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Developing mathematical literacy in an inquiry-based setting working with play-coins in a second-grade classroom

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In this paper we discuss how focusing on inquiry-based learning may contribute to the development of mathematical literacy in the early grades. We analyze a grade 2 lesson with play-coins with a particular view on what it means for students to have a critical orientation.

Keywords: Mathematical literacy, Numeracy, Elementary school mathematics, Number concepts, Inquiry-based learning

Introduction

The work reported in this paper is part of a longitudinal study on mathematical and scientific literacy and inquiry-based learning in two Norwegian primary schools. The project group consisted of university didacticians from both the mathematics and the science sections of the teacher education department working together with teachers at the two schools. At the start of the project, the students were in grade 1. The data analyzed in this paper was collected when the students were at the start of second grade, at ages 6- and 7-years old. Using data from an IBL-inspired lesson with students at their start of second grade, we explore affordances for students' critical meeting with mathematics in a 'realistic' context. While a lot has been said and written about literacy in general and also the particular topic of mathematical literacy, what mathematical literacy constitutes in the earlier grades has been less discussed (e.g., Askew, 2015, p. 708). In this paper our research question is: Which opportunities do students have to be critical in mathematics in a second-grade classroom, and how can an IBL approach foster this?

Literature review

The use of more inquiry-based learning has been launched as a way to make mathematics and science more relevant to students, preparing them for an uncertain future where the ability to ask questions, to reason, explore, explain and develop a creative and critical mind is seen as essential (e.g., Maaß & Artigue, 2013). Also for early years mathematics inquiry-based activities has been launched as beneficial to develop critical and creative thinking (e.g. Skoumpourdi, 2017). Through working inquiry-based, teachers will expand their teaching repertoire and will also be better able to meet different types of students, including students who struggle with mathematics. Inquiry-based learning has been shown to be effective in rich and real-life contexts and may have a positive effect on students who otherwise perform poorly (Kogan & Laursen, 2013). Through context-based approaches, students gain insight into meaningful use of mathematics and this results in improving students' motivation and attitudes towards learning, which in turn affects academic achievement (Bruder & Prescott, 2013). In this way, IBL appears as a framework that is productive in terms of building mathematical literacy. There is no universally agreed upon definition of inquiry-based learning. Core elements of IBL include developing a questioning mind and a scientific attitude. In

the EU funded Primas project (Maaß & Reitz-Koncebovski, 2013), aspects related to students, teachers, classroom culture, valued outcomes and learning environment were identified and summarized, see Figure 1.

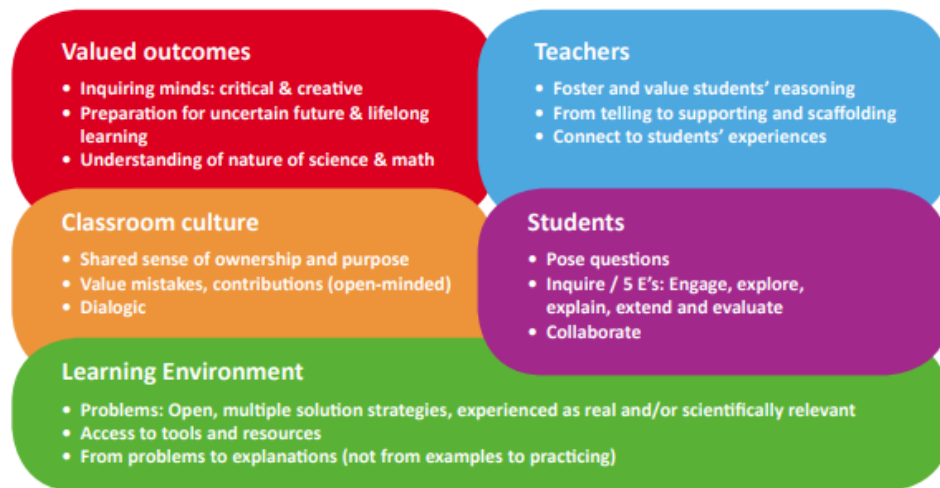


Figure 1. The multifaceted understanding of IBL (Maaß & Reitz-Koncebovski, 2013, p. 8)

