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Development of strategies for a combinatorial task by a 5 year old child

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The transition from kindergarten to first grade is a very special time for children. In a longitudinal research project over two years, we interviewed children up to 6 times individually. They solved several tasks from different mathematical areas. In this study, I will present one combinatorial problem and analyze approaches to solve it by one child over two years as a case study using the strategies of Hoffmann (2003). How does this child solve the combinatorial problem before school, just after the school start and at the end of grade 1? What kind of different information do we get if we analyze the data product-oriented or process-oriented? Observing and analyzing the child's approaches and development over time will be at the heart of this paper.

Keywords: combinatorics, combinatorial reasoning, Piaget, preschool education, elementary school

Introduction

The domain of combinatorics is a very interesting and fertile field for research in mathematics education. Even young children can discover the rich mathematical structure and can become acquainted with mathematics as the art and science of patterns (Devlin 1995). Still this topic is not the best recognized one in elementary school, as English (1991, p. 451), almost 30 years ago mentioned. Hoffmann (2003, p. 3) indicates that combinatorial problems are interesting because previous knowledge is not necessary. It is possible to solve combinatorial problems in an activity-oriented and material-oriented fashion; it is easy to build analogous problems on a higher level; one can choose different ways to arrive at the solution. Furthermore, problem-solving approaches can become apparent and children are encouraged to mathematical communication and argumentation skills.

Theoretical background

Piaget and Inhelder (1975) investigated in their studies the development of a combinatorial system connected to the Piagetian developmental stages. They find out that children in the preoperational stage search without a system, in the concrete operational stage they search for a system and in the formal-operational stage, they search systematically and use the odometer strategy. Their ability to reason and argument is increasing similar to their search for a system. Combinatorial problems play not only an important role in cognitive development but mathematics education researchers like Piaget and Inhelder (1975) found that they may promote students' reasoning and generalization processes. Further research showed that there are first steps to a systematical search and that the task should be in a more suitable and meaningful context and material should be more child appropriate (English 1991, p. 453).

Further studies investigated the strategies of the children when solving a combinatorial task. I refer to two studies explicitly because in my study I deal with almost the same age range and videotapes

like English (2005) and with a case study like Martino (1992). English (2005, p. 129) videotaped 50 children between 4.5 and 9 years, solving combinatorial tasks. She analyzed the video material without transcripts. In comparison to English, I observe the children several times over two years. I use transcripts to reconstruct the approach of the child when solving the combinatorial task. Martino (1992) dealt with the understanding and development of mathematical ideas using combinatorial tasks as one example. She looked at three case studies. She observed children in grades 2, 3 and 4 in pairs or a group of three without the intervention of a teacher. I observe children from 5 to 7 years old in one-to-one-interviews with an interviewer, as an observer, and so I examine a younger group of children. Looking at younger children is especially interesting because combinatorial problems also “facilitate the development of enumeration processes, as well as conjectures, generalisations, and systematic thinking” (English 2005, p. 122) and these are key components of mathematical understanding.

Hoffmann (2003), based on the two studies mentioned above, worked out that there are macro and micro strategies. Macro strategies are guiding actions for a structured finding of all combinations. One strategy is the odometer strategy, “holding one item constant while systematically varying each of the other items” (English 1991, p. 451). Other strategies are, the entire counterpart construction, and looking for the solution in phases. Micro strategies describe patterns of action to find the next combination or find some of the combinations. For my study, only the following seem relevant: 1- and 2-constant-principle, stair formation, counterpart construction and turn around. In the analysis part, I will describe them on more detail.

The study reported here made use of two levels of manipulative materials. The concrete external representation enables the children to handle the material and keep their focus on the solving process without representing their solution on paper. English (1991, p. 453) stressed that young children would display more sophisticated combinatorial approaches than suggested by the Piagetian studies. Further on English (2005, p. 128) assumes that preoperational children (like the children in my study) “generate combinations only in an empirical manner by randomly associating elements” and her conclusion is that “there is a lack of systematic method”. This study addressed this assumption in the course of a longitudinal study over two years where each child solved the same combinatorial task up to six times in a one-to-one-interview. The idea of repeating always the same task without teaching is that the children need time to become acquainted with this task. In addition, the verbalization competence is restricted with young children, but they have the possibility to show their thinking by using materials, gestures, deictic expressions (Werner 2018) and also by us observing their actions in process. Previous studies looked at the products children developed when they solved combinatorial tasks and because of the young children, I take a process-oriented view on the children’s actions. Even more, I could not find any studies, which had a longitudinal view on the solving of combinatorial tasks of young children. English (2005, p. 130) observed the learning across the set of tasks dealing with combinatorics at one interview point.

The present report examines the following research questions focused on one case study for a first small analysis of all collected data:

What kind of approaches does a child, aged 5 to 7 years, use in solving the same combinatorial problem several times over two years?

