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Preschool children's understanding of length and area measurement in Japan

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This study assesses preschool children's understanding of length and area measurement through the activities of a mathematics program in Japan. Japanese preschool children do not formally learn mathematics; thus, we designed mathematical measurement activities for young, less-experienced preschool teachers to implement for nine five- to six-year-old children. We conducted structured clinical interviews with the children individually, both before and after the measurement activities, and qualitatively analyzed the results by comparing their answers and connecting them to the activity contents. Results indicate that children learn to understand direct comparison through activities; however, it is more difficult to establish understanding of measurement by non-universal units.

Keywords: Preschool children, mathematical activity, length measurement, area measurement

Research on measurement in preschool

Much of the current research focuses on elementary mathematics education for preschool children and younger elementary school students (e.g., Brandt, 2013); however, measurement has not been sufficiently included in these studies (Sarama, Clements, Barrett, Van Dine, & McDonel, 2011; Smith, van den Heuvel-Panhuizen, & Teppo, 2011), even though it is an important real-world area of mathematics used in everyday life. In addition, Hachey (2013) states that mathematical conceptual change in young children is as effective as a substantial change in early childhood mathematical teaching practice, as a large body of developmental research advocates that young children are born mathematicians, and therefore, early childhood mathematics education is vital.

It is known that the development of children's understanding is related to foundational or key ideas of measurement such as comparison, unit iteration, number assignment, and proportionality (Lehrer, Jaslow, & Curtis, 2003; Wilson & Osborne, 1992), and several kinds of teaching methods for measurement have been developed (Battista, 2006; Kamii, 2006). Although Piaget (1968) assumed that children up to seven years old are unable to consider more than one stimulus dimension in their judgments, subsequent research has demonstrated that preschoolers can consider two dimensions; for example, they can consider the width and length of rectangles to estimate their area (Wilkening, 1979). Further, Ebersbach (2009) addressed the question of whether children can also take three stimulus dimensions into account and showed that preschoolers have the cognitive competencies required for multidimensional reasoning. There have also been numerous other important studies on preschool children's understanding of measurement. Sarama et al. (2011) proposed a learning trajectory for length and area in the early years and evaluated it. Zöllner and Benz (2013) concluded that four- to six-year-old children could compare directly and indirectly, but cannot measure using a non-standardized unit. Tzekaki (2017) indicated that a seven-month intervention helped preschoolers improve their abilities to reflect on their own activities and express their ideas regarding measurement. Skoumpourdi (2015) verified that preschoolers do have some strategies for length

measurement, but confuse the concept of length with perimeter and area. Finally, Kotsopoulos (2015) showed that a free exploration approach in the context of a play-based learning environment is more effective in teaching length measurement than guided instruction and center-based learning.

This article examines whether preschool children can improve their understanding of length and area measurement, especially direct and indirect measurement, as well as measurement by non-universal units. It does so with an elementary mathematics play-based program, which can even be implemented by young, less-experienced teachers who have not been trained in mathematics instruction, thereby fostering children's understanding of measurement. These play-based activities are intended as a further contribution to the existing research. The article also seeks to explain the specific reasons the program is effective. First, we compare pre- and post-interview results for five- to six-year-old children. Next, we intentionally choose two children with different degrees of change between their pre- and post-interview results. Finally, we discuss the reason the change occurred, scrutinizing each child's actions and attitude during the activities, as well as his/her post-interview.

The early childhood mathematics program in Japan

Elsewhere, we have proposed a framework for constructing an early childhood mathematical curriculum (Matsuo, 2016). The theoretical and methodological underpinnings of our program are social constructivism (Ernest, 1994), mathematical guided-play (Weisberg et al., 2013), and the Structure of the Observed Learning Outcome (SOLO) taxonomy (Biggs, 1982). This program presents a scope for the content of preschoolers' activities, while the learning process of those activities are explained from the viewpoint of sequence, based on the relationship between age and modes of representation and focusing on activities for five- to six-year-old children to create a program tailored to these concerns. Based on this framework, we have proposed a mathematical education program to enable a smooth transition from preschool to elementary school mathematical education (Matsuo, 2017). Less-experienced preschool teachers should be able to use these activities to better incorporate mathematical content into children's play and recognize activities that will make children think and act mathematically. This practical program was designed for preschools and elementary schools.

