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BAND DIAGRAM AND FORCED RESPONSE ANALYSIS OF PERIODIC AND QUASI-PERIODIC PANELS

S. Timorian^{1,2}, M. Ouisse², N. Bouhaddi², S. De Rosa¹ and F. Franco¹

¹ Laboratory for Promoting experience in Aeronautics Structures,
Department of Industrial Engineering —Aerospace Section, University of Naples Federico II, Napoli,
via Claudio 21, 80125, Italy.
e-mail: safiullah.timorian@unina.it,
web page: <http://www.pastalab.unina.it/home.html>

² Femto-ST Institute, Department of Applied Mechanics,
Univ Bourgogne Franche-Comté, Besancon, 25000, France
e-mail: safiullah.timorian@femto-st.fr
Web page: <https://www.femto-st.fr/fr>

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Abstract. The vast majority of structures introduce symmetry in one or two dimensions, which can be a periodicity in the case of constructing several identical unit cells. Periodic structures have found a big interest in engineering applications because they introduce frequency band effects, due to the impedance mismatch generated by periodic discontinuity in the geometry, which can improve the vibroacoustic performances. However, the presence of imperfections i.e. (defects, irregularity) in the structure, leads to a partial loss of regular periodicity (called quasi-periodic structure) that can have a noticeable impact on the vibrational and/or acoustic behaviour of the elastic system. The tailored irregularity, which is a designed loss of periodicity, can be used, therefore, to improve the dynamic behaviour of structures. In the present paper numerical studies on the vibrational analysis of two-dimensional non-planar periodic and quasi-periodic structures have been performed. The content deals with finite and infinite systems. The finite element models of solid structures focused on the band diagram analysis of the infinite systems and the forced responses of the finite structures. The quasi-periodicity is defined by invoking the Thue-Morse sequence for building the assigned variations (geometry) along the domain of finite element model. The wave characteristics in quasi-periodic panels, present some elements of novelty and could be considered for designing structural filters and controlling the properties of elastic waves.

1 INTRODUCTION

Periodic metamaterials are one of the central topics in vibration and acoustics domain. A degree of regular periodicity in arrays of engineered metamaterials, always leads to design and create frequency stop bands (i.e. waves cannot freely propagate in these frequency ranges). One of the main features associated with the periodic distribution of mechanical devices is the ability of the system to block the propagation of waves in specific frequency ranges [1]. Such structures behave as filtering systems, since in some ranges of frequencies waves propagate without attenuation (if damping is neglected), while in

others decay exponentially [2, 3]. They are also very well known metamaterials and phononic materials for exhibiting frequency band gaps, that can improve the vibroacoustic response of structures [4, 5]. Recent studies show interesting topics on architected material like phononic crystals, metamaterials, and auxetic sandwich cores [6] to control the propagation of elastic waves. These architected material rely on two mechanisms for forming band gaps. Phononic crystals can form Bragg type band gaps, but are limited in the low-frequency regimes because their unit cell size scales with wavelength. In contrary metamaterials overcome this size, because they rely on resonances, but the resulting band gaps are narrow [7, 8].

Numerical analyses lead to the design of nominally perfect periodic structures. However, the presence of imperfections i.e. defects or irregularities in post-manufacturing are highly likely to be occurred in regular periodic designs. Recent studies show that there are effects on random variability due to the additive manufacturing process on the performance of the elastic band gap behaviour in creating a stop band effect in the forced response of the waveguide [9]. On the contrary, the present work is focused on tailored irregularities, which is called (quasi-periodicity), which are considered to have impacts on dynamical behaviour of the structures. The first challenge is how to predict the frequency band gap in infinite quasi-periodic systems. In fact, the design of periodic configurations is usually based on the use of Floquet-Bloch (FB) theorem which considers a single cell with adequate boundary conditions[10, 11].

To design an enhanced quasi-periodic structure and analyse the impact of this design in terms of dynamics response, two methods are under investigation. A forced response of the finite quasi-periodic structure and a periodic panel which has commonality in terms of in-plane dimensions. In the second method in order to predict the frequency stop band zones a dispersion analysis of the infinite periodic structure is designed. In this design two meso-scale unit cells, which represent the dispersion analysis of the infinite periodic structures are considered. The predicted dispersion curves can be directly compared with its finite periodic counter parts for frequency band gaps prediction. In the forced response, those two meso-scale cells are combined using Thue-Morse morphism series in order to tailor a quasi-periodic panel. The FRFs of the quasi-periodic panel show effective results compared to strictly periodic panels in terms of band gaps width enlargement and level of attenuation.