To what extent does the analysis method, product-oriented or process-oriented, show a different view on the way of finding all combinations for a combinatorial task of one child (5-7 years)?

Research project

Sample

The study is called “mathematics and first day at school – talking with children in the transition from kindergarten to primary school” (MaScha). The key aspect of this study is to investigate the (re)interpretation and strategy diversity of children from the last year of kindergarten (preschool) to the end of grade one in several different mathematical areas. It includes finger patterns, one problem solving task, one combinatorial task and the attitudes of children to mathematics teaching and mathematics as a teaching content. In this paper, I will only focus on the combinatorial problem.

The study involved 14 children (8 boys, 6 girls) from a rural area. They all attended the same kindergarten and the same class in primary school. At the beginning of the study, they are all in their last year of kindergarten (5 or 6 years old). The end of the study is at the end of first grade and they are almost all 7 years old. Surveys were conducted every three months in the last year of kindergarten (December 2015, February 2016, May 2016) and every 5 months at the beginning, in the middle and at the end of the first school year (September 2016, February 2017, June 2017). Each child is interviewed for about 30 minutes in a one-to-one-interview. The interviews are all video documented and interesting parts are transcribed for analysis. The interviews take place during school time but are additional to teaching. The interviewer is either a pre-service teacher who attended a seminar on talking with children about mathematics or the researcher. It follows a semi-structured interview guide, which is competence-oriented and can be widened with questions to better understand the comprehension of the child and with additional analogous tasks if necessary. The semi-structured interview guide remains the same over the two years because I can only recognize changes or process of development in the (re)interpretation and strategy use of the children when they were familiar with the task and knew what to do. Especially the assisted problem solving with the interviewer could help the children, and like Freudenthal (1973, p. 110) said, “the best way to learn an activity is to perform it”. On the other side, the tasks have to be sufficiently challenging so the children have to solve the tasks every time from scratch and cannot remember the solution like an easy calculating task.

The competence of the interviewer influences the results of the interviews with the children. Therefore, the same interviewer interviews most of the children over the two years to keep the influence constant. The interviewer sometimes gives instructions or intervenes (Bräuning & Steinbring 2011) to establish a “common base of understanding” (Bräuning & Steinbring 2010) between the child and the interviewer and to give the children the chance to deal with unfamiliar tasks. The kindergarten teacher and the primary teacher were only told that the interviewer is talking with the child about mathematics for around 30 minutes. They were not given the tasks so that the teacher is not able to train the children and cannot intervene.

The combinatorial task called bears task is: “You have three bears, a yellow, a blue and a green bear. They move into a tower. Each bear lives on one level. How many different ways can they live in the tower?”. The children were given eight towers (without a roof to avoid confusion) drawn on paper and several yellow, blue and green plastic bears (more than 20 bears).



Figure 1: Solution of the bears task interview 1 with Lena final product

We offered the children more material than needed so that they have to decide if they found all combinations on their own. Similar to the bears task the problem is often presented with blocks which the child puts together. The advantage for the bears task is that through the tower on paper and the placing bears, two dimensions for representing the child's approach are available. Especially if a very detailed process-oriented view on the approach is taken the researcher can identify on which level at what time the child is placing a bear and where the child is looking at to find a new combination. Through the video documents and the transcripts, eye and head movements are observed. This would be easier using eye tracking, but it probably would be an artificial situation for the child, therefore we did not use it.

Realization of the interview

When the child solved the task with material there was preferably no inquiry. At the end the question, “How many ways did you find” is posed. If the child found the same combinations several times, or less than three combinations the interviewer asked: “Can you find double combinations in how you have bears living in towers? Can you find additional combinations? How often can you find yellow (blue, green) on the lower (middle, upper) level?” I think these questions are very important because during the analysis there occurred the phenomena that some children had a lot of trouble with understanding equality. Some of them interpreted equal as identical so that every tower constellation even with the same order of colors was a new combination for the child.

In the reflection phase, the children were asked: “What can you notice if there is yellow twice on the lower, middle and upper level? Are you sure you found all combinations? Please explain”. Hoffmann (2003, p. 260) observed that her youngest participants (7-9 years old) could not justify repositioning their combinations. As this challenge was too much for them, I decided for the 5 to 7 years old children not to pose this demand. Instead, we asked them to explain why they think that they completed the task. Some of the children developed their argumentation skills and were able to explain their approach. By means of these explanations, the macro strategy can probably be identified.

