Annular acoustic black holes to reduce propagative Bloch- Floquet flexural waves in periodically supported cylindrical shells

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ABSTRACT
Current designs of acoustic black holes are mainly intended for straight beams and flat plates. However, many structures of interest in naval and aeronautical applications essentially consist in a periodically stiffened cylindrical shell, which could benefit from the acoustic black hole (ABH) effect to reduce vibrations and noise radiation. In this work, we suggest the design of annular ABHs to that purpose. To test the feasibility of such annular ABHs we consider the idealized case of a periodically simply supported cylindrical shell of infinite length. Such a structure allows for the propagation of Bloch-Floquet flexural waves at some passbands, which can play an important role in the radiation of noise at the far field. By means of wave finite element models, we show that the proposed annular ABHs constitute an effective way of reducing the shell flexural motion.

Keywords: Acoustic black holes, Periodic structures, Bloch-Floquet waves, Wave finite element, Cylindrical shells
I-INCE Classification of Subject Number: 42

1. INTRODUCTION

It is well-known that the vibroacoustic behavior of infinite structures, like beams or plates, with periodic supports or stiffeners, is characterized by the formation of wave frequency stopbands and passbands [1,2]. In this work we are interested in the case of

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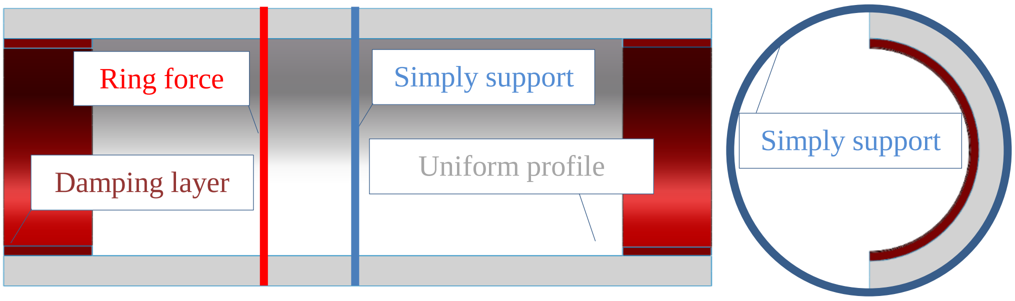

Figure 1 Unit cell for the uniform cylindrical shell.

periodically simply supported cylindrical shells [3], which are of importance in many naval and aeronautical applications. In particular, we address the problem of trying to suppress propagative Bloch-Floquet (BF) in the shell. Those may result in the radiation of noticeable noise at the acoustic far-field [4]. To that purpose, we suggest the design of annular acoustic black holes (ABHs) and check their efficiency in reducing vibrations by means of finite element simulations.

Acoustic black holes consisting of power-law profiles tending to zero thickness have been proposed for beam terminations to reduce flexural vibrations [5], and several methods have been investigated in literature to improve their performance [6-9]. ABHs in the form of circular cuneate indentations have been also intended and tested in plates [10-12]. Besides, rectangular ABHs were analyzed in [13-14], while ring-shaped ABHs for vibration isolation in plates have been recently advocated in [15]. Beams and plates with periodic ABH distributions were reported in [16,17]. It is worthwhile mentioning that the ABH concept has been also used for acoustic wave attenuation at duct terminations [18,19].

In the current work, we embed annular ABHs with power law profile in the longitudinal axis of an infinite cylindrical shell with periodic simple supports. A wave finite element model is employed to test their performance in the reduction of BF waves, by comparison with a uniform cylindrical shell without ABHs. Dispersion curves are presented for the infinite shells, as well as transmissibility functions for points within a finite structure consisting of ten unit cylindrical cells.

