - (\bar{\phi}_i)^T J_n \sigma_s] \\ &\quad \times [(\bar{\phi}_i)^T (\bar{S}^T - \bar{\mu}_i I_{2n})^{-1} J_n (\bar{S} - (\bar{\mu}_i)^{-1} I_{2n}) - (\bar{\mu}_i)^{-2} (\bar{\phi}_i)^T J_n]^{-1} \end{aligned} \quad (7)$$

similarly, the standard deviation of the eigenvectors (σ_{ϕ_i}) can be written as

$$\sigma_{\phi_i} = -[\bar{S} - \bar{\mu}_i I_{2n}]^+ [\sigma_s - \sigma_{\mu_i} I_{2n}] \bar{\phi}_i \quad (8)$$

where + is pseudo inverse.

Using equation (7) and (8), the statistics of the wave characteristics can be expressed in the standard deviation of the propagation constant.

3 Results

This section shows the validation of SWFEM formulation considering binary periodic rod and binary periodic beam respectively. Binary rod and beam consist of section A of length l_1 and section B of length l_2 as depicted in the Fig. 3. Here cells A and B are made of different materials. In the first case, the longitudinal waves are studied while in the second case the flexural waves in beam of circular section are studied.

3.1 Longitudinal waves in binary periodic rod with uncertainty

In this subsection, the SWFEM is used to study the effect of parametric uncertainty on the dispersion relation of the longitudinal wave in binary periodic rod with sections A and B made of epoxy and aluminum respectively. The length of l_1 and l_2 are 1m each with circular cross section of radius of 0.0644m.

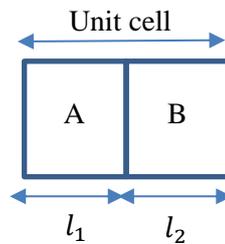


Figure 3: Symmetric unit cell

For the validation purpose SWFEM results is compared with the analytical solution of the wave number K expressed [14]:

$$\cos(Kl) = \cos\left(\frac{\omega}{c_a}l_a\right)\cos\left(\frac{\omega}{c_b}l_b\right) - \frac{1}{2}\left(\frac{\rho_a c_a}{\rho_b c_b} + \frac{\rho_b c_b}{\rho_a c_a}\right)\sin\left(\frac{\omega}{c_a}l_a\right)\sin\left(\frac{\omega}{c_b}l_b\right) \tag{8}$$

Where c_a, c_b is the wave velocity in the section A and B respectively and expressed as $c_a = \sqrt{\frac{E_a}{\rho_a}}$ and

$$c_b = \sqrt{\frac{E_b}{\rho_b}}$$

The uncertainty effect is studied considering variation of the Young’s modulus with four percentage. The study frequency range are up to 1800 Hz. The analytical results are treated as reference result for validation purpose. The sampling method with 10000 samples are used to get the wave characteristics of the analytical wave number obtained using equation (8).

In SWFEM formulation two-node rod elements are considered with the stiffness and mass matrices as following:

$$K_{rod} = \frac{ES}{d} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$M_{rod} = \frac{\rho S d}{6} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Where E is the Young's modulus, S is cross sectional area and d is element length.

The global stiffness and mass matrices are assembled in the MATLAB environment with 100 elements in the unit cell of binary periodic rod. The material and geometric properties are shown in Table 1.

Geometry/Property	Value
Rod length (A)	1 m
Rod length (B)	1 m
Radius of rod	0.0644 m
Young's modulus (A)	4.35 Gpa
Young's modulus (B)	77.56 Gpa
Mass density (A)	1180 kg/m ³
Mass density (B)	2730 kg/m ³
Loss factor (A) and (B)	0.001

Table 1: Geometric parameters and material properties of binary periodic rod

Validation of SWFEM result in terms of dispersion equation is also computed using the MCS method with 10000 samples. As shown in Fig. 4, there exist one stop band in the considered frequency range according to the results obtained with SWFEM, analytical sampling and MCS in the considered frequency range. Also, the non-dispersive nature of propagation can be observed for the standard deviation. The comparison of the standard deviation is presented in the Fig. 5, obtained from analytical sampling, SWFEM and WFEM MCS. The results are in good agreement. The SWFEM standard deviation is computed considering loss factor on the contrary the analytical sampling is based on without damping.