Analysis of one case study

I will present as a case study Lena. She is interviewed six times. I chose her because five interviews were conducted by the same interviewer, and only the fifth interview was by another interviewer. Therefore, the influence of the interviewer is kept almost constant. This is especially important

because the interviewer posed different questions, which I distinguish in subject-specific questions and non-subject-specific questions. In another paper, I will analyze the influence of these two types of questions for the explorative investigation of the child's thinking. I will present the analysis of three interviews of three specific moments. The analysis of her first interview represents the first contact of Lena with this task. Her fourth interview is just one month after the first day of school and so she is still getting used to this new institution. At the end of first grade, the sixth interview shows the end of development in her approaches because it is the final interview of this study.

In the product-oriented analysis, I always present the final product of the solving process in the interview and in the process-oriented analysis I describe each change and the whole process precisely in order the child placed the bears in the tower and the order of the towers. The eight towers on paper are represented in two rows of four towers. The researcher when transcribing the video document writes down the order. In the coding table, you will find the order of towers with T1, T2, etc. and the order of placed bears in an extra column for each tower. The researcher and his team watch the video several times to identify each hand and eye movement and analyse the transcript qualitatively.

The following micro strategies referring to Hoffmann (2003) are identified in the previous analysis:

1-constant-principle: the color of one level will be fixed. For three-dimensional combinatorial tasks, an implicit 1-constant-principle occurred in the process-oriented analysis where at first two colors will change places and then the constant will be placed implicitly or automatically the third color keeps constant.

Stair formation: the colors "walk" through the levels (entire or single stair formation)

Counterpart construction: changing the two possible colors between two combinations (only for multiples of two-dimensional combinatorial tasks). With three-dimensional combinatorial tasks, this corresponds to the 1-constant-principle or the implicit 1-constant-principle.

Turn around: two combinations are the same when rotated 180°.

The macro strategy is reconstructed out of the most frequent occurrence of micro strategies and the verbal justification of the child.

In the first interview with Lena in December 2015, she found six combinations. Tower 6-4 means that Lena found this combination after rejecting three other possibilities. I present the solution in this table because I want to compare Lena's solutions at different interview points and to give more consistency (you can see the real picture in Figure 1).

T5 Blue Yellow Green	T6-4 Yellow Blue Green		
T4 Green Yellow Blue	T3 Yellow Green Blue	T2 Green Blue Yellow	T1 Blue Green Yellow

Table 1: Solution of the bears task interview 1 with Lena final product

From T1 to T2 yellow remains constant and so the 1-constant-principle can be identified. The 1-constant-principle also occurs as from T3 to T4 blue is constant and from T5 to T6-4 green is fixed. T2 to T3 is an entire stair formation. T4 to T5 is a “turn around” or a 1-constant-principle where yellow remains in the middle. In consideration of the identified micro strategies, a kind of odometer principle is inherent, because Lena uses a lot the 1-constant-principle. Analyzing the same approach with a process-oriented view provides another insight into strategies Lena used.

T5 Blue Yellow Green	3 2 1	T6-4 Yellow Blue Green	10 9 8 (I)				
T4 Green Yellow Blue	3 1 2	T3 Yellow Green Blue	2 3 1	T2 Green Blue Yellow	3 1 2	T1 Blue Green Yellow	1 2 3
		T6-1 Blue Green Yellow	3 1 2	T6-2 Green Blue Yellow	5 4 -	T6-3 Green Yellow Blue	- 7 6

Table 2: Solution of the bears task interview 1 with Lena process

From T1 to T3 there is a single stair formation and between T2 and T3 even an entire stair formation. T3 to T4 is not identified and T4 to T5 is a “turn around”. T5 to T6-1 is an implicit 1-constant-principle and T6-1 to T6-2 a 1-constant-principle where Lena detect the identity to T2 and rejects this combination. T6-2 to T6-3 is also the 1-constant-principle and identical to T4; Lena also rejects this combination. From T6-3 to T6-4, there is an implicit 1-constant-principle. In consideration of the identified micro strategies we can recognize that Lena’s’ approach is very inconsistent and cannot identify a macro strategy. English (1991, p. 458) named this strategy trial-and-error-procedure with random item selection and rejection of inappropriate items whereupon in this study the interviewer asked the child if there are identical combinations. In addition, I present some of the interviewer’s questions posed to Lena. “Are they all different (the towers) which you posed here?”, “How often does a yellow bear live in the lower level?”, “Compare these two (towers)”, ... These questions influenced of course the way of Lena’s solving process, but these affects occur in the product-oriented and process-oriented analysis and nevertheless there is a difference. Even more the interviewer wanted to give stimuli to deepen the mathematical thinking of the child.

