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8 A hybrid numerical approach to predict the vibrational
9 responses of panels excited by a turbulent boundary
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28 **Abstract**

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30 In this work, a hybrid numerical approach to predict the vibrational responses
31 of planar structures excited by a turbulent boundary layer is presented. The
32 approach combines an uncorrelated wall plane wave technique with the finite
33 element method. The wall pressure field induced by a turbulent boundary
34 layer is obtained as a set of uncorrelated wall pressure plane waves. The
35 amplitude of these plane waves are determined from the cross spectrum den-
36 sity function of the wall pressure field given either by empirical models from
37 literature or from experimental data. The response of the planar structure
38 subject to a turbulent boundary layer excitation is then obtained from an
39 ensemble average of the different realizations. The numerical technique is
40 computationally efficient as it rapidly converges using a small number of
41 realizations. To demonstrate the method, the vibrational responses of two
42 panels with simply supported or clamped boundary conditions and excited
43 by turbulent flow are considered. In the case study comprising a plate with
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63 simply supported boundary conditions, an analytical solution is employed
64 for verification of the method. For both cases studies, numerical results from
65 the hybrid approach are compared with experimental data measured in two
66 different anechoic wind tunnels.
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68 *Keywords:* Uncorrelated wall plane wave, finite element method, turbulent
69 boundary layer, flow-induced vibration
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71 72 73 **1. INTRODUCTION**

74
75 The vibrational response of elastic structures subject to flow excitation is
76 a seminal problem in a variety of technical applications, for example, an air-
77 craft fuselage excited by turbulent boundary layer, a hydrofoil operating in
78 turbulent flow or a telecommunications tower excited by wind. In such cases,
79 the correct prediction of the vibrational response is crucial to minimise struc-
80 tural fatigue as well as structure-borne radiating noise (Leibowitz, 1975; Boily
81 and Charron, 1999; Ciappi et al., 2014, 2018). To predict the vibrational re-
82 sponses for such problems, the forcing function arising from the turbulent
83 flow field on the elastic structure should be obtained. This can be achieved
84 by solving the Navier Stokes equations for given geometry and flow condi-
85 tions. Numerical approaches such as direct numerical simulation (DNS) or
86 large eddy simulation (LES) can be implemented to solve the Navier Stokes
87 equations as well as take into account interaction of the flow with the body.
88 However, these methods are computationally demanding as their implemen-
89 tation for simulation of realistic scenarios often become impractical due to
90 the significant range of spatial and temporal scales of the turbulence that
91 need to be resolved (Shtilman and Chasnov, 1992). An alternative approach
92 involves a steady-state Reynolds-averaged Navier Stokes (RANS) solution to
93 predict the turbulent boundary layer (TBL) parameters (Bailly et al., 1997;
94 Peltier and Hambric, 2007). RANS is an attractive approach as it is capable
95 of predicting TBL parameter mean values with good fidelity. These param-
96 eters can then be used as an input to analytical or semi-empirical models
97 to predict the wall pressure field under the turbulent boundary layer (Blake,
98 1986; Lee et al., 2005; Chen and MacGillivray, 2014).
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101 A vast number of studies have been carried out to predict the vibrational
102 responses of plates excited by a turbulent flow field in air, including analytical
103 models of infinite and finite plates (Strawderman, 1969; De Rosa and Franco,
104 2008), numerical models (Birgersson et al., 2003; Hambric et al., 2004; Birg-
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120 30 ersonsson and Finnveden, 2005; De Rosa and Franco, 2008; Hong and Shin, 2010;
121 31 Ciappi et al., 2016), and from experiments (Ciappi et al., 2016). Birgersson
122 32 et al. (2003) proposed a wavenumber domain approach to investigate the
123 33 response of structures to TBL excitation. The cross spectral density of the
124 34 TBL wall pressure was expressed as a finite Fourier series. The structural
125 35 response to each term in the series was calculated using the spectral finite
126 36 element method (FEM) and the total response was then obtained using the
127 37 superposition principle. A spectral super element formulation for modelling
128 38 plate vibration excited by distributed forces was developed by Birgersson
129 39 and Finnveden (2005). Ichchou et al. (2009) employed "rain-on-the-roof"
130 40 excitation represented by statistically independent point sources, to excite a
131 41 flat plate in the mid frequency range, showing good agreement with results
132 42 obtained using an FEM model. Turbulence induced vibration of aerospace
133 43 composite plates was numerically and experimentally investigated by Ciappi
134 44 et al. (2016). It was demonstrated that at high Mach number, the fluid-
135 45 loading effect on the composite panels cannot be neglected. More recently,
136 46 Marchetto et al. (2017) compared the responses of a panel excited by a TBL
137 47 as well as by a diffuse acoustic field.

140 48 De Rosa et al. (2015) presented a pseudo deterministic excitation method
141 49 to evaluate the dynamic response of a linear system excited by a turbulent
142 50 load. The method achieved a significant reduction in computational time
143 51 compared to the full stochastic solution. Errico et al. (2018) proposed an
144 52 approach to model structures excited by aerodynamic and acoustic sources
145 53 based on a wave finite element method. The approach was employed to pre-
146 54 dict flow-induced vibrations of periodic flat and curved structures (Errico
147 55 et al., 2019). Similitude laws for scaling the vibration response of flat pan-
148 56 els to a turbulent boundary layer excitation were proposed by Franco et al.
149 57 (2019). These laws remove the necessity of repeating experiment or numer-
150 58 ical simulations due to the change in flow speeds, dimensions and material
151 59 properties of panel.

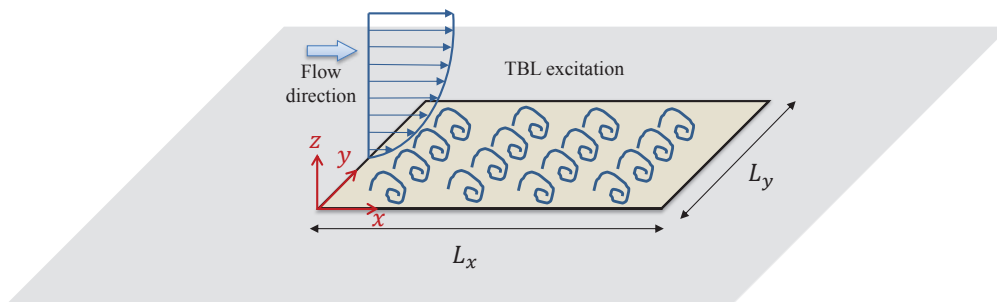
153 60 In many numerical approaches to predict vibrational responses of struc-
154 61 tures excited by turbulent flow, the system response depends on the cross
155 62 spectrum density (CSD) function of the wall pressure fluctuations. In order
156 63 to correctly describe the partial correlation of the excitation, a large number
157 64 of points distributed on the structural surface from which the frequency re-
158 65 sponse functions are calculated need to be considered (Hambric et al., 2004;
159 66 Hong and Shin, 2010). The coupling between a statistical model to describe
160 67 the wall pressure fluctuations and a deterministic numerical model of the