2 MODEL AND LEXICON

2.1 Infinite structure

The model consists of a metamaterial waveguide. Metamaterials in the present investigation are an engineered sets of multiple elements to show some filtering properties. The waveguide consists of two different unit cells in meso-scale. The unit cells are made of polymers materials and the mechanical properties are given in Tab.1.

The unit cell is made of two sub-parts, on-board panel, and a resonator. The dimension of the meso-scale wave-guide is detailed in the Fig.1. The information of the control volume of the both unit cells are taken into account for modelling a lightweight, easy-to-manufacture metamaterial [12]. The on-board panel has a thickness of $2mm$ with an area of $35 \times 35 mm^2$. The resonator shape has a height of $17.5mm$ and a lateral and horizontal span of $30mm$. Cell A and cell B have two different shapes with approximated equal control volumes. The wall thickness of the cell A is $4.8mm$ and the circular hole in cell B has a radius of $8mm$.

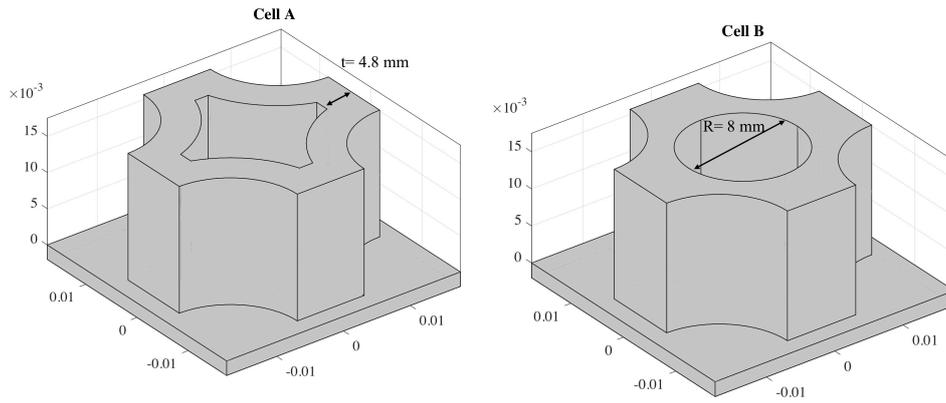


Figure 1: Meso-scale unit cells of infinite periodic panels

Table 1: Material properties of the metamaterial

Material	Polymer
Modulus of elasticity(Pa)	2.32×10^9
Density ($\text{kg} \cdot \text{m}^{-3}$)	1050
Poisson's ratio	0.34

2.2 Finite structure

In the finite panel two perfectly periodic and two quasi-periodic structure are considered. In the last two models specific Thue-Morse sequences are used to generate impedance mismatches in the view of desired degree of quasi-periodicity. The degree of quasi-periodicity might be controlled with mathematical rules that are introduced in the next section.

2.2.1 Thue-Morse sequence

A Thue-Morse sequence is binary sequence (an infinite sequence of 0s and 1s). The sequence starts with 0 then 01, 0110, 01101001, and so on, [13]. For the simplicity, let us define the Thue-Morse morphism to be the function s from the set of binary sequences to itself by replicating every 0 in a sequence with 01 and every 1 with 10 [14]:

$$s(x, y) = s(x)s(y) \quad (1)$$

Eq.1 defines a map s for all strings x, y . Defining the Thue-Morse morphism $s(0) = 01, s(1) = 10$ leads to the relations for increasing the quasi-periodicity pattern in two directions:

$$\begin{aligned} s(0) &= 01 = AB \\ s^2(0) &= s(s(0)) = 0110 = ABBA \end{aligned}$$

$$s^3(0) = s(s(s(0))) = 01101001 = \text{ABBABAAB}$$

Each 0 and 1 represents cell A and cell B respectively. In the present finite element model only 8 combined unit cells are used, it means that the Thue-Morse morphism map stops at s^3 .

