



HAL
open science

A Criterion for Piece-wise Mid-Frequency Vibro-acoustic Modelling Using the Condensed Transfer Function Approach

Zhongyu Hu, Laurent Maxit, Cheng Li

► **To cite this version:**

Zhongyu Hu, Laurent Maxit, Cheng Li. A Criterion for Piece-wise Mid-Frequency Vibro-acoustic Modelling Using the Condensed Transfer Function Approach. *Internoise 2017*, Sep 2017, Hong Kong, China. hal-02414819

HAL Id: hal-02414819

<https://hal.science/hal-02414819>

Submitted on 16 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



A Criterion for Piece-wise Mid-Frequency Vibro-acoustic Modelling Using the Condensed Transfer Function Approach

Zhongyu HU¹; Laurent MAXIT²; Li CHENG³

^{1,3}Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hong Kong, China

²Laboratoire Vibrations-Acoustique, INSA, Lyon, France

ABSTRACT

Analyses of vibro-acoustic systems in the mid-to-high frequency range are technically challenging due to the complex wavelength composition involved in different vibro-acoustic components. The problem needs to be tackled in order to cope with the crying need in various applications fields such as vehicles and buildings. In the present paper, an improved condensed transfer function (CTF) approach is proposed, using a benchmark model consisting of a plate-cavity system. In the improved CTF approach, the uncoupled subsystems are firstly modelled separately, by decomposing the force and velocity on the coupling surfaces between subsystems over a set of CTFs. The entire coupled system can then be assembled and solved by applying the force equilibrium and velocity continuity conditions over the coupling surfaces. The modelling efficiency is shown to be greatly increased by properly selecting the condensation functions in terms of their type and size. It is shown that a better spatial match between the wavy feature of the condensation functions and the wavelength of the system would allow a more efficient modelling within a prescribed frequency range. Given a set of truncated CTF series, a piece-wise convergence behavior is observed, leading to the proposal of a convergence criterion. The proposed convergence criterion allows the entire system to be modelled in any prescribed frequency band, one at a time in a piece-wise manner, thus resulting in a great reduction in the computational time. The proposed criterion is then tested using various excitation and damping configurations in one-third octave bands.

Keywords: vibro-acoustic, substructure method, mid and high frequency I-INCE Classification of Subjects Number(s): 76

1. INTRODUCTION

The development of dedicated methods for the mid-to-high frequency vibro-acoustic modelling has always been a challenge and arousing vast interest in the vibro-acoustic community. Methods based on sub-structuring philosophy are often employed in such cases. Upon the proper handling of the interfaces between the sub-systems, these methods allow reducing the number of degrees of freedoms involved in the calculation. Among existing methods, the Patch Transfer Functions (PTF) method is a representative example(1). The PTF method allows the subsystems to be modelled separately, before being assembled over the coupling surface based on continuity conditions. The coupling surface can be either the interface between a structural domain and an acoustic domain(2), or between two acoustic domains(1). The method consists in dividing the coupling surface into patches using a criterion based on the wavelength of the propagative waves on the coupling surface. Then, for each uncoupled subsystem, the transfer functions between each pair of the patches should be calculated. In the final stage, the patch transfer functions are assembled to predict the behavior of the global system. The method has been applied to airborne noise predictions(3), sound transmission problems in buildings(4), and transmission loss predictions in silencer designs(5) etc.. The method was extended and cast into a more general form, referred to as the Condensed Transfer Function (CTF) approach(6). The CTF approach extends the patches to any orthogonal function sets over the coupling surface such as the

¹ Zhongyu.hu@connect.polyu.hk

² laurent.maxit@insa-lyon.fr

³ li.cheng@polyu.edu.hk

complex exponential functions and the Chebyshev polynomials, referred to as the Condensation Functions (CF). This method has been applied to the modelling of non-axisymmetric internal frames for the investigation of the radiation efficiency and energy distributions (7). However, the research on CTF approach, has only focused on the cases of line coupling like frame-shell connections. This paper examines the use of different types of condensation functions to the problem of structure-acoustic coupling over a surface. The choice of different types of CFs as well as their impact on the modelling efficiency and accuracy will be the focus of the paper.

