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Recovering underlying graph for networks of 1D waveguides by reflectometry and transferometry

Geoffrey Beck^{1,*}, Maxime Bonnaud², Jaume Benoit²

¹POEMS (CNRS-INRIA-ENSTA Paristech) Palaiseau, France

²CEA, LIST, 91191 Gif-sur-Yvette CEDEX, France

*Email: geoffrey.beck.poems@gmail.com

Abstract

We present a method for blind recovery of network made out of a tree of 1D homogeneous waveguides with the same physical characteristics using reflectogram and transferogram(s).

Keywords: Inverse problem, Topology recovering, Quantum graph, Reflectometry, Wire analysis

1 Introduction

We consider an unknown quantum graph \mathcal{G} (see [2]) equipped with a wave operator along its branches and some transmission conditions on its nodes connecting together the quantities evaluated on the branches. Our graph is a rooted tree graph, where all branches are oriented from a root-point Inp to end-points Out_{*k*} ($k = 1 \dots K$). We will consider a maximum of two consecutive nodes between Inp and any Out_{*k*} and at least a node in \mathcal{G} .

We will now explain how a wave V propagates along the graph \mathcal{G} :

- On each branch of \mathcal{G} the wave satisfies a homogeneous wave's equation

$$\partial_{tt}^2 V - c^2 \partial_{xx}^2 V = 0,$$

where t denotes the time and x the abscissa along the considered branch. The celerity c of the waves is supposed to be a known constant.

- Following Kirchhoff's rules, V is continuous on \mathcal{G} and at each node J

$$\partial_x V|_{\mathbf{e}_{j_0}}(J) = \sum_{j_k=j_1}^{j_K} \partial_x V|_{\mathbf{e}_{j_k}}(J),$$

where \mathbf{e}_{j_k} are the branches connected to J , with \mathbf{e}_{j_0} the branch closest to Inp.

- On Inp, we have an impedance boundary condition

$$\partial_t V(\text{Inp}, t) - c \frac{Z_u}{Z_c} \partial_x V(\text{Inp}, t) = (\partial_t u)(t)$$

where the constant Z_u and $u \in H_{\text{loc}}^1(\mathbb{R})$ are known. The unknown characteristic impedance Z_c is supposed to be constant.

- At each Out_{*k*} we have an impedance condition

$$\partial_t V + c \frac{Z_k}{Z_c} \partial_x V = 0,$$

where Z_k is an unknown constant.

2 Graph recovery problem

Reflectometry and transferometry methods can be applied to any practical electrical or acoustic network. The reflectogram is the following Steklov operator :

$$u(t) \mapsto \mathfrak{R}(t) := V(\text{Inp}, t),$$

whereas transferograms are operators for $k = 1 \dots K$:

$$u(t) \mapsto \mathfrak{T}_k(t) := V(\text{Out}_k, t).$$

We suppose that we can control u . With a known celerity c and the input load Z_u , the reflectogram and optionally some transferograms, we want to recover \mathcal{G} that is to say to determine

- the number of nodes and end-points, and their ordering (topology),
- the length ℓ_j of all branches,
- the end-points load Z_k ,
- the characteristic impedance Z_c .

3 Injectivity

There can exist several quantum graphs with the same reflectogram, so we will make two hypotheses. Firstly, no scatterer (node or end-points) have the same distance from Inp, to ensure they can be dissociated. Secondly, no Z_k is equal to Z_c to ensure waves are reflected on the end-point. We will suppose that $Z_k > Z_c$ is always satisfied.

4 Scattering

We can choose the excitation signal u such it is a peak function (sole local maximum). It propagates at celerity c along a branch until it meets a scatterer. \mathfrak{R} is the sum of attenuated u -shaped peaks, i-e of the form $\sum_p S_p u(t - t_p)$ where where each (t_p, S_p) - called echo - corresponds to the duration and the amplitude attenuation of a propagation of u in \mathcal{G} through transmissions T and reflections Γ looping on Inp. T_k is similarly generated, with propagations from Inp to Out $_k$. An echo amplitude S_p gives the nature of its contained scatterers (order for a node, load for an end-point), its abscissa the path length between the observation point (Inp for \mathfrak{R} and Out $_k$ for \mathfrak{T}_k) and the scatterer.