2. WAVE FINITE ELEMENT MODEL

We consider the case of an infinite cylindrical shell with periodic simple supports, its geometry and physical parameters being detailed in Table 1. The supports are separated apart a distance of 1.35 m and the shell is submitted to a ring force distribution, 1 N in amplitude. A thin damping layer is attached at the inner side of the shell to attenuate BF waves. To analyze the performance of such a structure we focus on a unit cell, the support being placed on its center (see Figure 1). Given that the damping layers have only limited impact on the propagative waves, we propose the design and insertion of annular ABHs in the uniform shell to attenuate wave transmission, which, as said before, have power law profiles in the longitudinal direction. The unit cell for the cylindrical shell with an ABH indentation is shown in Figure 2, and its parameters are also described in Table 1. The same amount of damping is included in both, the uniform and ABH shell configurations for a fair comparison.

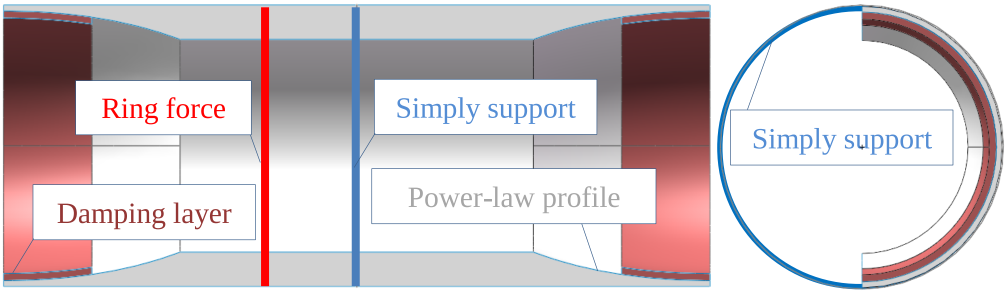

Figure 2 Unit cell for the ABH cylindrical shell.

The condensed unit cell mass and stiffness matrices **M** and **K** for the uniform and ABH cylindrical shells can be obtained by means of the finite element method (note that **K** will be complex if structural damping is assumed). At a given angular frequency ω, the equation of motion of the cell reads

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{q}=\mathbf{f}\ , \tag{1}$$

with $\mathbf{q}=[\mathbf{q}_L\ \mathbf{q}_R]^{\mathrm{T}}$ being the displacement vector after condensation of the inner nodes, and $\mathbf{f}=[\mathbf{f}_L\ \mathbf{f}_R]^{\mathrm{T}}$ the force vector at the FEM nodes (the subscripts L and R refer to the left and right end sides of the unit cell). Applying the periodic boundary condition

$$\begin{aligned}\mathbf{q}_R&=\lambda\mathbf{q}_L\\ \mathbf{f}_R&=-\lambda\mathbf{f}_L\end{aligned}\ , \tag{2}$$

where $\lambda=\exp(-\mathrm{i}kL)$ and k represents the wave vector propagating in the structure, we get

$$\begin{aligned}\mathbf{q}&=\mathbf{\Lambda}_R\mathbf{q}_L\\ \mathbf{f}&=\mathbf{\Lambda}_L\mathbf{q}_L\end{aligned}\ , \tag{3}$$

with $\mathbf{\Lambda}_R=[\mathbf{I}\ \ \lambda\mathbf{I}]^{\mathrm{T}}$ and $\mathbf{\Lambda}_L=[\mathbf{I}\ \ \lambda^{-1}\mathbf{I}]^{\mathrm{T}}$. Here **I** stands for the identity matrix. One can then pre-multiply both sides of Equation 1 by $\mathbf{\Lambda}_L$ to obtain

Table 1 Geometry and physical parameters of the uniform and ABH cylindrical shells

Parameter	Value	Description
R	5 m	Radius of the cylindrical shells
t_{uni}	0.03 m	Thickness of the uniform shell
t_0	0.006 m	Residual thickness of the ABH center
L	1.35 m	Length of 1 cell
L_{abh}	0.325 m	Length of the ABH portion
L_v	0.1625 m	Length of the damping layers
t_v	0.006 m	Thickness of the damping layers
E	210 GPa	Young modulus of the shell
ρ	7800 kg/m^3	Density of the shell
E_v	5 GPa	Young modulus of the damping layers
ρ_v	950 kg/m^3	Density of the damping layers
η	0.5	Loss factor of the damping layers

$$\mathbf{\Lambda}_L\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{q}=\mathbf{\Lambda}_L\mathbf{f}\ . \tag{4}$$