Methodology

Research participants

The preschool kindergarten in an urban area of Kanagawa Prefecture of Japan participated in our pilot study. This article focuses on nine five- to six-year-old children in the oldest classes. The children are all Japanese, coming from middle- and upper-middle-class families in Japan. There were six boys and three girls. The school's focus was on physical activities, as well as entrance to candidate schools for the International Baccalaureate (IB). Children started to learn Japanese letters and English to prepare themselves for primary education in September 2017.

The oldest classes of the preschool had two teachers: Teacher A, a male teacher with three years of experience teaching preschool, and Teacher B, a female teacher with two years of experience teaching preschool. Teacher A taught the first three sessions of measurement activities in the first cycle of the program and Teacher B taught the latter three sessions in the second cycle. For the analysis, we intentionally chose two children, T1 and T7, based on the results of pre- and post-

interviews, to intensively examine their outcomes and learning processes, as well as to determine the relations between the interview results, their actions, and the detailed observations of the teachers during the activities. T1 was actively engaged in the activities, but provided incorrect answers in the post-interview. T7 was not actively engaged in the activities, but made correct observations and provided correct answers in the post-interview. To compare the two children’s understanding, we discuss the level of understanding of direct and indirect measurement, as well as measurement with non-universal units, in connection with the young teachers’ instruction.

Data collection and analysis

In the project, we developed six play-based activities related to numbers, measurement, and shapes. Among these activities, we carefully chose those that were better tailored to relate to numbers, shapes, and measurements, things that are necessary for our curriculum to focus on and develop. To evaluate the improvement of children’s mathematical skills after these activities, we developed pre- and post-interview questions. Pre-interviews were conducted in June 2017 and all six activities were implemented twice; therefore, twelve activities were implemented after the pre-interview. After all activities had been completed, a post-interview was conducted with each child in February 2018. Sixteen questions were developed for the pre-/post-interviews, corresponding with the planned activities and previous research by the author (Matsuo, 2016), as well as the SOLO taxonomy. Four of the sixteen questions were in the area of measurement, as shown in Table 1.



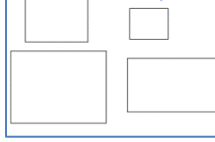

Q12	Prepare four different pencils, each of a different length, as shown in the diagram, and ask, “Which do you think is the longest pencil of the four? And tell me the order of the length, from longest to shortest.”	
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Table 1: Description of the interview questions and the problem contexts

Q13	Show four different pencils, each of slightly different length as shown in the diagram, and ask, “Which do you think is the longest pencil of the four? And tell me the order of the length, from longest to shortest.”	
Q14	Prepare four different rectangles as shown in the diagram, and ask, “What do you think is the biggest rectangle of the four? And tell me the order of the area, from biggest to smallest.”	
Q15	Prepare four different rectangles as shown in the diagram, and ask, “What do you think is the biggest rectangle of the four? And tell me the order of the area, from biggest to smallest.”	

Methodologically, the authors conducted structured clinical interviews (Goldin, 1998). To alleviate children’s anxiety, the teachers conducted the interview themselves, while one of the authors sat next to the child to record. The assigned teachers rehearsed the interviews to be consistent with mathematical words and expressions. During the implementation, each child came to the room randomly and the teacher sat in front of the child. The pre-interviews took fifteen to twenty-five minutes, and the post-interviews took ten to fifteen minutes. During the interview, the children could use a ruler, transparent paper, grid paper, and clips to answer questions 12 to 15. The results were qualitatively analyzed.

Description of measurement activities in the intervention

Teacher A taught the children the first course in how to measure the length of something carefully using clips as non-universal units. They measured assorted items by connecting clips (e.g., measuring the edge of the desk, the mat, the height of the teacher). Teacher B taught the second course, in which the children tried to connect eight clips in order to use them to measure shorter objects than the ones measured in the first course. After the children used the connected clips to measure a building block or a box, the teacher asked the children to find objects with lengths equivalent to eight clips. Next, they looked for objects with lengths equivalent to sixteen clips. They lined up sixteen red magnets and set out to measure them using the clips.