Whereas mathematical literacy has been propagated by policy makers and educational authorities, the term is not well defined within the scientific literature. Terms like numeracy and quantitative literacy have been used, more or less synonymously with mathematical literacy, without any determinate definition of what the term entails (Geiger, Goos, & Forgasz, 2015). Niss and Jablonka (2014) pointed out that researchers from non-English speaking countries may have reservations towards the use of the term based on the fact that it is not easily translated and lack counterparts in many languages, including German and the Scandinavian languages. For the PISA assessments, OECD made a working definition that has been slightly amended through the years. In its latest version it reads

Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens. (OECD, 2017, p. 4)

An aspect that is downplayed by this definition is the critical aspect. Some authors see the development of a critical disposition as a primary aspect of mathematical literacy. In the model developed by Goos, Geiger and Dole (2014) a critical orientation is seen as an overarching element. D'Ambrosio (2003) used the term matheracy along with literacy and technoracy as essential for the "providing, in a critical way, the communication, analytical, and technological instruments necessary for life in the twenty-first century" (p. 237). Matheracy is not about counting or measurements, but about thinking and philosophy, about the ability to derive, suggest hypotheses and draw conclusions from data. Literacy, according to D'Ambrosio, is the ability to process information, such as the use of spoken and written languages, of characters and gestures, of codes and numbers. Ole Skovsmose (1998) introduced the term mathemacy as a parallel to the way Paulo

Freire has used literacy, namely by linking it to being able to function in society, and thus not only to number sense or basic skills in arithmetic (p. 199). To Skovsmose, mathemacy is about the competence to be able to function in society structured by mathematics. Against this we can ask to what extent mathematics education educates students to become critical citizens who can challenge the authorities. Askew (2015, p. 708) argued that being critical can be addressed in at least three ways: 1) being critical within a particular mathematical model; 2) being critical of the choice between mathematical models; and 3) being critical of what is modelled mathematically. Askew emphasized that these three forms of being critical are not to be seen as separate to each other or mutually exclusive, but rather “co-construct each other” (p. 709). Being critical within a particular mathematical model is the approach in most papers about young students, this is about reflecting on the chosen methods/approaches used by the students towards finding the required answers or results. It is less clear what being critical to the choice of mathematical model or being critical to what is being modelled may constitute in the lower grades. Does being critical presuppose mathematical knowledge to a certain degree? And which comes first, Askew asked, the mathematical knowledge that enables you to be critical, or the critical orientation towards practice? The problems posed in mathematics classes has a long tradition of being ‘made up’ to look like real problems, and posed to be solved by the algorithms of mathematics. On the other hand, “real-life problem solving is pragmatic and seeks effective solutions to particular problems, rather than the generality and abstraction sought in formal mathematics” (Askew, 2015, p. 709). There are, however, no quick fixes according to Geiger et al. (2015): “The question of how to best promote numeracy capabilities remains an open question. (...) there appears to be little research that outlines the characteristics of effective numeracy tasks or how these tasks promote student learning” (p. 544).

Methodology

The data was collected as part of a seven-year longitudinal study where four groups/classes in two different primary schools and their teachers partake in lesson study cycles focusing on inquiry-based approaches to mathematics and science. Each group of children with their teachers partake in two lesson study cycles each year, in science or mathematics, starting with the children’s first year in school and until they leave primary school after 7th grade. At the time of the lessons reported on here the students were at start of their second year at school. Skoumpourdi (2017) pointed out that “it is not easy for teachers to design inquiry-based activities and implement them in the classroom” (p. 1906). To implement new pedagogical and didactical ideas, it is beneficial for teachers to work collaboratively and with support from outside ‘experts’. Lesson studies in the form of planning lessons together and observing each other’s teaching in cyclic processes of design and redesign is a particular efficient way of professional development (e.g., Yang & Ricks, 2013). In our project, each lesson study cycle is initiated by the in-service teachers choosing a topic or teaching goal, building on the national curriculum, in either mathematics or science. The research group and the teachers meet to develop a detailed lesson plan for the teaching. The planning meeting is audio-recorded. One of the in service-teachers then conducts the lesson, which is video-taped, while the other teachers and researchers observe the lesson, using observation sheets that have been developed as part of the planning sessions. In addition, the teacher wears an audio-recorder, and