169 structure represents a difficulty in the calculation process. The current study
 170 proposes a way of addressing this difficulty by employing a hybrid numerical
 171 approach involving the coupling between statistical and deterministic meth-
 172 ods, to predict the vibrational response of a panel under TBL excitation. To
 173 this end, the turbulent boundary layer excitation is modelled using a set of
 174 uncorrelated wall plane waves. The synthesized wall pressure corresponding
 175 to each realization within the set of uncorrelated wall plane waves is applied
 176 to a standard FEM model of the panel. This process is repeated for each
 177 realization of the wall pressure field. The vibrational response of the panel
 178 is then obtained from an ensemble average of the different realizations of the
 179 wall pressure. To demonstrate the technique, two case studies are considered
 180 corresponding to simply supported and clamped plates. The vibrational re-
 181 sponse is predicted numerically and analytically (for the simply supported
 182 plate). The results for both case studies are validated with experimental
 183 data.

193 2. Mathematical formulation

194 2.1. Hybrid numerical approach

195 Figure 1 shows an elastic rectangular baffled panel with arbitrary bound-
 196 ary conditions. The plate is excited by a turbulent flow field. It is assumed
 197 that the TBL is homogeneous, stationary and fully developed over the panel
 198 surface. Further, it is assumed that the wall pressure field (WPF) is not
 199 altered by the vibration of the panel.
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213 Figure 1: A baffled panel under TBL excitation

214
 215 The uncorrelated wall plane wave technique (UWPW) recently introduced
 216 by Maxit (2016) is used to simulate the pressure field beneath a turbulent
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232 boundary layer. The UWPW approach is summarized in what follows. The
233 space-frequency cross spectrum of the wall pressure fluctuations is given by
234 (Graham, 1997; Maxit, 2016)

$$235 \quad S_{pp}^{\text{TBL}}(\mathbf{x} - \mathbf{x}', \omega) = \Psi_{pp}(\omega) \left(\frac{U_c}{\omega} \right)^2 \tilde{S}_{pp}(\mathbf{x} - \mathbf{x}', \omega), \quad (1)$$

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239 where $\Psi_{pp}(\omega)$ and $\tilde{S}_{pp}(\mathbf{x} - \mathbf{x}', \omega)$ are respectively the auto spectrum density
240 (ASD) function and normalized CSD function of the pressure field. U_c is the
241 convective velocity, ω is the angular frequency, and \mathbf{x} , \mathbf{x}' correspond to point
242 locations on the plate. The CSD of the wall pressure field in the physical
243 space is related to the CSD of the wall pressure spectrum in the wavenumber
244 domain, denoted by $\phi_{pp}(\mathbf{k}, \omega)$, using a spatial Fourier transform as follows
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$$246 \quad S_{pp}^{\text{TBL}}(\mathbf{x} - \mathbf{x}', \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \phi_{pp}(\mathbf{k}, \omega) e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} d\mathbf{k}, \quad (2)$$

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250 where $i = \sqrt{-1}$ is the imaginary unit and \mathbf{k} is the wavevector with compo-
251 nents k_x and k_y in the streamwise and spanwise directions, respectively. The
252 cross spectrum of the wall pressure can be computed using different models
253 for the ASD of the pressure field and the normalized CSD of the pressure
254 field, denoted by $\tilde{\phi}_{pp}(\mathbf{k}, \omega)$, independently from each other as follows
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$$256 \quad \phi_{pp}(\mathbf{k}, \omega) = \Psi_{pp}(\omega) \left(\frac{U_c}{\omega} \right)^2 \tilde{\phi}_{pp}(\mathbf{k}, \omega). \quad (3)$$

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258 The improper integral in equation (2) can be approximated using the
259 rectangular method by truncating and sampling the wavenumber space as
260 follows (Maxit, 2016)
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$$264 \quad S_{pp}^{\text{TBL}}(\mathbf{x} - \mathbf{x}', \omega) \approx \frac{1}{4\pi^2} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi_{pp}(k_x^i, k_y^j, \omega) e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} \delta k_x \delta k_y. \quad (4)$$

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268 where δk_x , δk_y are the wavenumber resolutions in the streamwise and span-
269 wise directions, respectively. N_x and N_y are the number of points considered
270 along the k_x and k_y directions. The total pressure beneath a turbulent bound-
271 ary layer is now represented by a set of UWPWs. As the wall plane waves
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are uncorrelated, the CSD function of the wall pressure fluctuations induced by a set of wall plane waves can be written as

$$S_{pp}^{\text{UWPW}}(\mathbf{x} - \mathbf{x}', \omega) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \Lambda_{ij} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}, \quad (5)$$

where Λ_{ij} is the ASD function of the stochastic amplitude of UWPW with indices i and j . Equating equations (4) and (5) allows the CSD function of the pressure field by the TBL to be approximately equal to the CSD function of the UWPWs if the amplitudes of the UWPWs are

$$\Lambda_{ij} = \frac{\phi_{pp}(k_x, k_y, \omega) \delta k_x \delta k_y}{4\pi^2}. \quad (6)$$

The amplitude of each wall pressure plane wave is defined such that the set of UWPWs represent the statistical properties of the WPF generated by the TBL. The statistical model to describe the WPF can now be coupled to a deterministic model based on the FEM. This important step allows the WPF to be expressed as a deterministic load input to the FEM. The WPF for the l^{th} realization of the UWPW can be expressed by (Maxit, 2016; Karimi et al., 2019)

$$p_{\text{inc}}^l(\mathbf{x}, \omega) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sqrt{\Lambda_{ij}} e^{i(k_x x + k_y y + \varphi_{ij}^l)}, \quad (7)$$

where φ is a random phase uniformly distributed in $[0, 2\pi]$, expressing that the waves are uncorrelated. Substituting equation (6) into equation (7) and considering the q^{th} node of an FEM mesh, the nodal pressure can be written as follows

$$p_{\text{inc}}^l(\mathbf{x}^q, \omega) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sqrt{\frac{\phi_{pp}(k_x, k_y, \omega) \delta k_x \delta k_y}{4\pi^2}} e^{i(k_x x^q + k_y y^q + \varphi_{ij}^l)}. \quad (8)$$