Two types of quasi-periodic finite panels are considered. The first one is a pure quasi-periodic panel (8×8 unit cells) and the second quasi-periodic panel is designed similar to the first one with an addition of two bare panels without resonators in the right and left sides of the quasi-periodic one. These two bare panels are used as an input zone in the left and output zone in the right side of filtering domain shown in Fig.2.

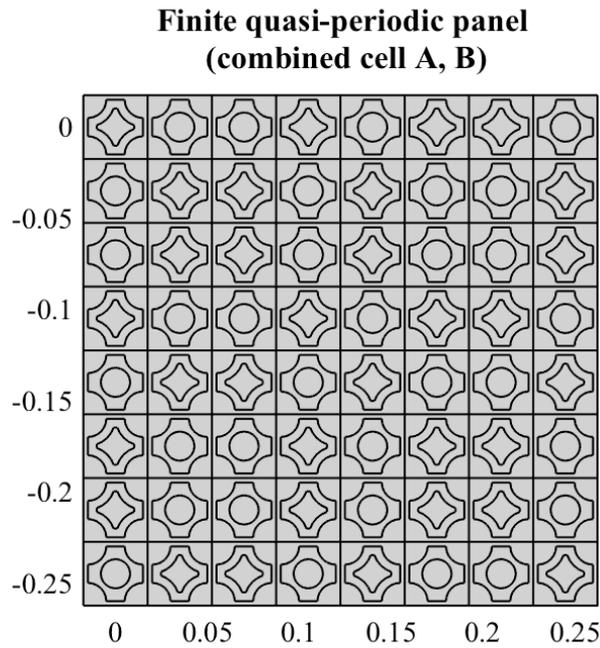


Figure 2: Quasi-periodic panel with the combination of cell A and B

2.2.2 Periodic sequence

The two periodic panels have a repetition of single cell A for first panel, and single cell B for second panel respectively with the dimensions of 8×8 shown in Fig.3. According to the Fig.3, the panel has a thickness of 2 mm, and the hollow resonator has a wall thickness of 4.8 mm.

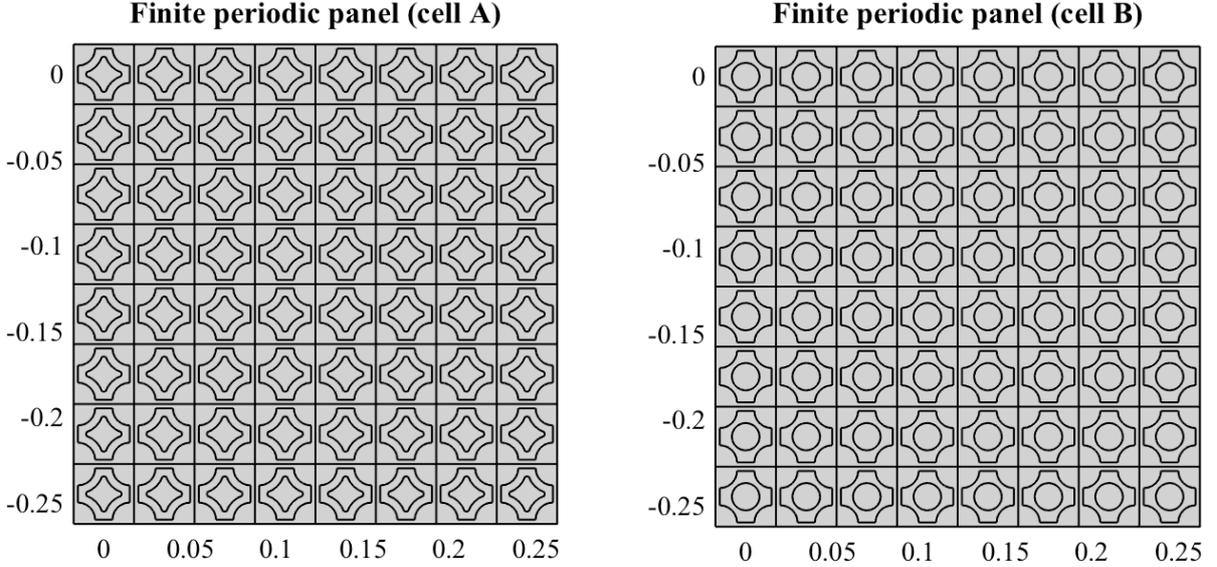


Figure 3: Two strictly periodic panels, left with cell A periodicity and right with cell B