2. Theoretical Analyses

2.1 Basic Theory

Consider a rectangular acoustic cavity with one of its walls covered by a vibrating plate while the others being acoustically rigid, as shown in Fig.1. The plate, simply supported along all four edges, is subjected to a prescribed harmonic sound pressure excitation P^e with an angular frequency ω . The two subsystems, structural (plate) and acoustic (cavity), are coupled over their interface Ω , denoted by superscript s and a , respectively.

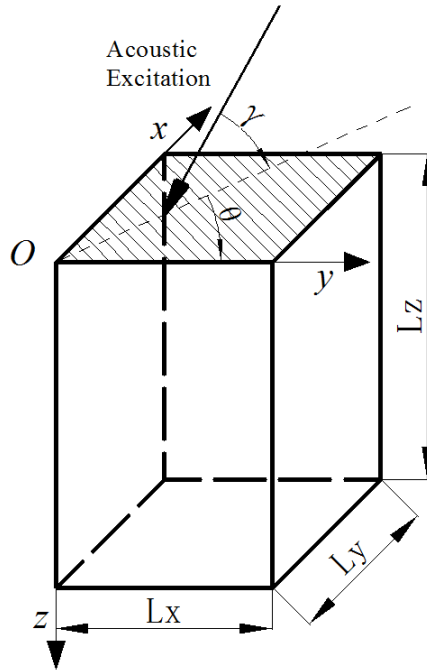


Fig. 1. The cavity-plate configuration and coordinate system

A set of N orthonormal functions $\{\varphi_i\}_{1 \leq i \leq N}$, referred to as *condensation functions* (CF), is employed to approximate the velocities and the forces on the coupling interface. The value of N should warrant the convergence of the results, to be discussed later. One assumes that the velocity $U^\alpha(x, y)$ and the force $P^\alpha(x, y)$ can be approximated by the condensation functions for subsystem $\alpha (\alpha \in [s, a])$ with a sufficient accuracy. Notice that the coupling interface is two dimensional.

For the structural subsystem, the condensed mobility between $\varphi_{ij}(x, y) = \varphi_i(x)\varphi_j(y)$ and $\varphi_{kl}(x, y) = \varphi_k(x)\varphi_l(y)$, is defined and calculated by applying an excitation pressure $P^s(x, y) = \varphi_i(x)\varphi_j(y)$ on Ω as:

$$Y_{kl,ij}^s = \frac{\langle \bar{U}_{ij}^s, \varphi_{kl} \rangle}{\langle P^s, \varphi_{ij} \rangle} = \langle \bar{U}_{ij}^s, \varphi_{kl} \rangle, \quad (1)$$

in which $\langle f, g \rangle$ is a scalar product defined as $\langle f, g \rangle = \int_{\Omega} f g^* dx dy$ with g^* being the complex conjugate of g , and \bar{U}_{ij}^s is the uncoupled velocity on Ω when the structural subsystem is subjected to an external excitation $P^s(x, y)$. Additionally, the uncoupled free condensed velocity \tilde{u}_{ij}^s of the structural subsystem is defined by

$$\tilde{u}_{ij}^s = \langle \tilde{U}^s, \varphi_{ij} \rangle, \quad (2)$$

where \tilde{U}^s is the uncoupled velocity of the structural subsystem at Ω when only external excitation exists.

For the acoustical subsystem, the condensed impedance $Z_{kl,ij}^a$ is defined by imposing a prescribed velocity $U^a(x, y) = \varphi_i(x)\varphi_j(y)$ on Ω as:

$$Z_{kl,ij}^a = \frac{\langle \bar{P}_{ij}^a, \varphi_{kl} \rangle}{\langle U^a, \varphi_{ij} \rangle} = \langle \bar{P}_{ij}^a, \varphi_{kl} \rangle, \quad (3)$$

where \bar{P}_{ij}^a is the uncoupled acoustic pressure on Ω when the acoustical subsystem is subjected to the prescribed velocity $U^a(x, y)$. Similarly, the uncoupled free condensed acoustic pressure of the acoustical subsystem is defined by

$$\tilde{p}_{ij}^a = \langle \tilde{P}^a, \varphi_{ij} \rangle, \quad (4)$$

where \tilde{P}^a is the uncoupled acoustic pressure of the acoustical subsystem at Ω when only internal sound sources exist. In the present case, this value is null.