Using equation (8) as the deterministic load, FEM is now implemented to compute the l^{th} realization of the structural displacement \mathbf{u}^l by solving the following linear system of equations

$$\mathbf{D}\mathbf{u}^l = \mathbf{f}^l, \quad (9)$$

where \mathbf{D} is the dynamic stiffness given by

$$\mathbf{D} = \mathbf{K} - i\omega\mathbf{C} - \omega^2\mathbf{M}, \quad (10)$$

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343 **K**, **C** and **M** are respectively stiffness, damping and mass matrices of the
344 structure, and \mathbf{f}^l is the force vector corresponding to the l^{th} realization of
345 the TBL pressure field given by equation (8). It should be noted that the
346 mass and stiffness matrices are not frequency dependent and need to be
347 constructed only once for a given geometry. After the inverse of the dynamic
348 stiffness matrix is obtained, the plate displacement response can be computed
349 for each realization as follows

$$352 \quad \mathbf{u}^l = \mathbf{D}^{-1}\mathbf{f}^l. \quad (11)$$

354 The ASD of the plate displacement due to the TBL excitation is then calcu-
355 lated from the ensemble average of the different realizations by

$$357 \quad S_{uu} = \text{E} [\mathbf{u}^l \overline{\mathbf{u}^l}]_l, \quad (12)$$

359 where $\text{E} [\]$ represents the ensemble average over the realizations and the
360 overline denotes the complex conjugate. This process is repeated for each
361 frequency to obtain the spectra of the structural response.

362 Figure 2 illustrates the computational sequences for the UWPW-FEM
363 approach. First, a mesh is created from the geometry. A RANS simulation,
364 theoretical formula or experimental data can be used to estimate the TBL
365 parameters over the surface of a structure for a given geometry and flow
366 condition. The cross-spectrum of the wall pressure is evaluated from the TBL
367 parameters using semi-empirical models. The spectra of the wall pressure
368 is then applied in conjunction with the UWPW technique to obtain the
369 WPF. The WPF is then used as an input to the FEM solver to compute
370 the vibrational response. This process is repeated for each realization of
371 the WPF. Finally, the structural response of the system is obtained from an
372 ensemble average of the different realizations of the wall pressure fields at
373 each frequency.

374 2.2. Analytical model

375 For verification of the hybrid numerical approach, an analytical model is
376 herein presented for the case of a plate with simply supported boundary con-
377 ditions on all its edges. The vibrational response of a panel due to excitation
378 by a pressure field can be obtained using the modal expansion method and
379 by identifying a panel sensitivity function. The ASD of the panel velocity is

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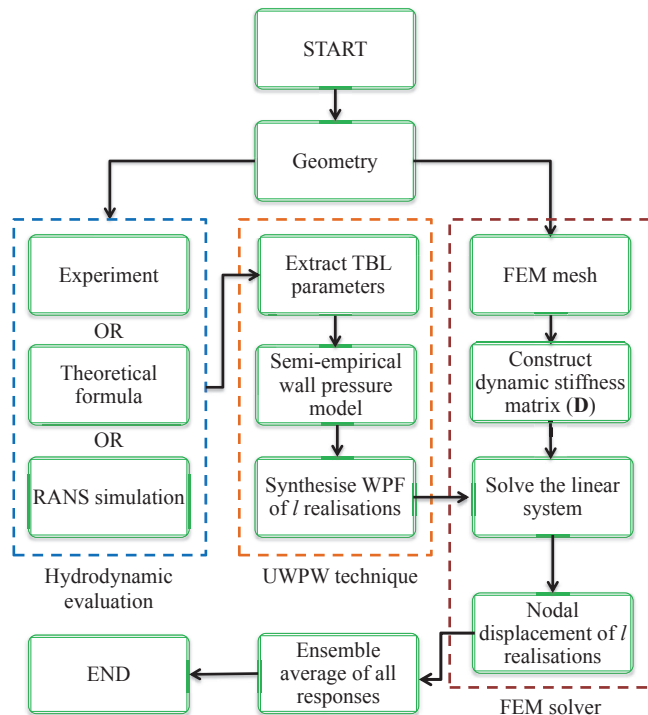


Figure 2: Flowchart outlining the computational process of the hybrid UWPW-FEM approach

120 given by (Maury et al., 2002)

$$S_{vv}(\mathbf{x}, \omega) = \frac{1}{4\pi^2} \int_{\infty} \phi_{pp}(\mathbf{k}, \omega) |H(\mathbf{x}, \mathbf{k}, \omega)|^2 d\mathbf{k}, \quad (13)$$

121 where $H(\mathbf{x}, \mathbf{k}, \omega)$ is the sensitivity function which corresponds to the velocity
122 at point \mathbf{x} when the panel is excited by a unit wall plane wave. $\phi_{pp}(\mathbf{k}, \omega)$
123 is the CSD of the wall pressure field defined previously. Using numerical
124 integration based on the rectangular method by truncating and regularly
125 sampling the wavenumber domain, equation (13) can be expressed as

$$S_{vv}(\mathbf{x}, \omega) \approx \frac{1}{4\pi^2} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi_{pp}(\mathbf{k}, \omega) |H(\mathbf{x}, \mathbf{k}, \omega)|^2 \delta k_x \delta k_y, \quad (14)$$

Using the normal modes of the panel, an analytical solution for the panel

sensitivity function can be derived as follows (Maxit, 2016; Marchetto et al., 2017)

$$H(\mathbf{x}, \mathbf{k}, \omega) = i\omega \sum_{m=1}^M \sum_{n=1}^N \frac{\psi_{mn}(\mathbf{k})\varphi_{mn}(\mathbf{x})}{\Omega_{mn}(\omega_{mn}^2 - \omega^2 + i\eta\omega\omega_{mn})}, \quad (15)$$

where M and N are the truncation number of modal orders in the x and y directions, respectively. For the modal expansion, the number of modes within the extended frequency band $[0 \ 1.5\omega_{\max}]$ were considered, where ω_{\max} represents the highest angular frequency of interest. η is the structural loss factor, $\Omega_{mn} = \rho_s h L_x L_y / 4$ is the modal mass, ω_{mn} and φ_{mn} are respectively the modal frequencies and mode shapes of the panel given by

$$\omega_{mn} = \sqrt{\frac{D}{\rho_s h} \left(\left(\frac{m\pi}{L_x} \right)^2 + \left(\frac{n\pi}{L_y} \right)^2 \right)}, \quad (16)$$

$$\varphi_{mn}(\mathbf{x}) = \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right). \quad (17)$$