3 STRUCTURAL DYNAMICS

3.1 Dispersion analysis

Classical FB method is used for the band diagram analysis of the strictly periodic infinite panel. The Floquet-periodicity is defined in the four side boundaries of the unit cell. The unit cell represents a bi-directional spatial periodicity in the given infinite structure. The band diagram is computed along the three symmetry directions of the first Brillouin zone shown in Fig. 4. Classical (FB) Floquet-Bloch theory is used as a periodic boundary condition using COMSOL Multiphysics for the analysis of dispersion curves. The periodicity boundary conditions in four sides of the lattice are

$$\mu_R = e^{-jk_x L} \mu_L \quad (2)$$

$$\mu_U = e^{-jk_y L} \mu_B \quad (3)$$

where, μ_R and μ_L are the displacement in the right and left side of the x axis and μ_U and μ_B are the displacement in the upper and bottom of the y axis. L is the length of reciprocal element, k_x and k_y are the wavenumbers in the x and y directions. Band diagrams are analysed by setting up eigensolver analysis as a parametric sweep using one parameter, k , which varies from 0 to π . The range of the k parameter spans three wave vectors starting from (A to R to S and back to A), covering the edges of the first irreducible Brillouin zone Fig. 4.

Two different unit cells are considered for the dispersion analysis. First cell A and second cell B. Both cells have commonalities in terms of mass, volume fraction, material and structural properties in order to be useful periodic tool to be compared with the quasi-periodic counter part. The results predicted by these two cells could lead to design a quasi-periodic lattice, which is a finite panel with a tailored irregularity that is invoked by Thue-Morse morphism deterministic approach. The results of the band diagram is plotted in Fig. 5 There are two complete frequency band gaps in each sub-figure. The first band gaps with small $\Delta\omega = 570$ Hz that represents the (resonance band gaps) following the larger Bragg

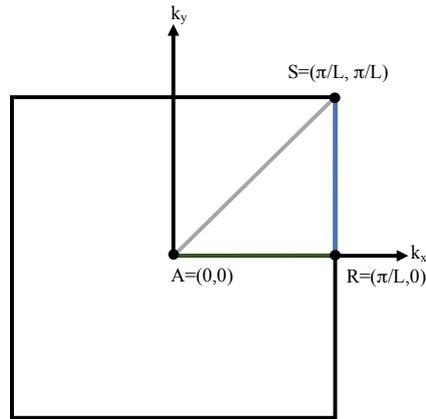


Figure 4: Diagram of the first irreducible Brillouin zone for a square lattice

band gaps $\Delta\omega = 2.9$ kHz around 11 kHz is predicted for the cell A. Cell B in the right hand side of the Fig.5 shows almost similar dispersion behaviour but with a wider resonance band gaps $\Delta\omega = 754$ Hz around 4 kHz and a slight wider Bragg band gaps 3.1kHz compared to cell A. According to a sensitivity analysis on both cells, the compromised wall thickness of the cell A = 4.8 mm and radius of the resonator shape cell B = 8 mm is selected to isolate the maximum vibration level.