Then one obtains the approximations of the uncoupled velocities and forces on the coupling surface for each subsystem:

$$\begin{cases} \tilde{U}^s(x, y) = \sum_i \sum_j \tilde{u}_{ij}^s \varphi_i(x) \varphi_j(y) \\ \tilde{U}^a(x, y) = \sum_i \sum_j \tilde{u}_{ij}^a \varphi_i(x) \varphi_j(y) \end{cases}, \text{ and } \begin{cases} \tilde{P}^s(x, y) = \sum_i \sum_j \tilde{p}_{ij}^s \varphi_i(x) \varphi_j(y) \\ \tilde{P}^a(x, y) = \sum_i \sum_j \tilde{p}_{ij}^a \varphi_i(x) \varphi_j(y) \end{cases}, \quad (5)$$

where u_{ij}^α and p_{ij}^α are the amplitudes of velocity and the pressure with respect to condensation function φ_{ij} .

On the other hand, the velocity continuity and force equilibrium principle over the coupling interface write

$$\begin{cases} U^s = U^a \\ P^s = -P^a \end{cases}. \quad (6)$$

According to the orthogonal property of the condensation functions, the following relationship can be deduced from Eq. (6):

$$\begin{cases} u_{ij}^s = u_{ij}^a \\ p_{ij}^s + p_{ij}^a = 0 \end{cases}. \quad (7)$$

Substituting Eq. (7) and Z^a into Eq. (5), the velocity on the coupling surface U^c can be finally calculated as:

$$U^c = [I + Y^s Z^a]^{-1} \tilde{u}. \quad (8)$$

2.2 CTF in the plate-cavity system

Following the definition of CTF, $Y_{kl,ij}^s$ can be calculated by applying an excitation pressure $P_{ij}^e(x, y) = \varphi_i(x)\varphi_j(y)$. Assuming $P_{ij}^e(x, y)$ is harmonic, the equation of motion of transverse displacement of the simply supported plate writes

$$-\omega^2 M^s w^s(x, y) + D^* \nabla^4 w^s(x, y) = \varphi_{ij}(x, y), \quad (9)$$

in which $w^s(x, y)$ is the plate displacement; M^s is the mass per unit area; ω is the angular frequency, $D^* = E^* h^3 / 12(1 - \nu^2)$ is the bending stiffness, and $E^* = E(1 + j\eta^s)$ is the complex Young's modulus with the structural damping taken into account with η^s being the damping loss factor. $w^s(x, y)$ can be decomposed over the *in-vacuo* plate modes as $w^s = \sum W_{rs}^s \psi_{rs}^s$, where W_{rs}^s and $\psi_{rs}^s = \sin\left(\frac{r\pi x}{L_x}\right) \sin\left(\frac{s\pi y}{L_y}\right)$ are the modal amplitude and displacement mode shape of the plate, respectively. One obtains the plate velocity:

$$U(x, y) = j\omega \sum_{r,s} \frac{\psi_{rs}^s \int_{\Omega} \varphi_{ij}(x, y) \psi_{rs}^s(x, y) dS}{M^s L_x L_y (\omega_{rs}^2 - \omega^2)}, \quad (10)$$

in which $\omega_{rs}^2 = \frac{D^*}{M^s} \left[\left(\frac{r\pi}{L_x}\right)^2 + \left(\frac{s\pi}{L_y}\right)^2 \right]^2$ is the natural frequency of the rs plate mode. Substituting Eq.

(9) into Eq. (1), one obtains the condensed mobility $Y_{kl,ij}^s$ for the vibrating plate,

$$Y_{kl,ij}^s = j\omega \sum_{r,s} \frac{\Gamma_{kl,rs}^{*s} \Gamma_{ij,rs}^s}{M^s L_x L_y (\omega_{rs}^2 - \omega^2)}, \quad (11)$$

in which $\Gamma_{ij,rs}^s = \int_{\Omega} \varphi_{ij} \psi_{rs}^s dS$ is the coupling coefficient between the condensation function φ_{ij} and the plate mode ψ_{rs}^s . Notice that $I_{ij,rs}^s$ is independent of the frequency. Therefore, the coupling matrix $[\Gamma^{*s}]$ and $[\Gamma^s]$ can be calculated beforehand to form a database before assembling at a later stage. In the present case, $[\Gamma^{*s}]$ and $[\Gamma^s]$ can be calculated analytically due to the simple geometry of the system. In a more complex scenario, numerical scheme or asymptotic approximations could be used, such as finite element method.

