Strategic Majoritarian Voting with Propositional Goals
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ABSTRACT

We study strategic behaviour in goal-based voting, where agents take a collective decision over multiple binary issues based on their individual goals (expressed as propositional formulas). We focus on three generalizations of the issue-wise majority rule, and study their resistance to manipulability in the general case, as well as for restricted languages for goals. We also study how computationally hard it is for an agent to know if they can profitably manipulate.

KEYWORDS

Computational Social Choice; Strategic Voting; Knowledge Representation; Preference Modeling

2 FORMAL FRAMEWORK

We recall the framework of goal-based voting by Novaro et al. [18]. A group of agents, represented by set \( N = \{1, \ldots, n\} \), has to take a collective decision over a number of issues, represented by set \( I = \{1, \ldots, m\} \) of propositional variables. We let \( L_I \) be the propositional language over the atoms in \( I \), with the usual boolean connectives. Agent \( i \) expresses her individual goal by a consistent propositional formula \( \gamma_i \) of \( L_I \). The languages \( L^* \) for \( \ast \in \{\land, \lor, \top\} \), defined by the following BNF grammars

\[
\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \varphi \lor \varphi
\]

represent restrictions on the language of goals. A goal-profile \( \Gamma = (\gamma_1, \ldots, \gamma_n) \) collects the goals of all \( n \) agents.

An interpretation or alternative is a function \( v : I \rightarrow \{0, 1\} \) associating a binary value with each variable in \( I \), where 0 means the issue is rejected and 1 that is accepted. We assume that there is no integrity constraint: all interpretations over the issues are allowed. We write \( v \models \varphi \) to indicate that interpretation \( v \) makes \( \varphi \) true (i.e., \( v \) is a model of \( \varphi \)) and the set \( \text{Mod}(\varphi) = \{v \mid v \models \varphi\} \) contains all the models of formula \( \varphi \).

We denote the choices of agent \( i \) for issue \( j \) in the models of her goal \( \gamma_i \) as \( v_i(j) = (m_{ij}^0, m_{ij}^1) \), where \( m_{ij}^v = |\{v \in \text{Mod}(\gamma_i) \mid v_j = x\}| \) for \( x \in \{0, 1\} \). Abusing notation, we let \( v_i(j) = x \) in case \( |\text{Mod}(\gamma_i)| = 1 \) and \( m_{ij}^0 = 1 \).

A goal-based voting rule is a function for any \( n \) and \( m \) defined as \( F : (L_I)^n \rightarrow \mathcal{P}((0, 1)^m) \setminus \{\emptyset\} \). If on every goal-profile \( F \) returns a singleton we call the rule resolute, and irresolute otherwise. The total number of acceptances and rejections of issue \( j \) in the outcome of \( F(\Gamma) \) are defined as

\[
F(\Gamma)_j = (F(\Gamma)_j^0, F(\Gamma)_j^1), \text{ where } F(\Gamma)_j^v = |\{v \in F(\Gamma) \mid v_j = x\}| \text{ for } x \in \{0, 1\}.
\]

In case we have \( F(\Gamma)_j^0 = 0 \), we write \( F(\Gamma)_j = 1 - x \) for simplicity.

The following are three variants of issue-wise majority defined for goal-based voting [18]. Firstly, \( EMaj \) interprets majority as the quota rule that accepts an issue if and only if more
As for irresolute rules the outcome may be a set of interpretations, \( EMaj(\Gamma) = 1 \) iff \( \sum_{i \in N} \left( \frac{u(i)_{\text{Mod}(\gamma_i)}}{|\text{Mod}(\gamma_i)|} \right) \geq \left\lceil \frac{n}{2} \right\rceil \). To guarantee equality among agents submitting formulas having a varying number of models, \( EMaj \) weights each model of an agent’s goal inversely proportional to the total number of models of her goal.