In the area measurement activity, the children employed indirect comparison and measured using non-universal units, such as length measurement. The activity was implemented like a code-breaking game, and when the code was solved, figures whose areas were objects for measurement were supposed to be arranged in descending order. A teacher directed the children to count the square grids covered in rectangular paper to measure the total area. First, the children copied the target onto plain paper, put this paper on grid paper, and counted the number of squares as non-universal units. The sizes of the three kinds of rectangles (thirty-two, forty, and forty-eight squares) were relatively large, and the children had been actively counting quite carefully. Finally, the paper with a number written on it was matched with a corresponding letter, which enabled them to solve the code. Letters with meaning were arranged in order so that the numbers became larger.

Results and Discussion

Table 2 shows that there is not a significant difference of the correctness toward questions between length and area measurement.

Child No.	Gender	Birthday	Q12		Q13		Q14		Q15	
			Pre	Post	Pre	Post	Pre	Post	Pre	Post
T1	F	MAY	1	1	0	0	1	1	1	0
T2	M	JULY	1	1	1	1	1	1	1	1
T3	M	NOVEMBER	1	1	0	1	1	1	0	1
T4	F	NOVEMBER	1	1	0	1	1	1	0	1
T5	F	AUGUST	1	1	1	0	1	1	1	0
T6	M	JUNE	1	1	0	1	1	1	1	0
T7	M	DECEMBER	0	1	0	1	0	1	0	1
T9	M	JULY	1	1	1	1	1	1	0	1

Table 2: Pre- and post-interview results for measurement of length and area

* 1 means correct, 0 means incorrect

Looking at the transition of the results from the pre- to post-interview, problems whose answers are incorrect in both pre- and post-interview cannot be seen except for T1. In the case of Q12 and Q14, children got the correct answers in the pre- and post-interview, excluding T7. Since the edges of some part of the figures in these problems are aligned, it is possible to judge the difference in size from the difference in other parts by visual observation. Children were thought to be mature, developable, and able to maintain natural demeanor. Regarding Q13 and Q15, about half of the children answered correctly in the pre-interview, but they had incorrect answers in the post-interview. T2 had correct answers for everything both pre- and post-interviews. T9 also had correct answers, except for Q15, in the pre-interview. T8, who was absent from the length and area measurement activity of the second course, was excluded from analysis.

Comprehensive analysis of eight children's understanding of measurement

Table 2 indicates that every child understood direct measurement in Q12 and Q14 in the post-interview, though in Q13 and Q15, a few children did not understand indirect measurement nor measurement by non-universal units. An erroneous view prevails that the area measurement is harder to understand than length measurement, as the number of dimensions is higher. As Ebersbach (2009) stated, not only length and area but also volume can be estimated by a young child, and the rate of correct answers does not change much between children and adults. It is possible for preschoolers to tackle activities that are not limited to length, area, and volume measurement, as we inferred from the results of our survey; however, even though children were working well on the measurement activities, most of them gave incorrect answers for area measurement.

T3, T4, and T7 answered incorrectly in the pre-interview but correctly in the post-interview for Q13 and Q15. Although they answered visually for Q13 and Q15, they were correct. We infer the program implemented had a positive effect, especially because the measurement using non-universal units is not commonly done at home or in preschools. While there were scenes in which they could not be regarded as proactively working on the program activities, in many cases, it is mentioned that they were working on collaborative and individual work well and could observe other subjects' behavior.

Conversely, T1, T5, and T6 had incorrect answers in the post-interview even if they were correct in the pre-interview, revealing that the influence exerted by the activities of the mathematics program are strongly related to this fact. Let us consider why they did not have correct answers for Q15, even if they were trying to answer based on measurements in non-universal units using tools and enjoyed it in the post-interview. It is probably because children could not distinguish between the measurement of length and area and did not associate the numerical value measured in non-universal units with the results of the comparison of length and area, or because they could not understand the meaning of the work and retain this understanding even if they acquired skills related to measurement. This can be judged from the state of the post-interview.