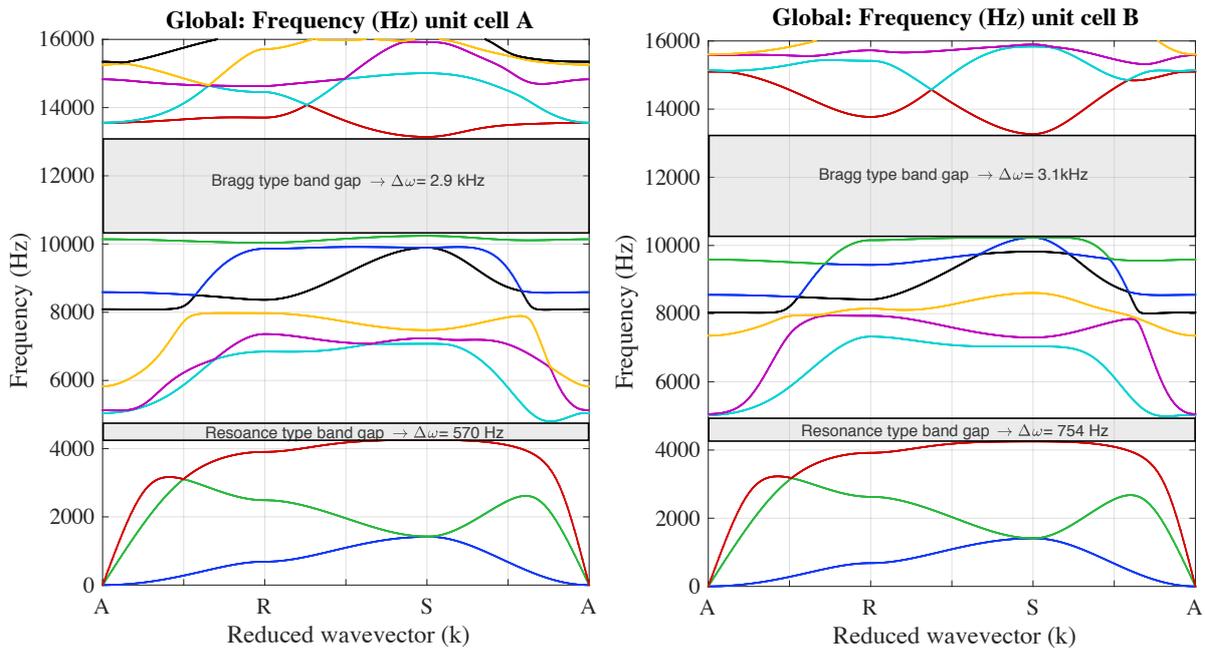


Figure 5: Band diagram of infinite panels with a representative, left cell A and right cell B

3.2 Finite element analysis

In this part, four types of investigations, on periodic and quasi-periodic panels; specifically, the aim is to apply on a finite structure the results obtained with a infinite periodic sequence.

Fig. 6 show three curves; by focusing on the Bragg type band gaps zone, the blue curve with dash lines corresponds to the cell B. It has thinner stop band wide compared to the other curves and there is also a slight shift in the stop bands towards the higher frequencies compared to the two cell A and combined (A, B). Despite of that cell A has a shift of the same band gaps zone towards the lower frequency ranges. Paying attention to the dark green solid curve that corresponds to the combined cell A and cell B, it has wider band gaps compared to both blue dash curve and dotted orange curve almost around 10.5 – 14.2 kHz. Also it should be mentioned that in the velocity amplitude response it has the lowest pick in comparison of two other curves. The highest picks could be seen at around 8.25 kHz, and the orange has the highest pick with the amplitude of $20dB$.