additional audio-recorders are placed among selected groups of students. After the lesson the researchers and the in-service teachers meet to reflect on the lesson and redesign it. A (possibly different) teacher conducts a new lesson with the new redesigned lesson plan in another group/class. A new reflection and redesign meeting is held after this second run-through. This is also audio-recorded. In addition, annual interviews are conducted. Students are interviewed in groups of 3-4, while teachers are interviewed individually and respond to questionnaires. For this paper the data analyzed comes from two lessons in the lesson study cycle. The first group of students comprised 16 students and the second 14 students. The same teacher, who is one of the two regular teachers sharing these two groups, conducted both lessons. The underlying mathematical goal concerned addition with natural numbers up to twenty. The mathematics was introduced in a context that involved Norwegian coins (values 1, 5, and 10 kroner), bringing the 'real' world into play. We use the word 'play' here deliberately to emphasize that the real world for children is as much a world of playing games and not only the real adult world. At the same time, using coins relates to the adult world and prepares students for meetings with, and they may already relate to, money in the real world. Thus, the tasks were partly presented as small 'mysteries' where the students were supposed to find solutions, as if solutions to real problems and not pure mathematical tasks. A particular aim in the design of the tasks was to focus on the different types of number of solutions a mathematical problem may have: one unique solution, several or infinitely many solutions, or no solution at all. For the students, working collaboratively, in pairs, was an explicit goal for the lesson. Students had access to number lines, play coins similar to Norwegian 1, 5 and 10 kroner coins, and A3-size sheets of paper, pencils and crayons. Each lesson was divided in three parts: 1) plenary gathering around the smartboard, where the topic was introduced; 2) students working in pairs with up to five different tasks. The tasks were printed on laminated A5-size paper sheets and handed to the students as they asked for it. 3) plenary session where students were asked to present their findings. Data sources used for the analysis in this paper are the audio- and video-recordings and transcripts of the lessons, observation sheets filled in by the authors during the lessons, and copies of the A3 sheets filled in by the students. The multiple data sources enhances the construct validity (Yin, 2018, p. 43). Data analysis started by the two of us watching the video recordings together. Whenever anything in the video was unclear, the audio recordings were consulted, and sometimes also copies of the student work sheets. Together with the lesson plan and the observation sheets this gave us an overview of the lessons. Subsequently, the transcripts were analyzed by each of the authors with respect to two lines of inquiry: 1) are any of the aspects of IBL as identified by Maaß and Reitz-Koncebovski (2013) apparent? and 2) are any of the three ways of being critical as identified by Askew (2015) apparent? We then sat together to exchange scripts and discuss our findings to enhance inter-rater reliability. Part of this also included consulting the video- or audio recordings or the students work sheets whenever anything needed to be clarified.

Findings

All together there were six available tasks for students to work on, labelled 1a, 1b, 1c, 2a, 2b, 2c. All students were able to work with at least the two first ones, and some students were able to finish all tasks. We arrange our findings according to the tasks, with an analysis of student answers to the first four tasks. The students started out on task 1a), that had a unique solution, namely: *Jens has*

four coins in his pocket. The value of the coins are 17 kroner. Which coins is it? In the conversation below we see an example where the students have already solved the mathematical problem by representing the answer with play-coins but without writing or drawing. The teacher asks them to write and draw the answer, and the calculation. They found a short plenary talk where the teacher reminded the class that they should collaborate and use the available resources when needed, as something that disturbed them:

24 Student 2: We are already finished with these coins. We managed to find out before you began with your talking-thing

30 Teacher: But, you, can't you say how you were thinking, and write it? Draw coins, write the addition? Write the answer!