	Product	Process
Micro strategies	T1/T2: <i>1-constant-principle</i> T2/T3: entire stair formation T3/T4: <i>1-constant-principle</i> T4/T5: turn around or <i>1-constant-principle</i> T5/T6-4: <i>1-constant-principle</i>	T1-T3 <i>single stair formation</i> T2/T3 <i>entire stair formation</i> T3/T4 ? T4/T5 <i>turn around</i> T5/T6-1 implicit 1-constant-principle T6-1/T6-2 <i>1-constant-principle</i> , identical to T2 T6-2/T6-3 <i>1-constant-principle</i> T6-4 implicit 1-constant-principle
Macro strategies	Mainly 1-constant-principle -> odometer principle	???

Table 3: Comparison of the product-oriented vs. process-oriented analysis of Lena's solution 1

I will only present the comparison of the product-oriented vs. process-oriented analysis of Lena's solution 4 and 6.

	Product	Process
Micro strategies	T1-T3: <i>entire stair formation</i> T4-T5: <i>entire stair formation</i> T6/T7 identical to T3 T8 identical to T1	T2/T4 <i>1-constant-principle</i> , yellow remains in the middle, blue and green change T3/T5 <i>1-constant-principle</i> , yellow remains in the lower, blue and green change T4/T6 <i>1-constant-principle</i> , green remains in the middle then see identical to T1 T5/T6 <i>1-constant-principle</i> , yellow remains in the lower, then blue and green change T6/T7 identical to T3 T8 identical to T1
Macro strategies	Mainly stair formation -> odometer principle	Mainly 1-constant-principle -> odometer principle

Table 4: Comparison of the product-oriented vs. process-oriented analysis of Lena's solution 4

In September 2016 some weeks after the first day of school, Lena had her fourth interview. She already used an emerging pattern in item selection. Interesting is that it is the same micro strategy for the whole solution process but a different one for product-oriented or process-oriented analysis. Lena did not identify the identical ones and she could not justify her approach.

	Product	Process
Micro strategies	T1/T2 turn around T2/T3 stair formation T3/T4 1-constant-principle, yellow remains in the upper level, green and red change T4/T5 stair formation T5/T6 1-constant-principle	T1/T2 implicit <i>1-constant-principle</i> T1/T2/T3 green in every level, <i>single stair formation</i> T3/T4 <i>1-constant-principle</i> T5/T3 <i>1-constant-principle</i> T6/T1 <i>1-constant-principle</i>
Macro strategies	???	Mainly 1-constant-principle -> odometer principle

Table 5: Comparison of the product-oriented vs. process-oriented analysis of Lena's solution 6

In the sixth interview with Lena in June 2017, the product-oriented analysis cannot find a macro strategy but if we look at the process-oriented analysis, there is evidence that Lena uses the odometer principle without rejecting any combination. She does not use the odometer principle visibly from the beginning but if she checks her solutions, one can observe it. Even more she justifies her approach after the question "how can you quickly check that they are all different?" with "these two have to change all the time (points to T6 and T1) and these also (points to T5 and T2) and also those (points to T3 and T4)". Looking at Lena's progression from the first to the sixth interview, she developed a more efficient solution procedure, which is only observable with the process-oriented analysis and is confirmed by Lena's verbal justification of her approach.

Closing remarks and outlook

To answer the posed research questions, I refer to the studies of English (1991, 2005). English (2005, p. 128) assumes that preoperational children (like the children in my study) "generate combinations only in an empirical manner by randomly associating elements" and her conclusion is

that “there is a lack of systematic method”. Looking at the analysis of the case study, Lena uses quite different strategies when analyzing her approaches product-oriented or process-oriented. In her first interview in the product-oriented interview, it seems that Lena is already “emerging pattern in item selection, with rejection of inappropriate items” (English 1991, p. 458). This could be traced back to the interviewer’s questions. If you compare it to the process-oriented analysis you can see that Lena uses the trial-and-error-procedure with random item selection and rejection of inappropriate items. In the sixth interview the interviewer does not pose any questions and in the process-oriented analysis it seems evident that she uses the “odometer principle without rejecting any combination” (English 1991, p. 461). There is an obvious development in Lena’s solving approaches when using the process-oriented analysis. It seems that the process-oriented analysis method is more appropriate to this young child and probably to all young children with restricted verbalization possibilities. So probably preoperational children are already able to evolve strategies from dealing several times with the same task and an explorative supervision of the interviewer. In the sixth interview, Lena is able to justify her conclusion by making the following statement: “these two have to change all the time (points to T6 and T1) and these also (points to T5 and T2) and also those (points to T3 and T4)”. Hoffmann (2003, p. 260) observed that her youngest participants (7-9 years old) could not reposition their combinations with justification. There will be further analysis of the children’s development over two years especially qualitative interpretative analysis of the communication and interaction between child and interviewer (Bräuning & Steinbring 2011). The possibilities of young children to express their mathematical thinking is quite different to that of older children and adults. Looking at the approaches of young children, researchers should seek methods to observe and to analyze strategies of children who have limited possibilities to verbalize their thinking.

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