Similarly, the CTFs for the acoustic cavity can be obtained by setting a boundary velocity $U^a(x, y) = \varphi_{ij}(x, y)$ on Ω and solve the Helmholtz equation:

$$\nabla^2 p^a + \left(\frac{\omega}{c_0^*}\right)^2 p^a = 0, \quad (12)$$

in which $c_0^* = (1 + j\eta^a)c_0$ is the complex sound speed with a damping loss factor η^a of air taken into account.

The resulted pressure response $p^a(x, y, z)$ may be calculated using Green's formula and the modal expansion method.

The whole procedure yields

$$p_{ij}^a(x, y, z, \omega) = j\omega \rho_0 c_0^2 \sum_{n,m,p} \frac{\int_{\Omega} \varphi_{ij} \psi_{nmp}^a(x, y, 0) dS}{\Lambda_{nmp} (\omega_{nmp}^2 - \omega^2)} \psi_{nmp}^a(x, y, z), \quad (13)$$

in which $\psi_{nmp}^a = \cos\left(\frac{n\pi x}{L_x}\right) \cos\left(\frac{m\pi y}{L_y}\right) \cos\left(\frac{p\pi z}{L_z}\right)$ is the acoustic mode shape functions of the rigid-walled cavity; Λ_{nmp} and ω_{nmp} are the generalized acoustic modal mass and the nmp th acoustic natural frequency, respectively. According to the definition in Eq. (3), the condensed acoustic impedance $Z_{kl,ij}^a$ writes:

$$Z_{kl,ij}^a = j\omega\rho_0c_0^2 \sum_{n,m,p} \frac{\Gamma_{kl,nmp}^* \Gamma_{ij,nmp}}{\Lambda_{nmp}(\omega_{nmp}^2 - \omega^2)}. \quad (14)$$

in which $\Gamma_{ij,nmp} = \int_{\Omega} \varphi_{ij} \psi_{nmp}^a dS$ is the coupling coefficient between condensation function φ_{ij} and acoustic mode ψ_{nmp}^a . Substituting Eq. (11) and Eq. (14) into Eq. (8), the response on the coupling surface can be obtained accordingly.

2.3 Examples of CFs

2.3.1 Gate functions

The gate functions are defined as:

$$\varphi_{ij}(x, y) = \begin{cases} \frac{1}{\sqrt{L_{gx}L_{gy}}} & \text{if } (i-1)L_{gx} \leq x \leq iL_{gx}, (j-1)L_{gy} \leq y \leq jL_{gy} \\ 0 & \text{elsewhere} \end{cases} \quad (15)$$

where L_{gx} and L_{gy} are the length of the gate function and i and j are the gate indices x and y directions, respectively. As illustrated in Fig.2, each condensed mobility term $Y_{kl,ij}^s$ physically corresponds to the velocity response U_{kl} when a unit excitation P_{ij} is applied. When the gate functions are employed, the CTF method retreats to the PTF method as a special case of the general CTF approach. Owing to the intuitive and explicit physical meaning of the gate functions, the corresponding condensed mobility and impedance can be obtained using various calculation schemes, or even from experimental measurements.

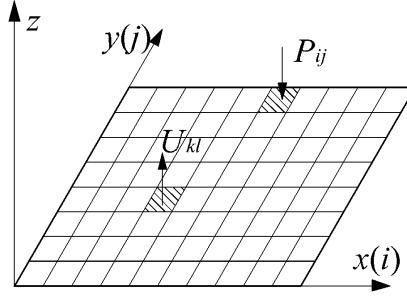


Figure 2. The working principle of gate functions

As to the convergence criterion, it is well established that at least two gate functions are needed to describe a wavelength (1), analogous to the spatial Shannon criterion used in signal processing. Therefore, for a given structural or acoustical wavelength λ , the length of the gate functions L_{gx} and L_{gy} should satisfy:

$$L_{gx} \leq \frac{\lambda}{2}, L_{gy} \leq \frac{\lambda}{2}. \quad (16)$$

2.3.2 Complex exponential functions

The complex exponential functions are defined by

$$\varphi_{ij}(x, y) = \frac{1}{\sqrt{L_x L_y}} \exp(\sigma \frac{2i\pi x}{L_x}) \exp(\sigma \frac{2j\pi y}{L_y}) \quad (17)$$

in which $\sigma = \sqrt{-1}$ is the imaginary unit, and $i \in [0, \pm 1, \pm 2, \dots, \pm I_{\max}^x]$ and $j \in [0, \pm 1, \pm 2, \dots, \pm I_{\max}^y]$ are the function indices in x and y directions, respectively. Different from the case of the gate functions in which the coupling interface is divided into more intuitive ‘patches’, the use of the exponential functions spatially decompose the velocity and the force over the entire coupling interface.