The algorithm presented in [1] identifies echoes in a complex reflectogram and associates them with unknown scatterers in \mathcal{G} , giving their nature and location. It runs iteratively, dispelling ambiguities from peaks overlapping and accumulated reflections.

But this method requires knowledge of Z_c and supposes that $Z_c = Z_u$ (no reflections at Inp). It can be enhanced by the use of transferograms.

5 Algorithm

5.1 Recovering Z_c

We simply recover Z_c from the reflectogram at origin where we see an echo (called mismatch echo) of amplitude $T_u = (1 - \Gamma_u)$ with $\Gamma_u = (Z_u - Z_c)/(Z_u + Z_c)$. Of course if $Z_c = Z_u$ then the mismatch peak is null.

5.2 Recovering the first node

The first echo observe in \mathfrak{R} after the mismatch echo have for abscissa $2\ell_0/c$ and for amplitude $T_u(1 + \Gamma_u)\Gamma_0$ with $\Gamma_0 = (2/m_0 - 1)$ where m_0 is the order of the first node J_0 . We thus recover ℓ_0 and m_0 .

5.3 Using the transferograms

If the amplitude of the first echo (t_1, S_1) of \mathfrak{T}_k is above $4T_u(\Gamma_0 + 1)/3$, then Out $_k$ is directly connected to J_0 . Thus we have $S_1 = T_u(2/m_0)(1 + \Gamma_k)$ with $\Gamma_k = (Z_k - Z_c)/(Z_k + Z_c)$, so we recover Z_k . The length of the J_0 to Out $_k$ branch is $(ct_1 - \ell_0)$. If S_1 is under $T_u(\Gamma_0 + 1)$, a node exists between J_0 and Out $_k$. This recovered topology

can remove branch location ambiguities for an unknown scatterer with the reflectogram.

5.4 Using the reflectogram

We use the algorithm developed in [1] to continue the analysis of \mathfrak{R} . We changed the procedure to use informations from the transferograms, and adapt to $Z_u \neq Z_c$. Indeed, we need to apply a $T_u(1 - \Gamma_u)$ factor to all \mathfrak{R} echoes amplitude and to consider reflexions on Inp when discriminating between echoes.

This method achieves an error-free topology reconstruction if some technical hypothesis on u are fulfilled. ℓ_j are retrieved with an accuracy decreasing when farther from Inp (relative error under 5% from 350 simulations), as are Z_c (under 0.1%) and Z_k (under 10%). Better determination is possible by optimizing the all lengths $\tilde{\ell}$ and all loads $\tilde{\mathbf{Z}}$ vectors such that they minimize the functional

$$J(\ell, \mathbf{Z}) = \int_0^{8\ell_{max}/c} \frac{|\mathfrak{R}(t) - \mathfrak{R}_{\ell, \mathbf{Z}}(t)|^2}{3\ell_{max}/c + t^2} dt$$

where ℓ_{max} is the maximum ℓ_j from previous steps and $\mathfrak{R}_{\ell, \mathbf{Z}}$ the simulated reflectogram using the recovered topology with ℓ and \mathbf{Z} . We look for the minimum of J with Newton's algorithm, initializing $\tilde{\ell}$ and $\tilde{\mathbf{Z}}$ with the previously recovered values.

6 Applications

This algorithm can be used for recovering unknown electrical networks, with one reflectometry device and optionnal transferometry transceivers on end-points. Removing the condition on Z_u makes the algorithm more resilient to real-life implementation limitations.

References

- [1] G. Beck, *Reconstruction of an unknown electrical network from their reflectogram by an iterative algorithm based on local identification of peaks and inverse scattering theory*. International Instrumentation and Measurement Technology Conference IEEE I2MTC, 2018 (to appear)
- [2] P. Kuchment, *Quantum graphs I. Some basic structures* Waves in Random Media 14, S107-S128 (2004)