Secondly, \( TrueMaj \) compares the total acceptances with the total rejections for an issue, setting the result to 1 (respectively, 0) if it is higher (respectively, lower) and to both 0 and 1 when tied. Formally, \( TrueMaj(\Gamma) = \Pi_{j \in J} M(\Gamma_j) \) where for all \( j \in I \):

\[
M(\Gamma_j) = \begin{cases} 
|x| & \text{if } \sum_{i \in N} \frac{m^x_{ij}}{|\text{Mod}(\gamma_i)|} > \sum_{i \in N} \frac{m^x_{ij}}{|\text{Mod}(\gamma_i)|} \\
0,1 & \text{otherwise}
\end{cases}
\]

As with \( EMaj \), the models of an individual goal are weighted inversely to the total number of models for that goal.

Finally, \( 2sMaj \) first applies majority to the models of the agents’ individual goals, and then again to the result of the first step of aggregation, i.e., \( 2sMaj(\Gamma) = \text{Maj}(\text{Maj}(\text{Mod}(\gamma_1)), \ldots, \text{Maj}(\text{Mod}(\gamma_n))) \).

### 3 MANIPULATION OF MAJORITY RULES

The induced preference relation on the alternatives is dichotomous: agents equally prefer any model of their goal to any counter-model. In the case of erosion it is manipulable in the general case, while if it is strategy-proof, agents can manipulately select between two options. The manipulation of voting rules has been extensively studied in the literature on judgment aggregation [16, 21]. Here, however, issues are not exclusively controlled by agents, a closer model being that of aggregation games [11].

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<td>EMaj</td>
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<td>TrueMaj</td>
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<td>2sMaj</td>
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Table 1: \( E \) stands for erosion, \( D \) for dilatation, \( SP \) for strategy-proof and \( M \) for manipulable.

### 4 COMPLEXITY OF MANIPULATION

We also study how computationally difficult would it be for an agent to find a goal allowing them to get a better outcome for the rules \( EMaj \) and \( 2sMaj \). The formal definition of the \( \text{Manip}(F) \) problem is in line with analogous work in judgment aggregation [8]: the input is a profile \( \Gamma \) and an agent \( i \), and the question is whether there is a \( \gamma_i' \) such that \( F(\Gamma_{-i}, \gamma_i') \prec_i F(\Gamma) \). Let \( \text{PP} \), for Probabilistic Polynomial Time, be the class of problems that can be solved in nondeterministic polynomial time with acceptance condition that more than half of the computations accept. We show that:

Theorem 4.1. \( \text{Manip}(2sMaj) \) and \( \text{Manip}(EMaj) \) are PP-hard.

### 5 RELATED WORK AND CONCLUSIONS

The literature on combinatorial voting (see, e.g., the chapter by Lang and Xia [16]) provides solutions to tackle the combinatorial explosion entailed by the structure of the alternatives, such as voting sequentially over issues using tractable voting rules. The work of Lang [15] on voting in multi-issue domains with compactly represented preferences is the starting point of our considerations. Propositional goals are perhaps the simplest compact language for preferences, linked to the literature on social choice with dichotomous preferences [6, 7]. The framework of belief merging [13, 14], also studies the aggregation of propositional formulas, focusing on aggregators satisfying a set of desirable properties inspired from belief revision. Closely related work is the study of strategy-proofness in judgment aggregation [3, 8], where the input is a complete binary choice over all issues rather than a propositional goal, as well as in belief merging [9]. Manipulation of voting rules has been amply studied in voting theory, starting from the seminal result of Gibbard and Satterthwaite [10, 19] to more recent studies aimed at finding barriers to manipulation (see, e.g., the survey by Conitzer and Walsh [1]). Propositional goals in a strategic setting have been extensively studied in the literature on boolean games [12, 21]. Here, however, issues are not exclusively controlled by agents, a closer model being that of aggregation games [11].

In this paper we studied the strategic component of the recent framework of goal-based voting for three variants of issue-wise majority \[18\]. We find that \( EMaj \), \( TrueMaj \) and \( 2sMaj \) are manipulable, even for erosion and dilatation strategies. As positive results, an agent with a goal in the language of conjunctions cannot manipulate, and for disjunctions \( TrueMaj \) and \( EMaj \) are only manipulable by erosion. While not strategy-proof in general, these resolute majority rules are PP-hard for an agent to manipulate.
REFERENCES