L_x and L_y are respectively the length and width of the plate. $D = Eh^3/(12(1-\nu^2))$ is the flexural rigidity, ρ_s is the density, h is the panel thickness, E is the Young's modulus and ν is Poisson's ratio. The modal forces ψ_{mn} are calculated by integration over the panel surface A as follows

$$\psi_{mn}(k_x, k_y) = \int_A e^{i(k_x x + k_y y)} \varphi_{mn}(\mathbf{x}) dA = I_m^x I_n^y \quad (18)$$

where

$$\{I_s^r | (r, s) = (x, m) \vee (y, n)\} = \left\{ \begin{array}{ll} \left(\frac{s\pi}{L_r} \right) \frac{(-1)^s e^{i(k_r L_r)} - 1}{k_r^2 - \left(\frac{s\pi}{L_r} \right)^2}, & k_r \neq \frac{s\pi}{L_r} \\ \frac{1}{2} i L_r, & \text{otherwise} \end{array} \right\}. \quad (19)$$

3. Results and discussion

To demonstrate the UWPW-FEM approach, two case studies comprising rectangular panels with different boundary conditions and excited by a TBL are examined. The first case study investigates a panel with simply supported boundary conditions. Numerical results for the simply supported

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 512 panel from the UWPW-FEM technique as well as analytical results obtained
 513 using the sensitivity function are compared with experimental data obtained
 514 in an anechoic wind tunnel. The second case study examines the vibrational
 515 response of a panel with clamped boundary conditions. Dimensions and ma-
 516 terial properties of both panels are given in Table 1. The fluid density and
 517 the kinematic viscosity were set to 1.225 kg/m^3 and $1.5111 \times 10^{-5} \text{ m}^2/\text{s}$, re-
 518 spectively. For each case study, the structural loss factor was experimentally
 519 estimated using the -3 dB bandwidth method for the first few resonances
 520 of the plate. The mean value of the loss factors was used in the numer-
 521 ical simulations. The simulations were conducted using Matlab on a desktop
 522 personal computer with 32 GB of RAM and a total of four physical cores.
 523 For the UWPW-FEM technique, the wall pressure field was synthesized in
 524 Matlab and then imported as a load to the FEM model of the panel in the
 525 commercial software COMSOL Multiphysics (v5.3a) using Matlab LiveLink.
 526

527 To obtain the panel response analytically or numerically, truncation of
 528 wavenumber domain is required. A truncated number of wavenumbers in
 529 the x and y directions need to be defined for equations (8) and (14). The
 530 criterion for defining the cut-off wavenumbers in the streamwise and span-
 531 wise directions must be chosen such that the significant contributions of the
 532 integrands of these equations are correctly taken into account. It has been
 533 previously demonstrated that in the vibration response, the wavenumbers
 534 below or close to the natural flexural wavenumber of the plate are dominant
 535 (Hambric et al., 2004; Maxit, 2016; Marchetto et al., 2018). Hence, a cut-off
 536 wavenumber of $2k_{p,\max}$ was used in both the streamwise and spanwise direc-
 537 tions, where $k_{p,\max} = (\omega_{\max} \sqrt{\rho_s h / D})^{1/2}$ is the flexural wavenumber of the
 538 plate at the maximum frequency of interest denoted by ω_{\max} . The wavenum-
 539 ber resolutions were set to $\delta k_x = \delta k_y = 0.25 \text{ (1/m)}$. Based on the cut-off
 540 wavenumber, a mesh size of $\Delta x = \Delta y = \pi / (2k_{p,\max})$ was selected in this
 541 work. A frequency resolution of 2 Hz was used in all numerical simulations.
 542 It is worth noting that if a very wide frequency range is considered, the fre-
 543 quency range can be divided into frequency bands. A different mesh size
 544 based on the highest frequency of interest for a given band can be employed,
 545 thereby increasing the efficiency of the method.
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549 3.1. Case study A - Simply supported plate

550 The first case study considers a simply supported baffled rectangular
 551 panel used in the experiment by Marchetto et al. (2018) which was con-
 552 ducted in an anechoic wind tunnel at the Université de Sherbrooke. The
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Table 1. Dimensions and mechanical properties of the panels

Parameter	Case A	Case B
	Simply supported plate	Clamped plate
Young's modulus E (GPa)	70	200
Poisson's ratio ν	0.3	0.29
Density ρ (kg/m ³)	2700	7850
Length L_x (mm)	480	470
Width L_y (mm)	420	370
Thickness h (mm)	3.17	1.59
Damping loss factor η	0.005	0.005

169 experimental fabrication method proposed by Robin et al. (2016) was used
 170 to replicate simply supported boundary conditions. The panel was made
 171 of aluminium and the edge of the panel was placed 1.8 m from the nozzle.
 172 The vibration of the panel was measured using an accelerometer located at
 173 ($x = 0.3$ m, $y = 0.33$ m, $z = 0$ m) on the panel surface as shown in Figure 3.
 174 The experiments were conducted at flow speeds of 20 m/s and 40 m/s.

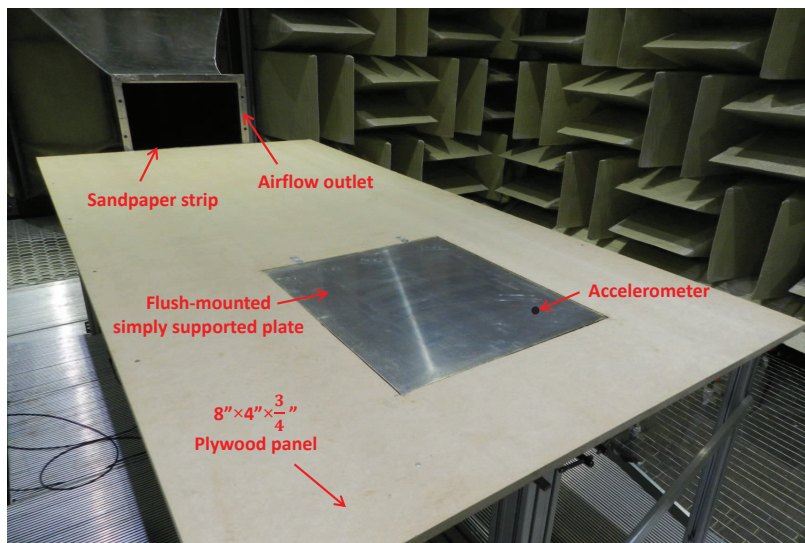


Figure 3: Experimental set-up in the anechoic wind tunnel at the Université de Sherbrooke (Marchetto et al., 2018). The accelerometer location is at ($x = 0.3$ m, $y = 0.33$ m, $z = 0$ m)