A truncation criterion has been proposed and has been validated in Ref. (7) for line coupled mechanical structures, expressed as:

$$I_{max}^x \geq \frac{2L_x}{\lambda} - 1 \quad \text{and} \quad I_{max}^y \geq \frac{2L_y}{\lambda} - 1. \quad (18)$$

3. Validation and Improvement

In the following numerical analyses, the dimension of the cavity is set to be $2.5\text{m} \times 2\text{m} \times 3\text{m}$ ($x \times y \times z$). The simply supported plate is 1.8mm thick, located at $z=0$ forming one wall of the enclosure. The plate has a Young's modulus $7.2 \times 10^{10}\text{Pa}$ with a Poisson's ratio 0.2 and mass density $2.53 \times 10^3\text{kg/m}^3$. Damping ratios η^a and η^p are set to 0.001 and 0.01 for the cavity and the plate, respectively. An oblique acoustic excitation with an amplitude of 1Pa is impinging on the plate, with both the dihedral angle and the intersection angle with x axis being 45° . The frequency band of interest is $[1, 1000]\text{Hz}$.

3.1 Pressure Validations

The sound pressure is calculated at a receiving point chosen at $(0.5, 1.3, 2)\text{m}$. The sound pressure level (SPL) results are given in Fig. 3, in which the CTF results using two different CFs are compared with the reference solution obtained from the reference results. The latter was calculated using modal method, which has been fully validated in a previous study. It can be seen that the pressure predictions by both types of CFs agree well with the reference result in the low frequency range. However, the performance of the gate functions slightly deteriorates as compared with the exponential functions in relatively high frequency range, especially at the troughs of the curves where system becomes less dynamic. Upon averaging the acoustic pressure within the entire cavity, the averaged overall SPL is shown in Fig. 6, leading to the same observation as that obtained in Fig. 5. Generally speaking, the truncation criteria allow acceptable calculation accuracy for both type of CFs, knowing however exponential functions perform better at non-resonance frequencies in both single point and overall predictions.

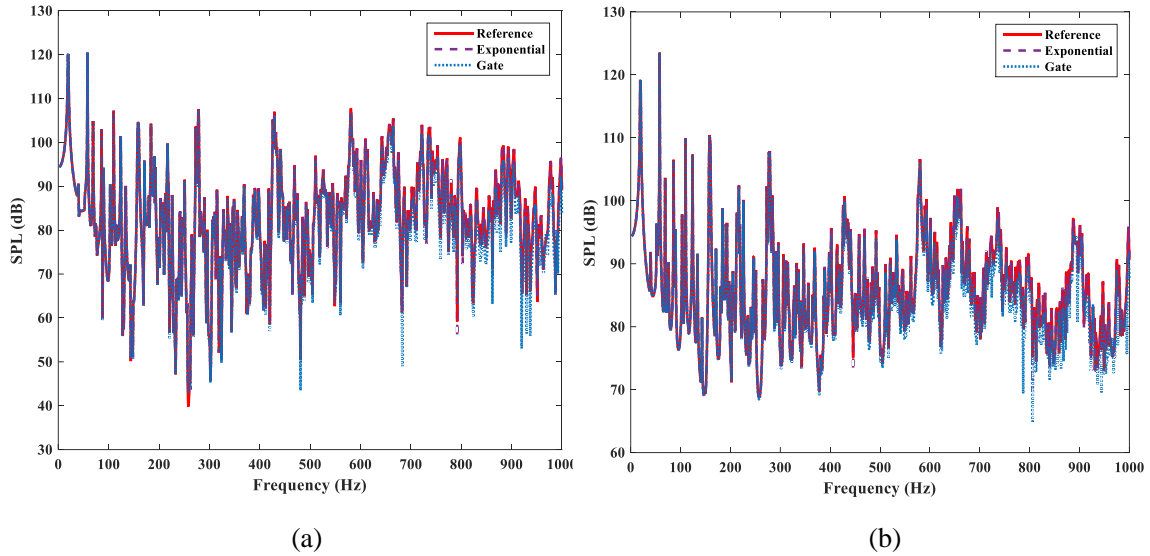


Fig. 3: SPL with reference to $2 \times 10^{-5}\text{Pa}$: (a) at $(0.5, 1.3, 2)\text{m}$; (b) overall within the cavity.