In this interaction with the teacher the students have a solution, and rather than giving the students an opportunity to explain their thinking with the play coins, they are encouraged to make a written record on paper. The teacher does ask them to draw the coins, while in the video it is clear that the students have used the play-coins to solve the problem. We see here that there is some discrepancy between the students' and the teacher's ideas about how to solve the problem. This can be interpreted as the students being critical within the mathematical model, the first of Askew's (2015) three ways of being critical.

Task 1b): Jens has 17 kroner in his pocket. Which coins can that be? This task has several solutions, however only a finite number using Norwegian coins.

66 Student 1: OK. 17 kroner. OK. One ten, one five, plus two. Like that. You can write it.

Student 2 asks whether to write the same answer as in 1a), but Student 1 says

70 Student 1: No, no, no. You do different ways.

The students explore different ways to make a sum of 17 kroner from the Norwegian coins, realizing that there are lots of solutions.

104 Student 2: Maybe we can fill the whole sheet only with b).

This type of task, with several solutions, is not very common in Norwegian text-books and classrooms. In their conversation, Student 1 sees only one possible solution, however Student 2 points out that there are in fact many solutions, and they continue to explore this together.

Task 1c): There are 4 coins in the bag. Which coins can it be and how many kroner? This task poses the same challenges as the previous one, as there are several solutions, though only a finite number using Norwegian coins. Many of the students did not realize that the coins did not need to hold the same value in this task. Several were happy with the solution $1+1+1+1$, and needed prompting from the teacher to realize that there were other possible solutions. Two of the students found several possibilities and started discussing whether there are more than already discovered. Further in the discussion a question arises:

416 Student 2: Is it allowed with minus? Because there are no more numbers.

This can be interpreted as the student questioning the choice of model in this setting, which is the second way of being critical according to Askew (2015). It is not within the model or given context natural to consider that you add some of the coins in the pocket and subtract any. These students are

in the early stages of learning addition and subtraction and so it is natural to also question which mathematical operation that is valid in a given situation. And indeed, considered as a purely mathematical problem, subtraction is also part of the additive context.

Task 2a): Kari has 5 coins in her pocket, in total 20 kroner. Which coins does Kari have in her pocket? This task does not have any solutions using Norwegian coins. The students' first reaction was

468 Student 1: OK, that is easy. No, what?

The two students discussed it, trying out possible solutions.

511 Student 1: We need one ... It doesn't work

513 Student 2: Yes, but it has to work, Hm ...

515 Student 1: This also does not work. Definitely not this. Kari has five in her pocket ...

527 Student 1: It would have worked if there was a 2-coin.

529 Student 2: OK. Oi, oi, oi. This is totally ... Hm ... Five ..

537 Teacher: What are you doing?

539 Student 2: We are working with this one, with 5 coins, it is not possible to divide in twenty or something. Ones, too few. All of them, too little. And fivers, too much, and tens, too much.

567 Student 2: Four. If we need, then,... If it was supposed to be 19, then we would have found the answer immediately. There should have existed 2-coins.

Not all mathematical tasks have a solution, and quite a few mathematical tasks do not have a solutions within the number set you are working with; e.g. $3 - 5$ does not have a solution in the natural numbers, $3 : 5$ does not have a solution in the integers, a set of linear equations represented by non-identical parallel lines does not have a solution; etc. In this case this fact is discovered by the students, when they realize that if you had a coin with value 2 kroner then the problem could be solved. Two other students, Student 3 and Student 4, also working with 2a), turned critical to the idea of having this type of tasks. Their relationship with the teacher is one of confidence and trust, allowing expression of honest opinions of frustration without being interpreted as rude or angry.

445 Student 3: So this is a nonsense task?

447 Teacher: Or an impossible task?

449 Student 3: Did you make this so that we should not be able to solve it, just sit there struggling and struggling?

These students questions the teachers motivation for choosing a task that does not have a solution. This can be seen as the students being critical to what is being modelled mathematically, the third way of being critical as identified by Askew (2015).