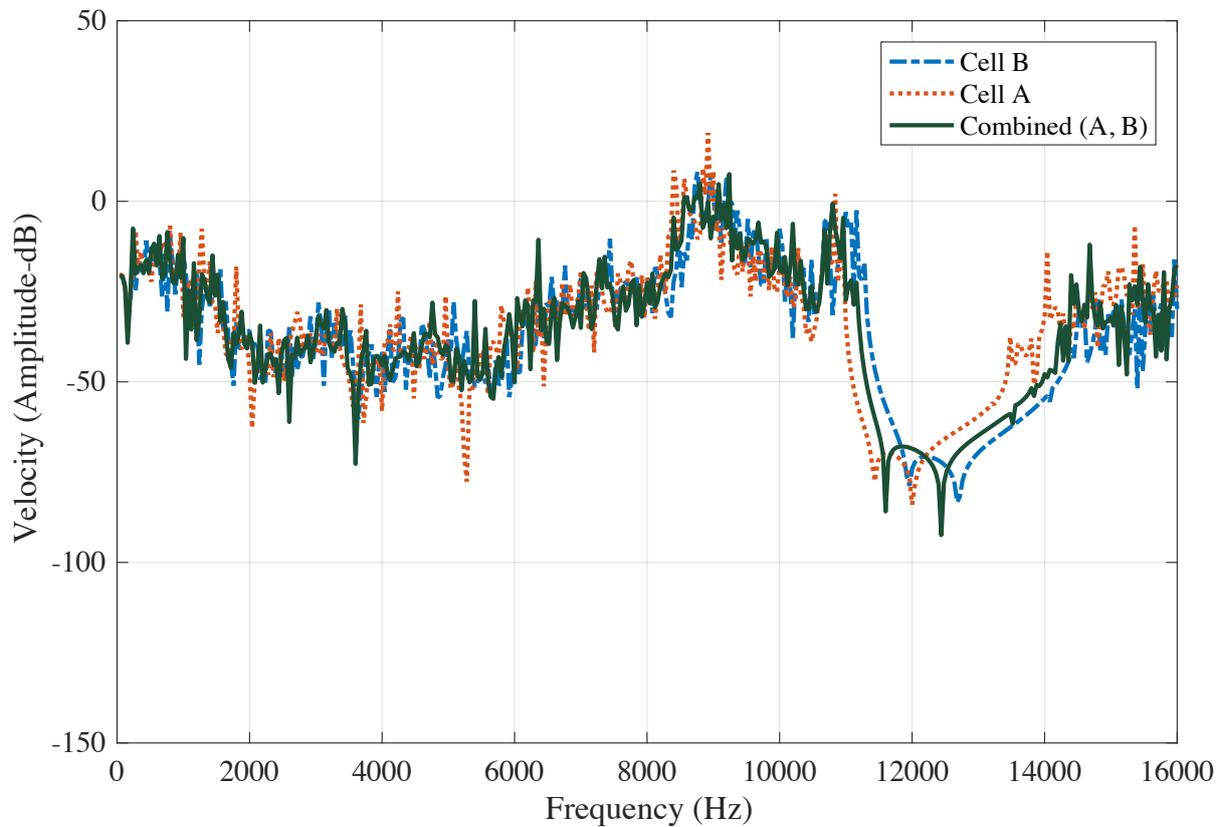


Figure 6: FRFs of three finite structures in terms of velocity amplitude in dB response

In conclusion of the first part, it could be predicted that the combination of two unit cells in finite quasi-periodic panel (8×8 unit cells) is a compromise model and it has impacts on the dynamic behaviour of panel compared to the two other strictly single periodic panels in terms of wide of band gaps and lower attenuation level.

In the second part of the finite element analysis, an FRF of the quasi-periodic structure with ($840 \times 280 \times 2 \text{ mm}^3$) is taken into account. This meta-structure shown in Fig.7 is made of three sub-parts. The middle part is a quasi-periodic panel of combined cell A and cell B and the two other parts in the left and right hand side bare panels with no resonators on it.

The panel has a cantilever constraint (fixed boundary condition) marked blue in Fig.7. A unit white spectrum force is applied to the input zone right left hand side bare panel (marked with red arrow) and the response is observed in terms of Root Mean Square (RMS) with the average node points in the input and output zones.

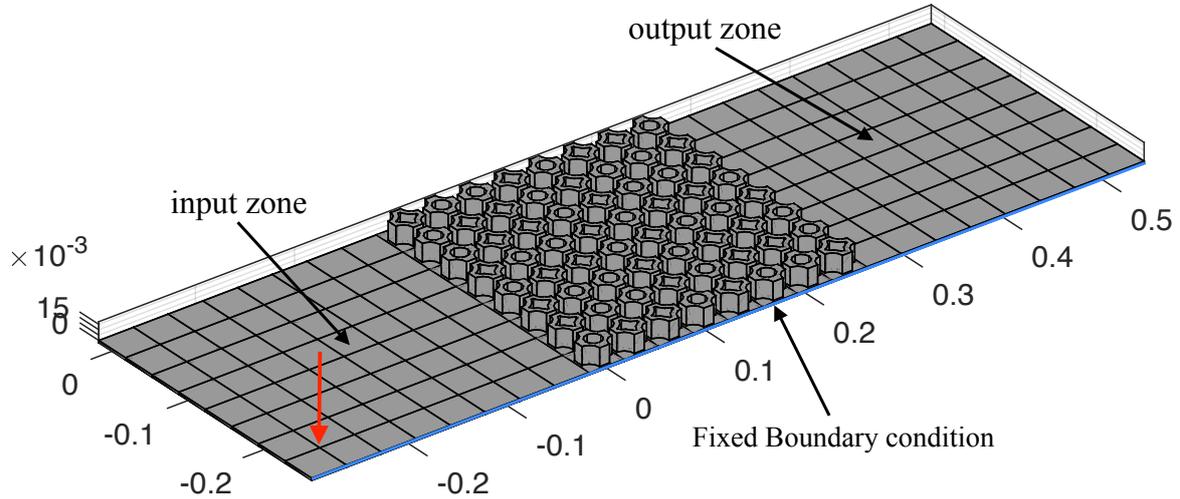


Figure 7: Geometry of meta-structure (filter)

Fig.8 shows FRF response in terms of velocity amplitude in dB. Practically, this type of panel is used as a junction filter between two bare panels. The induced vibration energy transfers through this junction, and it acts as a meta-material filtering property to the elastic waves due to the impedance mismatch in the shape of resonators embedded in the bare panels.