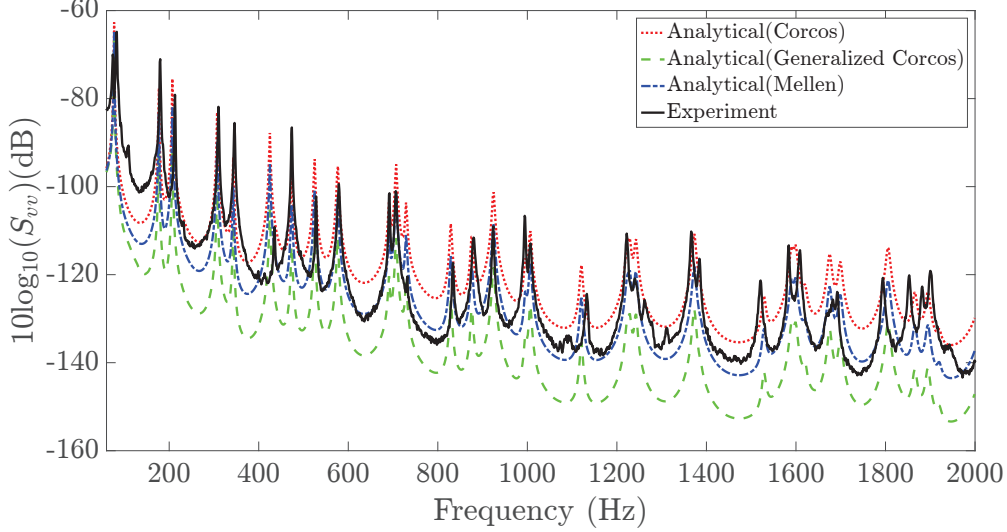


Figure 4: Predicted velocity spectra using different TBL models at flow velocity $U_\infty = 40$ m/s versus wind tunnel measurements (dB ref. $1 \text{ (m/s)}^2/\text{Hz}$).

3.1.1. Vibration response using the sensitivity function of the panel

Figure 4 compares the velocity spectra predicted analytically with the experimental data at a flow speed of 40 m/s, for a frequency range from 60 Hz to 2 kHz. The Goody model described in Appendix A was used to evaluate the ASD function of the wall pressure field. Note that $\Psi_{pp}(\omega)$ is a one-sided radial frequency spectrum. Hence $\Psi_{pp}(\omega)$ was multiplied by 2π to convert it to a one-sided cyclic frequency spectrum density $\Psi_{pp}(f)$. For the normalized CSD function, the Corcos, generalized Corcos and Mellen models described in Appendix B were employed, whereby analytical results for each model are compared. The TBL parameters were calculated based on theoretical formula for a flat plate from literature. The boundary layer thickness δ and the displacement thickness δ^* are given by (Çengel and Cimbala, 2006)

$$\delta \cong \frac{0.38x}{\text{Re}_x^{0.2}}, \quad \delta^* \cong \frac{0.048x}{\text{Re}_x^{0.2}} \quad (20)$$

where Re_x represents the Reynolds number and x corresponds to the distance from the nozzle to the centre of the panel. The wall shear stress τ_w was calculated using the following empirical relations (Hambric et al., 2004)

$$\tau_w \cong \frac{0.0225\rho_f U_\infty^2}{(8U_\infty\delta^*/\nu)^{0.25}} \quad (21)$$

where U_∞ is the free flow velocity, ρ_f is the fluid density and ν is the kinematic viscosity. The convective velocity U_c was approximated as follows (Bull, 1967)

$$U_c \cong U_\infty(0.59 + 0.3e^{-0.89\delta^*\omega/U_\infty}). \quad (22)$$

It can be observed from Figure 4 that analytical results using the Mellen model are in very good agreement with experimental data. Compared to experimental results, results obtained using the Corcos model are over predicted but under predicted using the generalized Corcos model. Over-prediction of the pressure spectrum at low wavenumbers using the Corcos model has been previously reported (Graham, 1997). Discrepancy between predicted results for the three TBL models is examined in further detail in Appendix C.

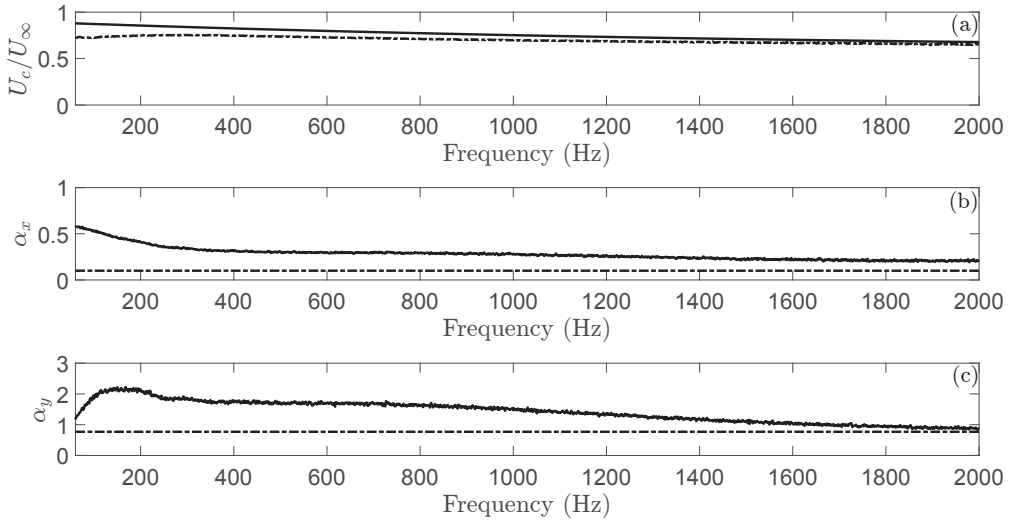


Figure 5: Comparison between TBL parameters extracted from measurements (solid line) and the standard TBL parameters from literature (dotted-dash line) at $U_\infty = 40$ m/s, (a) convective speed normalized by the flow velocity, (b) streamwise exponential decay rate α_x , (c) spanwise exponential decay rate α_y

The input TBL parameters for the Mellen and Goody models corresponding to the convective velocity, boundary layer thickness, displacement thickness and wall shear stress were obtained using theoretical and empirical equations from the literature given by equations (20)-(22). In the experiment, the Mellen model was fitted to the measured wall pressure field using the least square method to estimate the decay rates, α_x and α_y , and the convective

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736 189 velocity U_c (Marchetto et al., 2018). Figure 5 compares the convective speed
737 190 and exponential decay coefficients as a function of frequency for the Mellen
738 191 model extracted from the wind tunnel measurement with the standard val-
739 192 ues of these parameters from the literature for a flat plate. Figure 5 shows a
740 193 good match between the convective speed given by equation (22) and those
741 194 estimated from the experiment in the current frequency range. However, the
742 195 measured values of the decay coefficients are larger than those commonly
743 196 found in the literature. The experimental ASD function of the wall pressure
744 197 is also presented as a function of frequency in Figure 6 and compared with
745 198 predicted results by the Goody model. At low frequencies, the spectral level
746 199 is under-predicted by the Goody model. However above 600 Hz, the Goody
747 200 model produces larger spectral values compared to the measured data. The
748 201 maximum discrepancy occurs at very low frequency as can be observed in
749 202 Figure 6.