Discussion

The multi-faceted understanding of IBL consisting of five related areas displayed in Figure 1 was used as a guiding map during the planning of the lesson, and the analysis shows that several characteristic aspects of IBL could be discerned in the lessons. The learning environment was characterized by problems that afforded diverse possible solution strategies, and apart from task 1a)

they all had a number of possible solutions. Furthermore, the problems could be experienced as real for the students, and there was access to tools such as play-coins, number lines, paper and pencils. Explanation of solutions were encouraged, as made clear by the teacher both at the start of the lesson and during the students' problem solving. The realistic restrictions to the coin-problems by using only Norwegian coins afforded the students opportunities to discover firstly that not all problems are solvable with these restrictions, secondly that if there were other coins such as a coin with value 2 kroner the problem could be solved, and thirdly, in the real world there is actually no coin in Norway with value 2 kroner. Understanding what a solution means, under what circumstances and limitations solutions can be found, and being able to see that changing conditions may lead to other solutions, is an important part of learning mathematics, and in particular an important part of developing a critical orientation in mathematics, e.g. as identified by Goos et al. (2014) and Askew (2015). Additionally, it is certainly also an important part of being able to solve problems in the society. This brings us closer to those aspects of learning mathematics that are proposed by D'Ambrosio (2003), Skovsmose (1998) and which are some of the underlying ideas and valued outcomes of IBL (Maaß & Reitz-Koncebovski, 2013). For some students, such as students 1 and 2, we observed that they had to be encouraged specifically to do writing and drawing by the teacher. They had already solved the problem using play-coins but not made a written record. Learning to communicate your solution, making your thinking visible to others in a multitude of ways, is an important part of building mathematical literacy. At the same time, forcing children to represent their answers in particular ways may distract from their learning of the mathematical content, and prevent them from going into deeper thinking. One risks, as D'Ambrosio (2003) pointed out, to emphasize manipulation of numbers and symbols rather than developing matheracy. We believe that this tension between steering children towards particular representations and letting them explore more freely, is something we have to live with. By emphasizing the importance of writing or drawing as representations and as means for communicating results, one risks downplaying the importance of other, for instance oral communication (which is also something one would like to foster). From the empirical data we also see that the students engaged in the inquiry cycle with the five E's, engage, explore, explain, extend and evaluate while working collaboratively in pairs. The teacher encouraged students' reasoning by asking students about their ways of thinking, both in plenary and when students were working in pairs. From the data it is evident that it was necessary for the teacher to prompt students to collaborate and inquire, even if this prompting when done in plenary was seen as 'disturbing' for some students, as evident from transcript line 24 above. Developing a classroom culture that values questioning, exploration and where mistakes are seen as a necessary part of learning mathematics is emphasized as important in IBL (Maaß & Reitz-Koncebovski, 2013) and is essential for students to develop a critical disposition that is essential in being mathematically literate. In our data it is evident how the teacher tried to achieve this, both by her innate experience as a teacher and by using the IBL oriented lesson plan. The start of the lesson with the students all gathered around a smart-board where the coins and the problems were introduced, was used to create a classroom culture of shared ownership and purpose, and where the students were encouraged to explore and work together. Valued outcomes in an IBL setting include developing critical and creative inquiring minds. This echoes some of the ideas proposed by D'Ambrosio (2003) and Skovsmose (1998) and also the critical orientation

underlined by Goos et al. (2014). It is not straight forward to answer the question about what it means to be critical in a mathematics lesson, in particular at the lower grades of primary school. Our data shows that it is possible to identify the three ways of being critical (Askew, 2015) even in 2nd grade, if the setting and atmosphere in the classroom is encouraging, e.g. the classroom culture and learning environment are as described in the IBL model. Planning a lesson using an IBL model like the one used here may help fostering mathematical literacy, including developing a critical disposition.

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