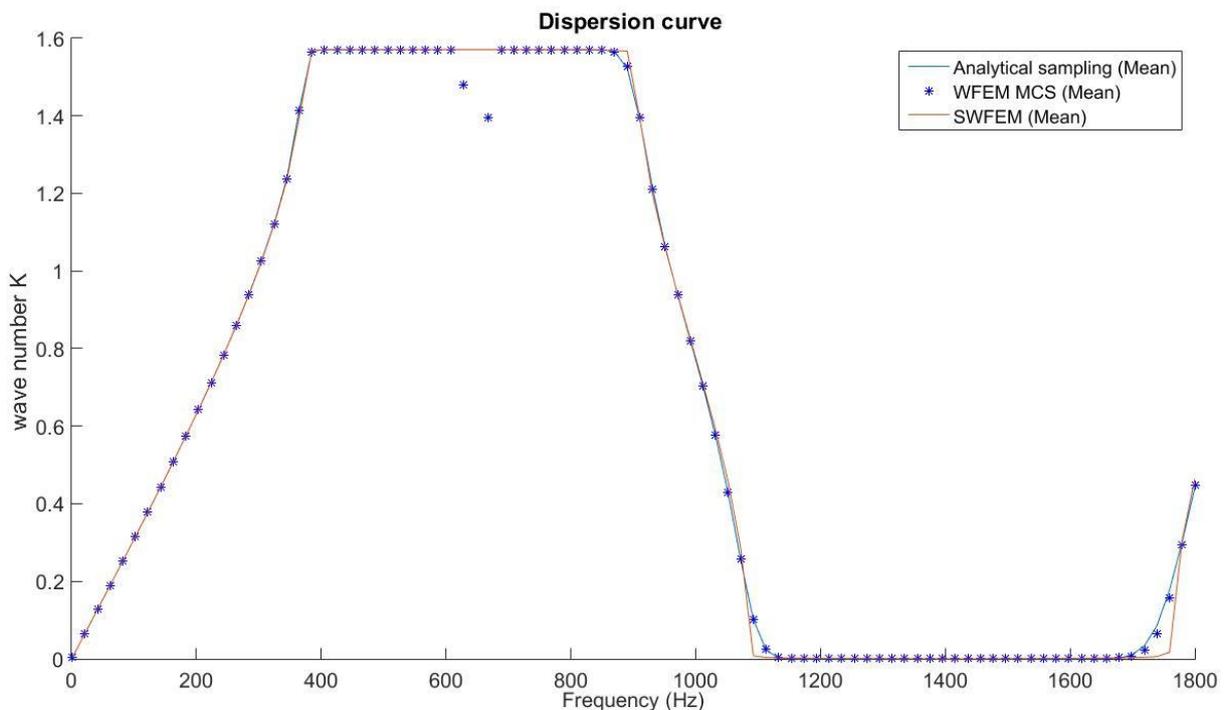


Figure 4: Comparison of mean of wave number

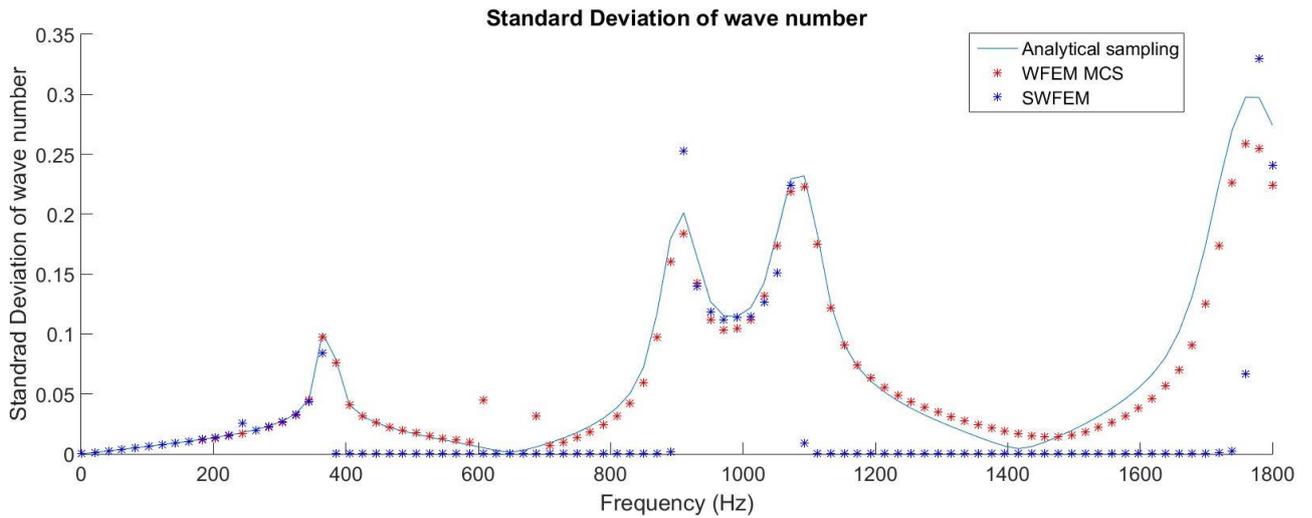


Figure 5: Comparison of the standard deviation of wave number

3.2 Flexural waves in binary periodic beam with uncertainty

In this subsection, the study of the uncertainty effect on the bending wave in binary periodic beam is performed. The Euler-Bernoulli beam theory is considered with material and geometrical properties listed in Table 2.

Geometry/Property	Value
Beam length	0.5m
Radius of beam	0.00945m
Young’s modulus (A)	210 Gpa
Young’s modulus (B)	0.72 Gpa
Mass density (A)	7800 kg/m ³
Mass density (B)	935 kg/m ³
Loss factor (A) and (B)	0.001

Table 2: Geometric parameters and material properties of binary periodic beam

For the numerical formulation the two-node beam element with two degrees of freedom per node is considered. The mass and stiffness matrices of the beam are as follows:

$$K_{beam} = \frac{EI}{d^3} \begin{bmatrix} 16 & 6d & -12 & 6d \\ 6d & 4d^2 & -6d & 2d^2 \\ -12 & -6d & 12 & -6d \\ 6d & 2d^2 & -6d & 4d^2 \end{bmatrix}$$

$$M_{beam} = \frac{\rho S d}{420} \begin{bmatrix} 156 & 22d & 54 & -13d \\ 22d & 4d^2 & 13d & -3d^2 \\ 54 & -6d & 156 & -22d \\ -13d & -3d^2 & -22d & 4d^2 \end{bmatrix}$$

The frequency range of computation is 1-1500 Hz. Considering the uncertainty in the Young’s modulus of the binary periodic beam with four percent about the nominal value, the numerical experiment is performed with SWFEM formulation. To validate the obtained result, MCS study is carried out with 10000 samples.

Fig. 6 shows the comparison of the mean value of the wave number obtained from SWFEM and MCS on the WFEM simulation. In Fig.7, the standard deviation comparison is shown from SWFEM and MCS results. The results are in good agreement which confirmed the validity of the SWFEM formulation applied to bending wave analysis.