Given that $\mathbf{q}=\mathbf{\Lambda}_R\mathbf{q}_L$ and that force equilibrium demands $\mathbf{\Lambda}_L\mathbf{f}=\mathbf{0}$, substitution into Equation 4 results in an eigenvalue problem for the FEM nodal displacements on the left side of the unit cell,

$$\left[\mathbf{K}(k)-\omega^2\mathbf{M}(k)\right]\mathbf{q}_L=\mathbf{0}\,, \tag{5}$$

where $\mathbf{K}(k)=\mathbf{\Lambda}_L\mathbf{K}\mathbf{\Lambda}_R$ and $\mathbf{M}(k)=\mathbf{\Lambda}_L\mathbf{M}\mathbf{\Lambda}_R$ denote the reduced stiffness and mass matrices. Limiting the range of k in $[-\pi/L,\pi/L]$ (i.e. to the first Bernoulli zone), one can obtain the dispersion curve for the periodically simply supported cylindrical shell.

3. RESULTS

3.1 Uniform shell

Let us first focus on the results for the infinite uniform shell with periodic supports. The dispersion curves for the frequency range [0, 1000] Hz in terms of the wavenumber are shown in Figure 3a. As observed, four stopbands and passbands are clearly identified beyond the shell ring frequency. In Figure 3b, we plot the transmission between two points located in a finite cylindrical shell of uniform thickness, consisting of ten unit cells. The points are placed in the first and last cells. As observed by direct comparison with Figure 3a, the transmissibility function strongly drops at the stopbands preventing vibration transmission. As opposed, BF waves get transmitted for the passband frequencies, the transmissibility reaching 25 dB. The inclusion of damping layer strips helps in reducing the peak values but do not result in substantial improvement. For better illustrating this phenomenon, in Figure 4a we have plotted the vibration shape of half the unit cell at 585 Hz, which corresponds to the central frequency of the third stopband. Figure 4b presents the vibration shape at 738 Hz, the central frequency of the third passband (see Figure 3a). The differences are apparent.

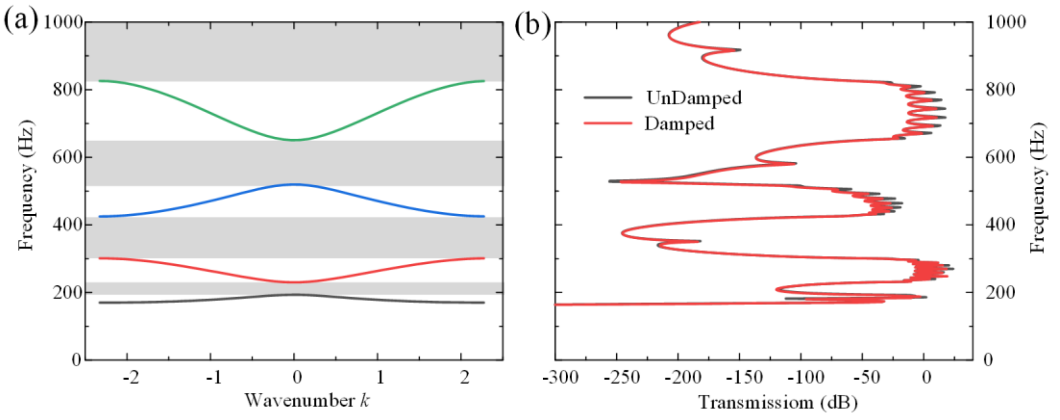

Figure 3 (a) Dispersion curves for the undamped infinite uniform shell (b) Transmissibility for the finite 10 cells uniform shell.