3.2 Approximations with CTF method

The approximation calculations will be focused on using the complex exponential functions, whose efficiency and advantage has already been shown in the last section. In this section, it will be seen that the high efficiency of the complex exponential functions is attributed to their wavy feature, which lead to a strong coupling to a certain wavelength range of the acoustic cavity and the vibrating structure.

Two generalized wavelengths λ_{ca} and λ_{cs} are defined to connect the condensation functions and the acoustic cavity and vibrating structural, respectively.

$$\lambda_{ca} = \frac{\lambda_c}{\lambda_a} \quad (19)$$

$$\lambda_{cs} = \frac{\lambda_c}{\lambda_s} \quad (20)$$

where λ_c is the wavelength of the exponential function $\varphi_{ij}(x, y)$, defined as

$$\lambda_c = \frac{2\pi}{\sqrt{\left(\frac{2j\pi}{L_x}\right)^2 + \left(\frac{2j\pi}{L_y}\right)^2}}, \quad (21)$$

λ_a and λ_s are the acoustic wavelength and structural wavelength, respectively, which are frequency dependent. When λ_{ca} is smaller than one, it refers to the stiffness modes of the cavity, otherwise, it refers to the inertia modes of the cavity. The same rule also applies to λ_{cs} .

For a targeted frequency band $[f_l, f_h]$ Hz with the wavelength range in $[\lambda_h, \lambda_l]$ m, where λ_l and λ_h are the longest and the shortest wavelength within all sub-systems, respectively, we include all CF terms satisfy $\lambda_h < \lambda_c < 2\lambda_l$. This selection criterion will be used to calculate the dynamic response of the plate-cavity system, and applied to calculate the overall pressure response of the cavity within the one third octave band [560, 707] Hz (with a central frequency 630Hz), with the results shown in Fig. 4. It can be observed that the system responses at the resonance frequencies are predicted well by using only 60 CF terms while the traditional convergence rule requires 80 terms. Slight discrepancies exist at some non-resonance frequencies especially in the higher frequency part of the band. The one-third octave band SPL error is calculated, giving only 0.06dB within the entire frequency band contained in Fig. 4. For the higher frequency band, the efficiency will be more evident because more long wavelength terms are neglected. Therefore, it is understandable that the proposed criterion is to guaranty the calculation result within a prescribed frequency band, instead of covering the entire low frequency range using traditional criterion, as shown in Fig. 5. It can be observed in Fig. 5 that the calculation is only focused on the targeted frequency band while the lower frequencies are abandoned. Nevertheless, the proposed criterion can be applied to each frequency band at a time in a piecewise manner so that the entire frequency range can be covered by moving the bands in a sequential way. This way, the calculation efficiency can be maximized in each frequency band of interest.

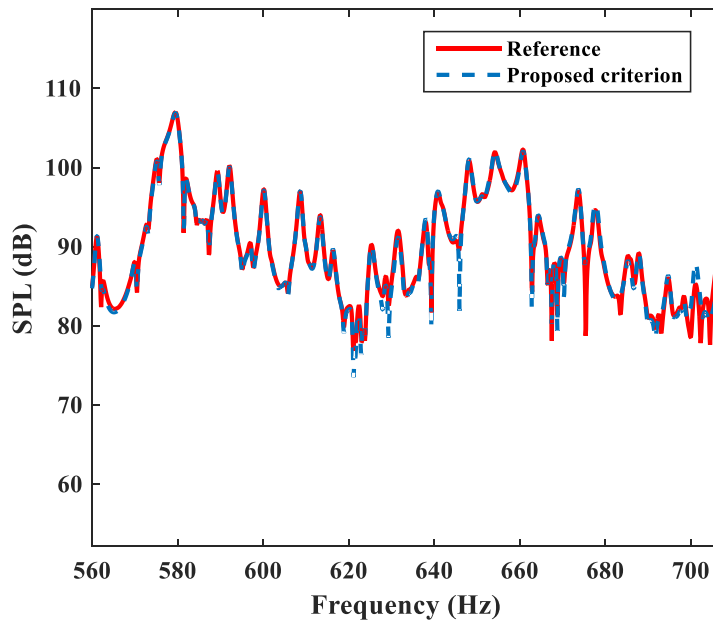


Fig. 4 SPL calculation under the proposed selection rule at the piecewise frequency band [560, 707] Hz.