The FRF shows two types of curves, the blue one corresponds to the the average of the 64 nodes in the input zone, where all the elastic waves are confined in the zone and there is no sign of frequency stop bands. The orange curve corresponds to the response of the output zone. The frequency stop bands starts from 10.36 kHz and continues to 13.6 kHz. There are some picks around 12 kHz inside the band gap in shaded area. Three frequencies are chosen inside the shaded area and their operative modes or forced deflections are plotted in Fig. 9.

A similar procedure for the numerical modelling of filtering effects has been done in the paper of Billon et al [10]. They basically selected 4 points in the FRFs. Two points are located in the inside the predicted band gap and two outside the band gap. One point inside the band gap does not really reflects good response despite of being in the predicted band gaps. In fact, in the present FRFs only three points are considered, two of them are on the lower and upper bound of the band gaps and the third one is taken in the middle of the attenuation zone.

As it can be seen from the results in the paper of Billon et al [10], the average squared amplitude velocity around 43.3 kHz which is inside the predicted band gap zone, has visible deformed shapes over $-20dB$. In contrary, although there are some picks in the middle of band gap in the present result but they are bellow $-25dB$ with less deflections.

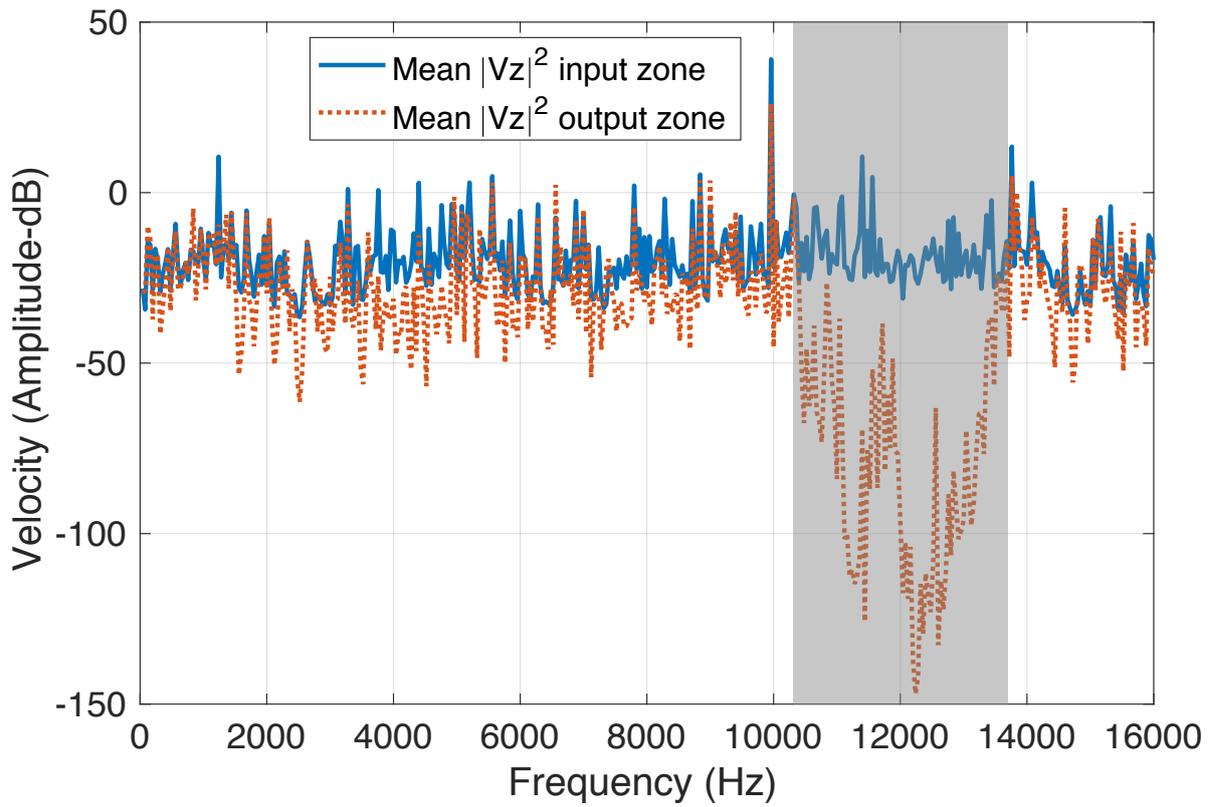


Figure 8: FRFs of the meta-structure in terms of RMS of velocity amplitude in dB

The operative modes in Fig.9 shows the deformation of the panel in 10.36 kHz, the start point of band gaps, 11.36 kHz middle of the band gaps, and 13.12 kHz almost at the finishing point of the band gaps. It can be seen that the panel has an efficient filtering effects by using quasi-periodic pattern.