751 203 Figure 7 presents two analytical results for the vibration responses of the
752 204 panel as well as the experimental velocity spectra as a reference solution.
753 205 Analytical¹ represents the vibration response of the panel when the standard
754 206 Mellen and Goody models are implemented. Analytical² corresponds to the
755 207 predicted velocity spectra using the experimental ASD function of the wall
756 208 pressure field and the Mellen model for the CSD function using the experi-
757 209 mental input parameters. For the latter, the measured TBL parameters in
758 210 Figure 5 were substituted into the Mellen model to evaluate the normalized
759 211 CSD function, which was then used with the experimental pressure spectrum
760 212 to calculate the forcing function for the panel. The two analytical results are
761 213 in very good agreement with the measured data. Better agreement with
762 214 the reference solution was observed at low frequencies using measured TBL
763 215 input parameters (Analytical²). This is due to the difference between the
764 216 experimental ASD function and the Goody model as well as discrepancy be-
765 217 tween standard Mellen model and experimentally fitted Mellen model (see
766 218 Appendix C.).

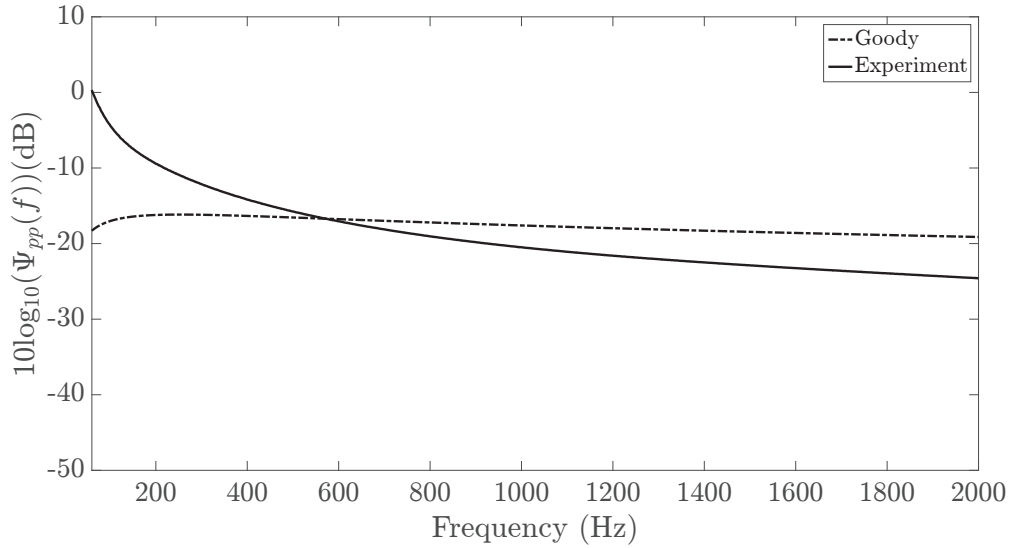


Figure 6: Measured ASD function of the wall pressure versus predicted results using the Goody model at $U_\infty = 40$ m/s (dB ref. $1 \text{ Pa}^2/\text{Hz}$)

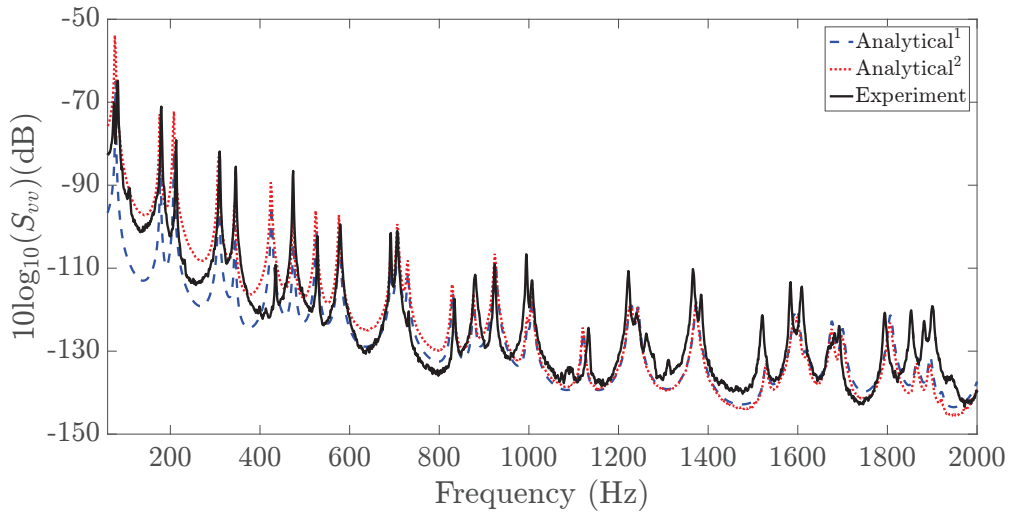


Figure 7: Velocity spectra obtained using the standard Mellen and Goody models (Analytical¹), using the measured pressure spectrum and experimentally fitted Mellen model (Analytical²), and from wind tunnel measurements (dB ref. $1 \text{ (m/s)}^2/\text{Hz}$)

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219 *3.1.2. Vibration response using the hybrid UWPW-FEM approach*

220 The UWPW technique described in Section 2.1 was used to synthesize the
221 pressure field at the surface of the panel. The standard Mellen and Goody
222 models were herein used to evaluate the CSD function of the pressure field.
223 Figure 8 shows the visualization of a realization of the surface pressure field at
224 two discrete resonance frequencies corresponding to 177 Hz and 1005 Hz, for
225 a flow speed of 40 m/s. The Goody and Mellen models were respectively used
226 for the ASD and normalized CSD functions of the pressure field. Figure 8(a)
227 shows that at low frequencies, a coarse mesh can resolve the waves as they
228 have larger wavelengths. However, at higher frequencies, a finer mesh is
229 needed to properly describe and synthesize the wall pressure field for plane
230 waves with short wavelengths (Figure 8(b)). In this work, the criteria used
231 for the mesh size ensures that the plane waves with the shortest wavelength
232 corresponding to the highest frequency of interest are adequately resolved.
233 A mesh of 24 elements in the streamwise direction and 21 elements in the
234 spanwise direction was used. The displacement of the panel at two discrete
235 resonance frequencies 177 Hz and 1005 Hz are respectively shown in Figure 9
236 and Figure 10, using the single realization of the wall pressure given by
237 Figures 8(a) and (b) as well as 30 realizations of the WPF. Whilst a similar
238 pattern for displacement fields can be observed using a single realization and
239 30 realizations of the WPF, at least 30 realizations of the WPF are required
240 to obtain a converged solution at the highest frequency of interest considered
241 here. The effect of the number of realizations on the structural response of the
242 panel is shown in Figure 11. The analytical solution described in Section 2.2
243 is also provided to verify the UWPW-FEM results. A close match between
244 the numerical and analytical results can be observed. Using 30 realizations,
245 the maximum estimated error in the calculation of the panel response was less
246 than 1 dB for the frequency range considered here. As such, 30 realizations
247 was used for all subsequent calculations.