Qualitative analysis for two children who had different results and processes of playing

We will now compare and examine the results of two children, T7 and T1, their post-interview responses, and the teacher's findings regarding them in Table 3.

T1 was engaged in acting as a group leader in length measurement. After the clip-connecting activity, she was looking for various items to measure, like building blocks, etc. She was highly interested in the length-measurement activity. Further, in the area-measurement activity, she actively worked on counting. She did the measurement work again, but her answer was not correct. She did not seem to have an opportunity to objectively review the meaning of the work after she had concentrated on it. It can be surmised that because the meaning of the work was unclear, she confused length and area. The result of this study is consistent with previous studies (e.g., Skoumpourdi, 2015). The length and the area are different in dimension, and in daily life, for a child, the length comparison is familiar and more understandable than the area comparison.

T1	Q12	She replied visually, pointing with her finger.
	Q13	She took the fine-grid tracing paper, and after a little measuring, she answered, "Oh, I understood it."
	Q14	She replied visually.
	Q15	She laughed and took the clips first, and asked, "Can I measure them sideways?" She joined the clips, measured the length of the rectangle sideways, adjusting the length of the joined clips. Next, using the tracing sheet (with a grid), she aligned the edges and measured the length of the rectangle.
	Post-test state	She said, "I was looking forward to it today," and "It's fun to find different lengths in measuring activities."
	Teachers' findings	She is not good at detailed work. She is good at drawing. She loves playing football and moving her body. She would rather move her body than sit and work. She is interested in the alphabet.
T7	Q12	He pointed in a sequential order visually.
	Q13	He pointed in the descending order visually and quickly without using tools.
	Q14	He pointed in the descending order visually and quickly.
	Q15	He pointed in the descending order visually and quickly.
	Post-test state	He worked silently. He did not use any tools at all in the measurement, and answered quickly and visually. He enjoyed the math activities and especially enjoyed using the clips to measure the room. He did not seem to be an noteworthy type.
	Teachers' findings	He is good at fine work and studying. Currently, he is learning division and fractions progressively. He is shy and not good at speaking in front of people.

Table 3: Results of T1 and T7 for Q12-15, post-interview results and teachers' findings

Conversely, T7 did not do much work and wandered around. He did not work as actively as the other children, though he observed the other children's activities well. During area measurement, everyone was absorbed in drawing figures and counting the squares, while he also worked and looked around. It is highly likely that his reply was affected by this behavior. As a result, T7 who did not actively work, gave the correct answer by visual observation. It can be inferred that this is due to the child's rich sense of length and area, fostered during the activities (Schoenfeld, 2016). Through these activities, it seems that the sense of long/short, wide/narrow is more sharpened. Preschool children may not have this vocabulary, but instead use "big" or "small" to express differences in quantity. Since children who observed better than doing their own work seemed to grasp the size correctly by visual observation in the post-interview, rather than just doing work, by sharing the work process of early mathematical activities, it can be said that the sense of quantity has been enhanced.

Further, even when it seems that a child is not actively working on a problem, or appears not to be thinking too much, we must accept the fact that the child considers silently and may still answer correctly. According to Shinohara (1942), since observation is an educationally effective activity, like experiments, it is important for children to observe others' behavior and imitate it. In the case of young children, it is often difficult for them to express in words; therefore, it is important for the teacher to observe the situation and guess from the usual activities or the activities with other children. From the countereffect by the program, it appears that by depending on teachers' questioning, summary, etc., children concentrate on the work without thinking about its meaning.

Conclusion

Children in this study showed different outcomes of learning in measurement. The children continued to understand direct comparison in all activities; however, it was difficult to establish understanding of measurement by non-universal units. The intervention activities seemed to affect their learning outcomes, although we only focused on how two children's actions related to their interview results. Play-based activities offered by the young teachers influenced children's understanding in both positive and negative ways. This suggests that play-based activities are effective, but should be meaningful and substantial mathematical activities, not superficial. This study only analyzed a small number of children's understanding of measurement, and therefore cannot be generalized further. Future research should analyze a larger number of children.

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