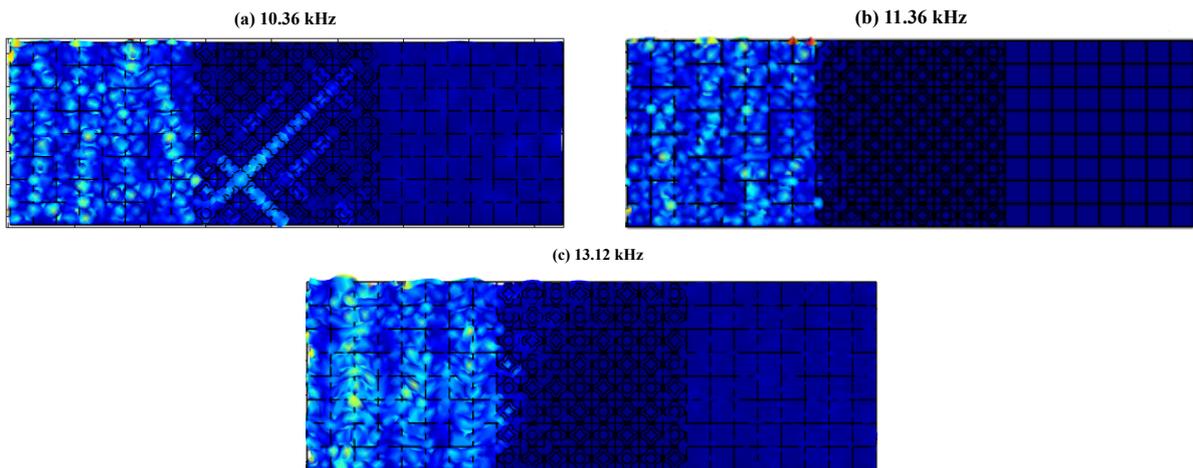


Figure 9: Operative modes of the meta-structure with three selected frequencies in the upper, lower and middle position of the attenuation level

4 CONCLUSIONS

The first aspect is formulating fundamental procedures to design a quasi-periodic meta-structure using Thue-Morse morphism sequence. The second aspect is dedicated to analysis of the impact of this design in terms of dynamical behaviour. The results of FRFs and dispersion analysis shows that quasi-periodic pattern have impact on dynamic behaviour of panels, especially in terms of isolating unwanted vibrational behaviour all over the metamaterial part of the span, specifically inside the attenuation level.

In this analysis, the presented tailored irregularity, which in higher extent is a designed loss of periodicity in finite structure has influential access to extend the attenuation level to lower velocity amplitude-dB and slightly increase the wide of the band gaps. These impact came due to the impedance mismatch of the combined (resonators) embedded on the bare panel. There are some points to be highlighted in the below:

The quasi-periodicity (Thue-Morse morphism) in two directions shows lower frequency band gaps compared to the one direction ones like Fibonacci sequences. Quasi-periodic panel in the present article has an efficient impacts on the vibrational analysis compared to the single periodic panels. In general quasi-periodic pattern is capable of confining the energy in some modes, and in higher extends there are possibilities to shift those localised modes in desired frequency bands.

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