248 Figure 12 presents the predicted velocity spectra using the UWPW-FEM
249 approach as well as measured velocity spectra for the simply supported panel
250 at flow speeds of 20 m/s and 40 m/s. The numerical results are in excellent
251 agreement with experimental data. As expected, with increasing flow speed,
252 the magnitude of the vibrational response of the panel increases. A slight
253 difference between resonant frequencies predicted numerically and obtained
254 experimentally is observed, whereby the first ten resonances are listed in Ta-
255 ble 2 for each case study. This can be attributed to differences in the panel

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 904 material properties as well as in the implementation of the boundary conditions in the numerical model and the experiment. To further demonstrate capability of the UWPW-FEM approach, velocity spectra at higher flow speeds of 60 m/s and 80 m/s are shown in Figure 13. Numerical results show good agreement with those obtained analytically. For the parameters chosen here, the aerodynamic coincidence frequency is 7.3 Hz, 29 Hz, 66 Hz and 117 Hz for flow speeds of 20 m/s, 40 m/s, 60 m/s and 80 m/s, respectively. At these frequencies, the flexural wavenumber $k_p = (\omega\sqrt{\rho_s h/D})^{1/2}$ equals the convective wavenumber $k_c = \omega/U_c$ and is given by $f_c = U_c^2\sqrt{\rho_s h/D}/(2\pi)$ and TBL strongly excites the structure (Marchetto et al., 2018).

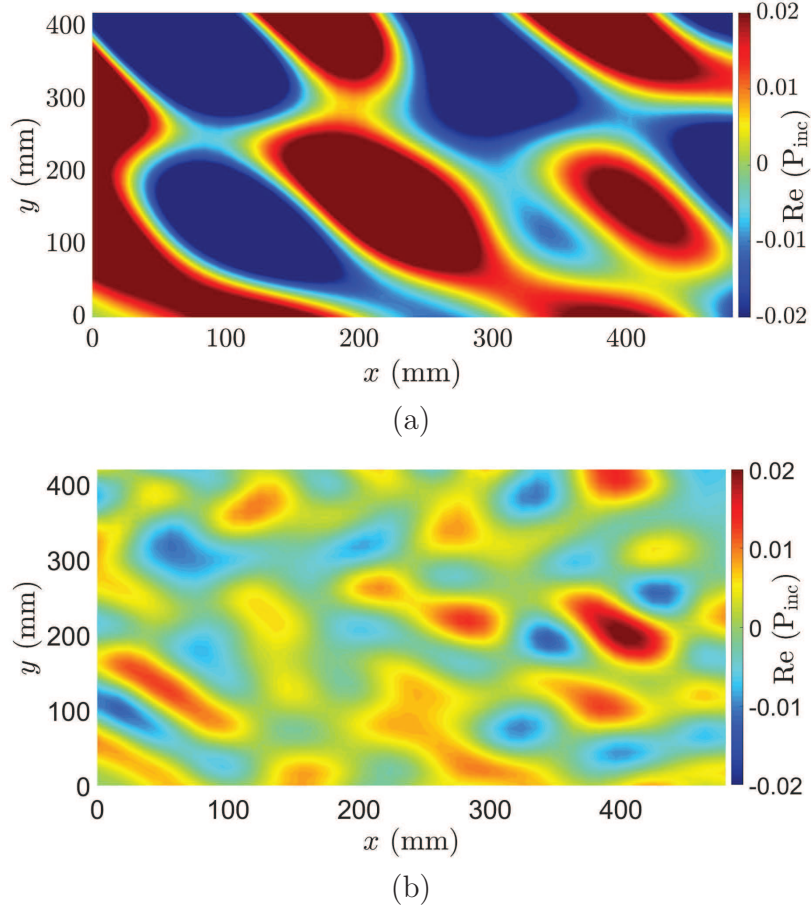


Figure 8: A realization of the wall pressure field using the Mellen and Goody models for a flow speed of 40 m/s at (a) 177 Hz and (b) 1005 Hz

