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► To cite this version:

S Timorian, S de Rosa, F Franco, M. Ouisse, N Bouhaddi. Wave and finite element analysis of quasi-periodic 2D phononic super-cell crystals. Medyna 2020: 3rd Euro-Mediterranean Conference on Structural Dynamics and Vibroacoustics, Jan 2020, Naples, Italy. hal-02415002

HAL Id: hal-02415002 https://hal.science/hal-02415002

Submitted on 16 Dec 2019

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WAVE AND FINITE ELEMENT ANALYSIS OF QUASI PERIODIC 2D PHONONIC SUPER-CELL CRYSTALS

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ABSTRACT

Phononic Crystals (PCs) are artificial materials with extensive application in the aerospace and aeronautical industries since they introduce frequency stop-bands (frequency ranges where wave cannot freely propagates). In this investigation 2D periodic and quasi-periodic pattern: instead of strictly repeating a pattern, a certain degree of irregularity is introduced. Quasi-periodic PCs in this paper are defined as assemblies of two different elements in two direction by using Thue-Morse and semi-Sierpinski carpet which results in asymmetry in both directions. Numerical studies on periodic and quasi-periodic lattices are performed. The spectral analysis in periodic waveguide is validated with the dispersion diagram of a reference model and vibration transmission of the lattices are studied. The wave characteristics in quasi-periodic super-cell configuration increases the wide of band-gaps about 22% compared to the reference model. Nevertheless, the associated level of vibration in the forced response of quasi-periodic lattices are predicted.

1 INTRODUCTION

The very first findings of PCs dates back to Sir Lord Isaac Newton who imagined that sound waves propagated through air in the same way that an elastic wave would propagate along a lattice of point masses connected by springs with an elastic force constant. Recent literature study shows theoretical, numerical and experimental works based on vibration reduction and acoustic transmission loss of small and large scale elastic structures using 2D PCs. The existence of band-gaps in PCs was predicted theoretically by [1]. There are two types of band-gap mechanism: the first one is Bragg scattering, this type of band-gaps can be shaped by the elastic wave reflection during the incidence with the scatterer of the neighbouring element which operates as a perfect non-absorbing mirror. The second one is local resonances mechanism that can provide a low frequency band-gap due to the coupling of two oscillators, say, the frequency is not dictated by the spacing of the periodic arrays but by the frequency of the local oscillator. This property could be an asset in plenty

of applications, from vibration filtering to acoustic shielding and from seismology to the dynamic behaviour of space stations. In this investigation quasi-periodic 2D PCs is considered for passive broad band-gaps analysis. The PCs which is embedded in a super-cell is composed of a periodic combination proposed by [2]. The model is optimised by introducing a small degree of irregularity by using Thue-Morse and semi-Sierpinski carpet. The proposed model has same mass as in the reference [2], but with different filling rates of resonator and different cylindrical radius. Dispersion analysis of infinite PCs and forced response of finite panels are analysed. The content of this paper is structured as follows: in section 2 quasi-periodic lexicon is presented. Section 3 introduces results of the infinite super-cells and finite forced response. Closing section brings the conclusion of this study.

2 QUASI-PERIODIC LEXICON

The quasi-periodic model is defined by a small degree of irregularity in the PCs super-cells and finite panels [3]. Two unit cells are modelled for wave finite element analysis, beside that two finite periodic and one quasi-periodic panels are modelled for forced response analysis shown in Fig. 1

2.1 Thue-morse sequence

The Thue-Morse sequence is based on binary arrays. Starting from a 1D example, the sequence can be derived by a function s from the set of binary sequences to itself by replacing every A in a 1D sequence with AB and every B with BA [4]:

$$s(x,y) = s(x)s(y) \tag{1}$$

Eq.1 defines a map s for all strings x, y. Defining the Thue-Morse morphism s(A) = AB, leads to the relations for increasing the quasi-periodicity pattern in one direction. The associated sequence starts with A then AB, ABBA, ABBABAAB, and so on, [5].

2.2 Semi-Sierpinski carpet PCs

Sierpinski carpet has a technique of subdividing a shape into 9 smaller copies of itself by removing one copy from the middle, and continuing recursively for the rest of copies [6]. In the present model Semi-Sierpinski carpet is used to create PCs with local resonators.