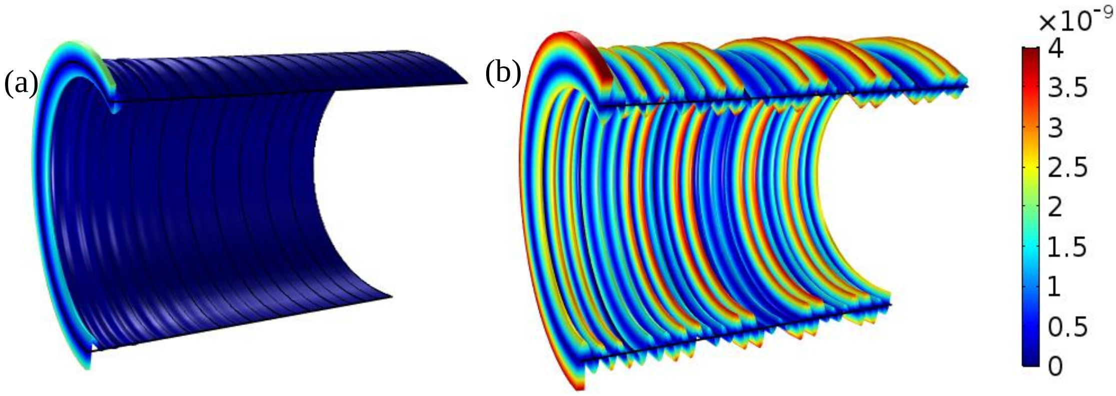

Figure 4 Vibration shape at (a) 585 Hz (center of the third stop band), (b) 738Hz (center of the third pass band)

3.2 ABH shell

To mitigate the transmission of BF waves we set annular ABHs in the infinite cylindrical shell (see Figure 2). The dispersion curves for the new periodic structure are shown in Figure 5a. The stiffness of the ABH unit cell is less rigid than that of the uniform cell so the number of stopbands and passbands in the analyzed frequency range increases. On the other hand, the transmissibility for the finite 10 ABH cell structure is presented in Figure 5b. Analogously to the uniform case, there is negligible transmission at the stopbands but some for the passbands. Note that as one could expect from the ABH physical principles, it is seen in the figure that including damping remarkably improves the transmissibility, especially at high frequencies. The great advantage of resorting to annular ABHs to reduce BF wave propagation becomes finally very clear in Figure 6, where we compare the transmissibilities of the uniform and the ABH 10 unit cell shells, for the damped and undamped cases.

To end with, the vibration shapes at the central frequency of the third stopband and third passband are plotted in Figures 7a and 7b, analogously to what was done in Figures 4a and 4b for the uniform shell. The figures confirm the efficiency of the annular ABHs in reducing BF waves. One should bear in mind, however, that the residual thickness at the center of the ABH has been reduced to one fifth of the uniform one (see Table 1). In practice, this will weaken the shell and one should check whether that is admissible or not in terms of structural resistance.

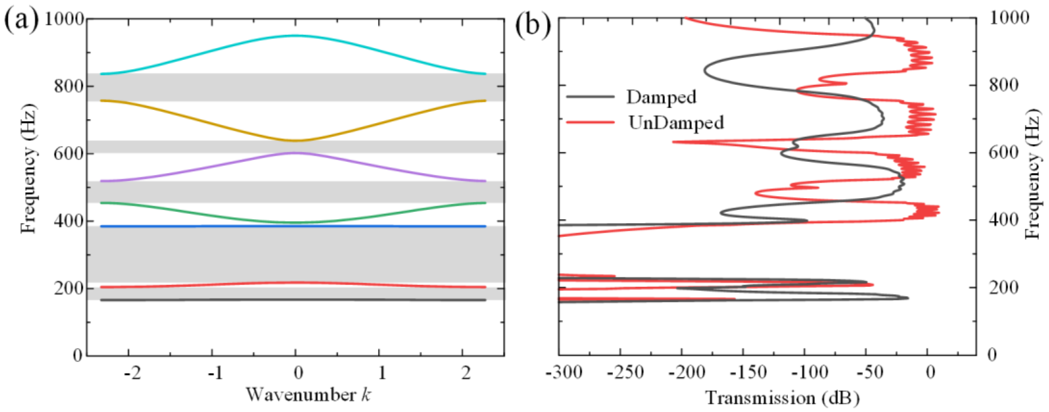

Figure 5 (a) Dispersion curves for the undamped infinite ABH shell and (b) Transmissibility for the finite 10 cells ABH shell.