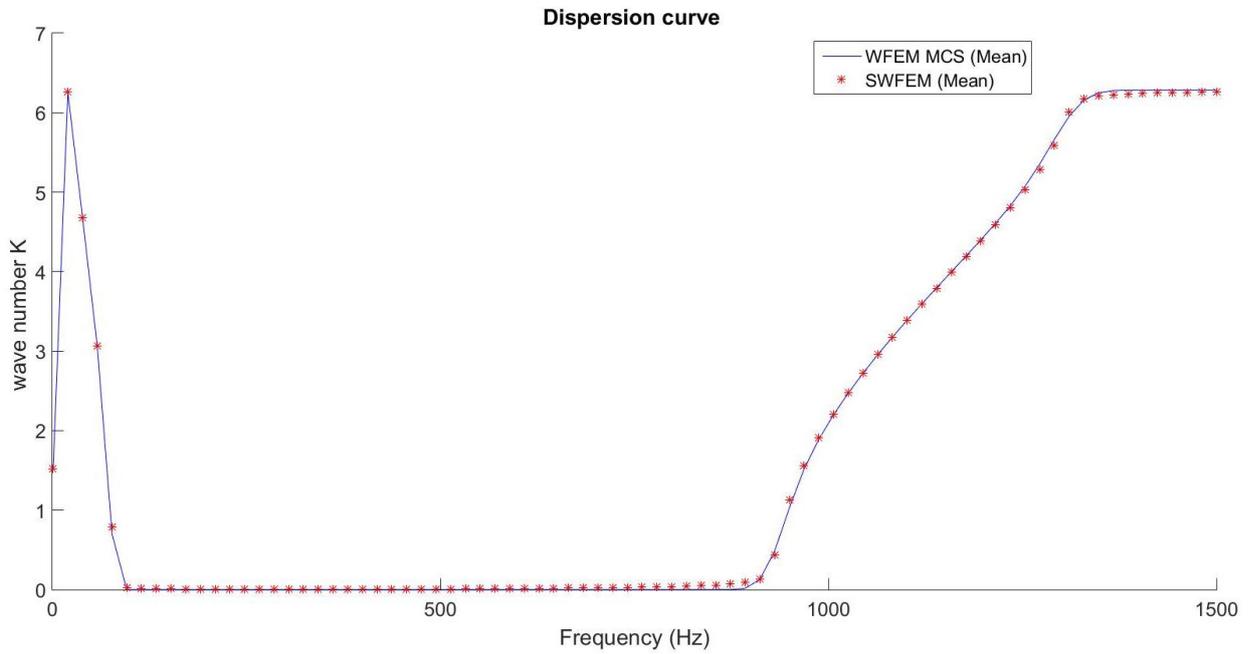


Figure 6: Comparison of mean of wave number

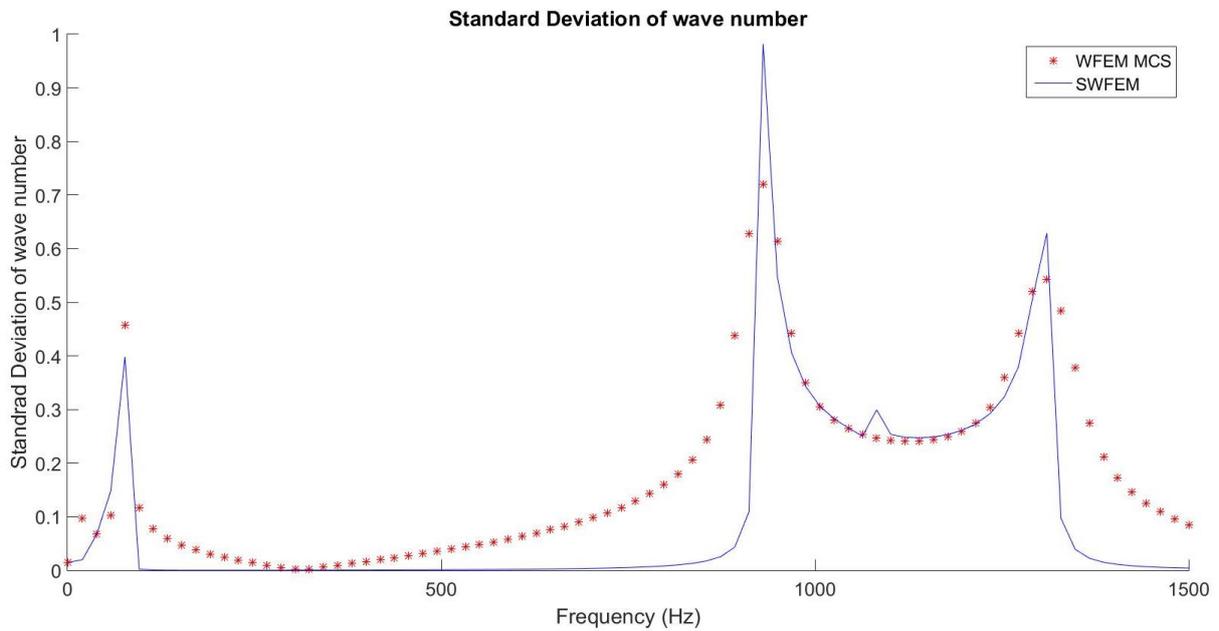


Figure 7: Comparison of the standard deviation

For the elapsed time computation, the MCS result obtained using homemade WFEM code which exploits the resources of the workstation with following characteristics, Intel® core™i7 7820 HQ CPU@2.90GHz with 32 GB RAM and data storage using solid state drive. The comparison is presented in the Table 3.

	WFEM MCS (10000 Sample)	SWFEM (single run)
Elapse time (Seconds)	3840	9.56

Table 3: Elapsed time comparison

4 Conclusion

This paper provides a computationally inexpensive stochastic numerical approach to study the uncertainty effect on free vibration of the 1D periodic structures. This formulation allows wave characteristic to be defined by stochastic finite element model using parametric probabilistic approach and which is easy to implement as it considers finite element model of the sample structural element. The SWFEM shows that the wave characteristic of the longitudinal and flexural wave can be obtained precisely. Main conclusion can be derived as follows:

- (1) The formulation of the SWFEM is applied for the binary periodic rod and beam.
- (2) Analytical sampling and numerical result showed the effectiveness of the formulation to predict the mean and standard deviation of the wave propagation.
- (3) The SWFEM has computational advantage over MCS results.

The saving the computational cost can be a good point for the optimization and reliability study under uncertainties of complex waveguides. The statistics of the response can also be useful for the damage detection and sensitivity analysis. Future investigation is under progress for the forced response computation, using the present formulation.

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