2.3 Super-cell

The super-cell is made of 25 cells including square matrices with circular scatters, in which 9 of the circles of these square unit cells are filled with metal scatters and the rest are hollow air holes all around the super-cell. Cell-A and cell-B are designed in two different cases. In case 1 (quasiperiodic 1) the Thue-Morse sequence is applied by considering cell-A (r = 5 mm), cell-B (r = 3 mm), cell-C (r = 5.5 mm), and cell-D (r = 2.5 mm). The 9 middle cells are associated to Semi-Sierpinski carpet model and the other circles with the air holes follows Thue-Morse sequence (Fig. 1). The semi-Sierpinski carpet model is adapted to the variation of volume fraction of resonance material by reducing the inclusion (45 steel) of cell-A and at the same time increasing the inclusion (45 steel) of cell-B, using different circular radius in order to keep the same mass as the reference model. On the contrary, case 2 (quasi-periodic 2) deals with a super-lattice of cell-A and cell-B (r = 4 mm) as in the reference model but the position of coordinate of the circles in the entire lattice is changed with only two combined elements, using Thue-Morse sequence. For instance the coordinate of the constant cell-B is at base position $p(\vec{i}, \vec{j})$, in this method unit cell-A is displaced to $\rho(2\vec{i}, -2\vec{j})$. In this case cell-A and B is replicated by Thue-morse sequence in two directions (Fig. 1). Material properties are introduced in the Tab. 1

3 RESULTS OF THE INFINITE SUPER-CELLS AND FINITE FORCED RESPONSE

The dispersion diagram of the super cell quasi-periodic 1 in Fig. 2 shows a complete stop-band effects in the ranges of 139 - 208 Hz compared to the reference model 153 - 196 Hz [2], which is 22% larger than the reference model shown in the Fig. 2. On the contrary, the quasi-periodic 2



Figure 1: From the right, quasi-periodic 1 and quasi-periodic 2 configurations are graphically presented, in the left, first Brillouin zone reduced by all symmetries in the point group of the lattice

Material type	Modulus of elasticity (Pa)	Poissons Ratio	Density $(kg \cdot m^{-3})$
Matrix (vulcanized rubber)	21.06×10^{10}	0.26	7780
Scatter (45 steel)	1×10^{6}	0.47	1300

Table 1. Physical parameters



Figure 2. Dispersion diagram of periodic and quasi-periodic PCs

has less effects at around 4% increment compared to the strictly periodic model. The finite panels are made of 2×2 super-cells. In periodic pattern a total of four super-cells exist, whereas for the quasi-periodic panel 2 periodic super cells and 2 quasi-periodic super cells are combined (Fig. 3). Forced response of the panels are computed by applying fixed constraint boundary conditions in the upper and lower sides of the panels (Fig. 3) and a unit boundary load is applied with the frequency range of 0 - 300 Hz with a bandwidth of 1 Hz. The vibration transmission is observed as $TL = 20log(a_1/a_2)$. The response of finite panels give four types of band-gaps (BG1, BG2, BG3, and BG4), in BG1 the response is the same almost for periodic B and quasi-periodic cases. It is the largest one for both cases but possesses a localised mode in the last part of the attenuation level. In BG2 the band-gap is very narrow, and it is only in periodic 2, while BG3 is narrower compared to BG2 and it is in periodic 2 and quasi-periodic case. BG4 is almost two times of BG3 and it is in quasi-periodic with a great fall and partially it could be seen in periodic 2 too. In conclusion periodic B has more band-gaps following quasi-periodic case.



Figure 3. Vibration transmission of two types of periodic and one quasi-periodic panel

4 CONCLUSION

Dispersion diagram of quasi-periodic super-cell enlarges the wide of band-gap more than 22% in lower frequency range 0-300 Hz compared to the reference model. Quasi-periodic 2 configuration has less effects. Despite its low effects, it gives a good prediction of the associated level of vibration. Finite panels verify the predicted band-gaps for quasi-periodic cases. Good dynamic behaviour is seen in periodic 2 and quasi-periodic with wider band-gaps and extra stop bands above 250 Hz.

ACKNOWLEDGEMENTS

The work is carried out in the framework of the VIPER project (VIbroacoustic of PERiodic media). The project has received funding from the European Union's Horizon 2020 research and innovation program under Marie Curie grant agreement No 675441 and EUR EIPHI (ANR 17-EURE-0002) project.

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