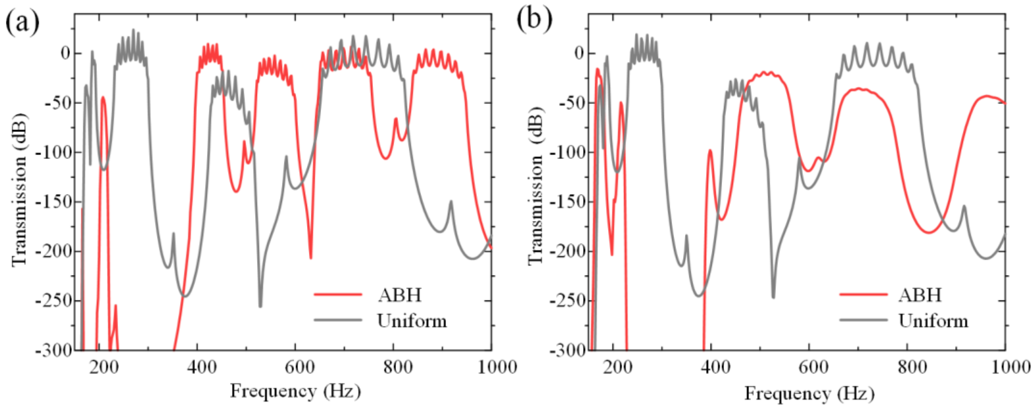

Figure 6 Comparison between the transmissibilities of the uniform and ABH cylindrical shells, (a) undamped shells, (b) damped shells.

4. CONCLUSIONS

In this paper it has been proposed to reduce propagative Bloch-Floquet waves in periodically supported cylindrical shells by means of annular ABHs. The influence of the ABHs in the dispersion curves of shells of infinite length has been reported. Besides, we have checked the efficiency of the annular ABHs analyzing the transmissibility function between points in a finite cylindrical shell, consisting of ten simply supported unit cells. The new annular ABH design opens the door to benefit from the ABH effect in curved structures. Future work will involve developing a semi-analytical model for a quick parameter analysis, as well as inserting stiffeners to compensate for the weakening effect of the ABH residual thickness.

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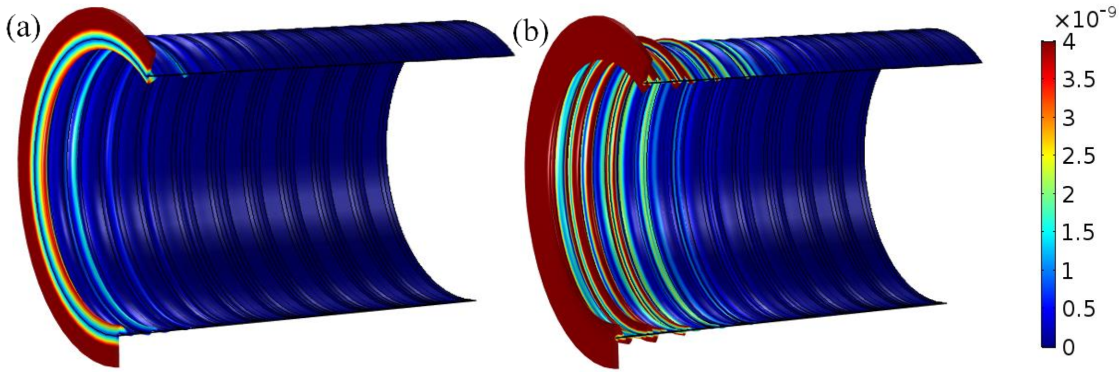

Figure 7 vibration shape at (a) 486 Hz (center of the third stop band), (b) 560 Hz (center of the third pass band).

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