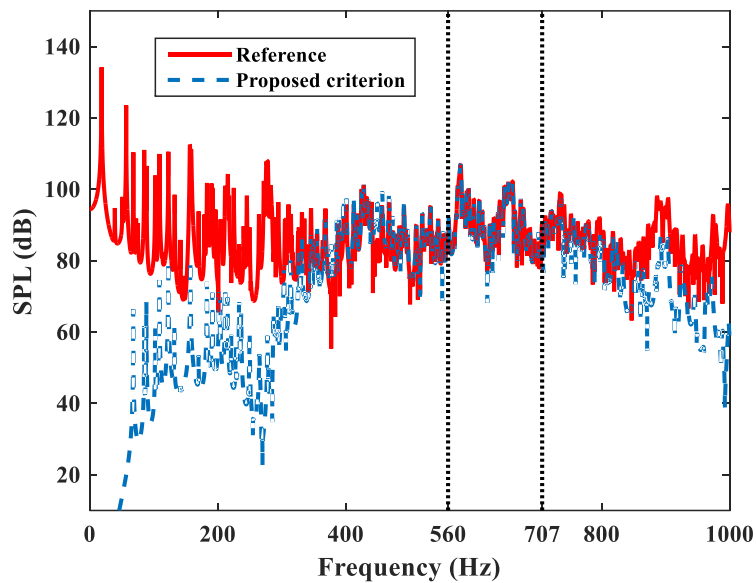


Fig. 5 The calculated result in [1, 1000]Hz with the selection criterion targeting to [560, 707]Hz.

4. Conclusions

Aiming at the modelling of the coupled vibro-acoustic system at mid-to-high frequency range, the condensed transfer function method has been re-examined. The method uses an orthonormal set functions to describe the force and velocity on the coupling surface of each subsystem, before being coupled together through the force equilibrium and velocity continuity. The condensation functions can be any functions satisfying the orthogonal condition. For a given targeted frequency band $[f_l, f_h]$ Hz, all exponential functions with their wavelengths λ_c delimited by $\lambda_h < \lambda_c < 2\lambda_l$ are advised to be included in the calculation so that the efficiency can be increased with a good accuracy being kept. The most significant advantage of the proposed criterion is that it allows accurate modelling of the system in a piecewise manner in terms of frequency bands at a much reduced calculation cost. Therefore, the criterion brings a significant increase in calculation efficiency when only the mid-to-high frequencies are focused. More discussions on the choice of the series sub-set, as well as the impact on the calculation accuracy and the frequency band to be covered will be given during the presentation.

REFERENCES

1. Ouisse M, Maxit L, Cacciolati C, Guyader J-L. Patch transfer functions as a tool to couple linear acoustic problems. *Journal of vibration and acoustics*. 2005;127(5):458-66.
2. Maxit L, Aucejo M, Guyader JL. Improving the patch transfer function approach for fluid-structure modelling in heavy fluid. *Journal of Vibration and Acoustics*. 2012;134(5):051011.
3. Maxit L, Cacciolati C, Guyader J. Airborne noise prediction using patch acoustic impedance. *Proceedings of ICSV*. 2002;9.
4. Chazot J-D, Guyader J-L. Prediction of transmission loss of double panels with a patch-mobility method. *The Journal of the Acoustical Society of America*. 2007;121(1):267-78.
5. Yu X, Cheng L, You X. Hybrid silencers with micro-perforated panels and internal partitions. *The Journal of the Acoustical Society of America*. 2015;137(2):951-62.
6. Meyer V, Maxit L, Guyader JL, Leissing T, Audoly C. A condensed transfer function method as a tool for solving vibroacoustic problems. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2015;230(6):928-38.
7. Meyer V, Maxit L, Guyader JL, Leissing T. Prediction of the vibroacoustic behavior of a submerged shell with non-axisymmetric internal substructures by a condensed transfer function method. *Journal of Sound and Vibration*. 2016;360:260-76.