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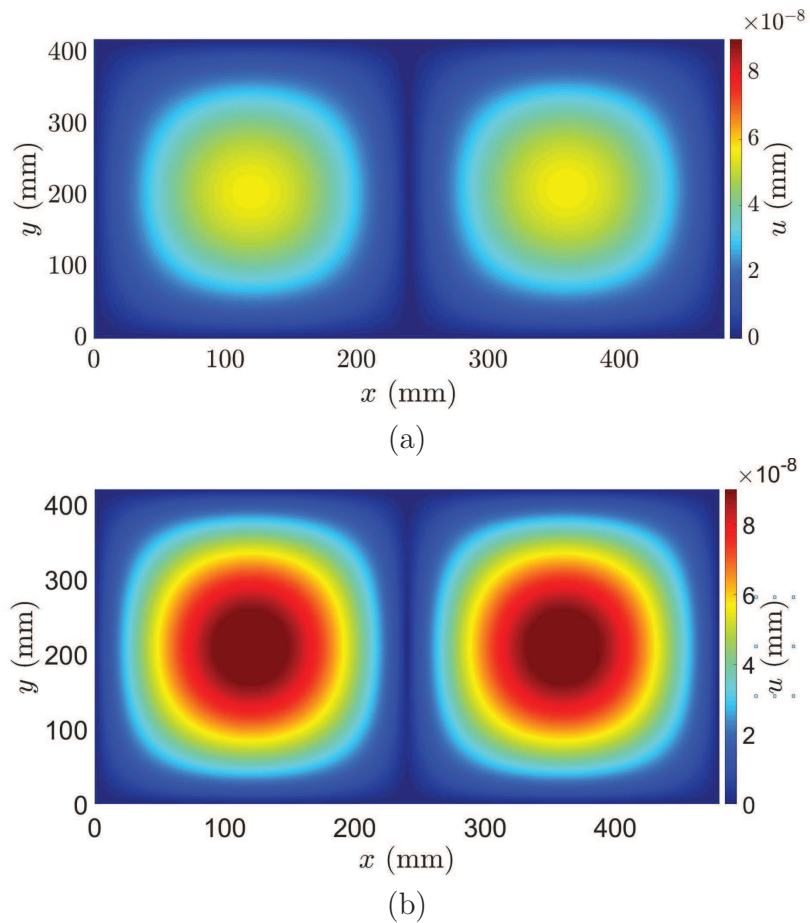
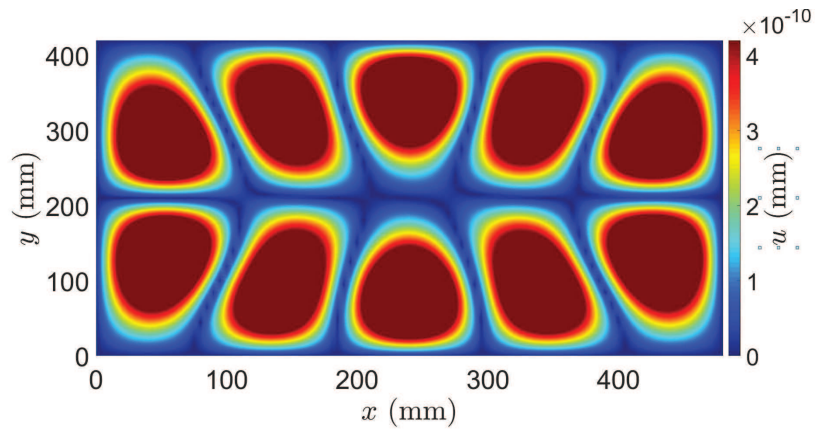
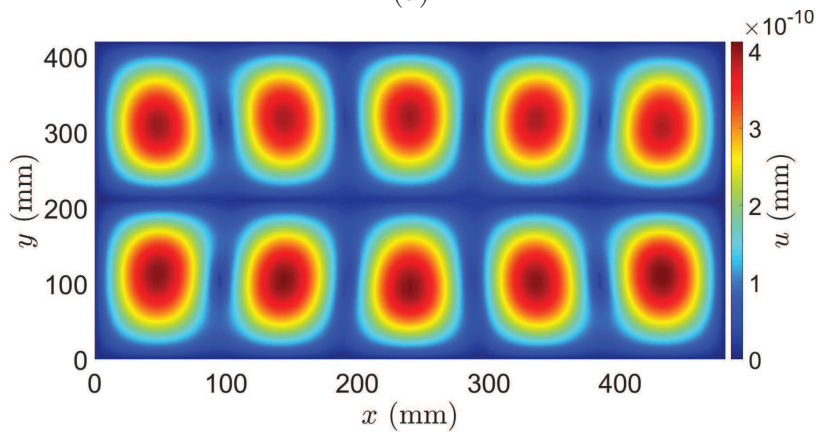


Figure 9: Panel displacement field for a flow speed of 40 m/s at 177 Hz using (a) one realization of the WPF corresponding to the WPF shown in Figure 8(a) and (b) 30 realizations of the WPF

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(a)



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Figure 10: Panel displacement field for a flow speed of 40 m/s at 1005 Hz using (a) one realization of the WPF corresponding to the WPF shown in Figure 8(b) and (b) 30 realizations of the WPF

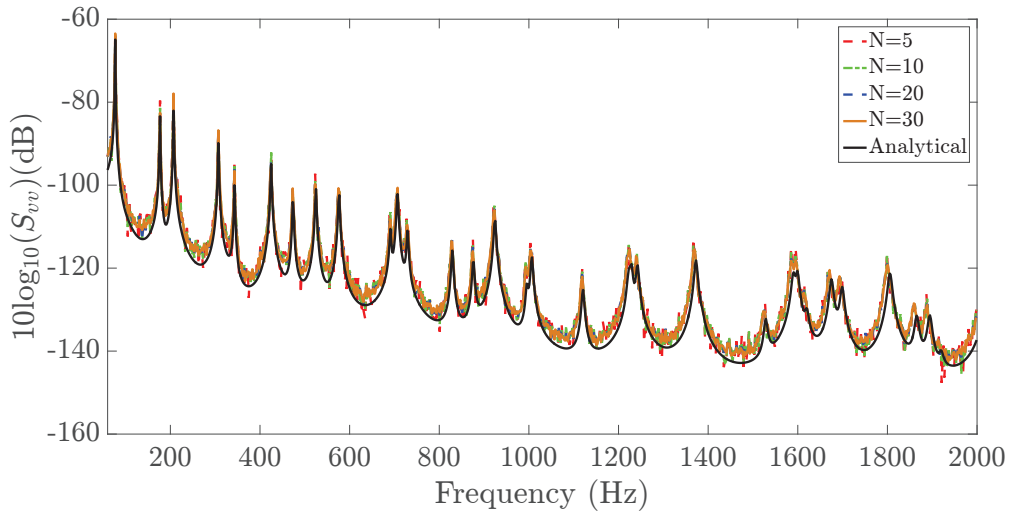
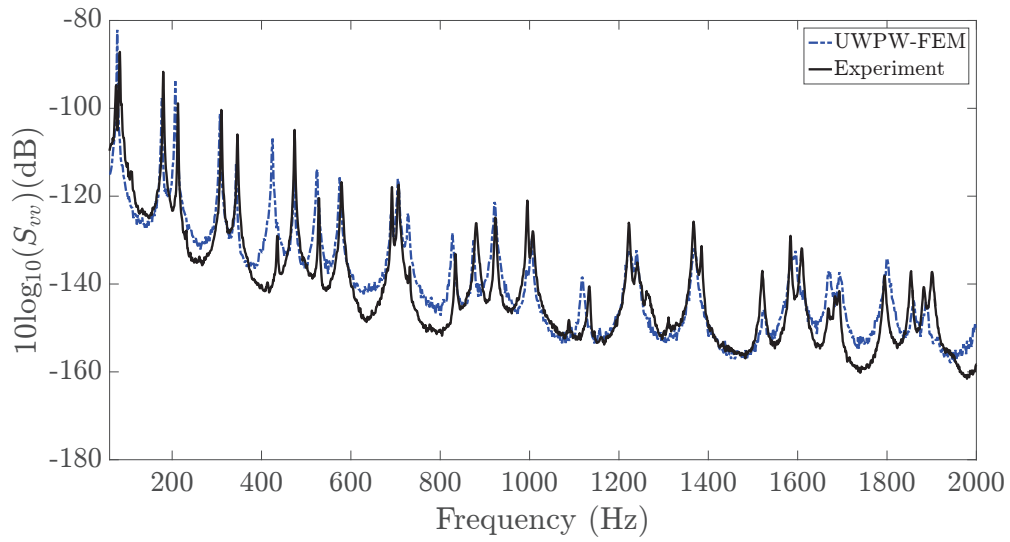
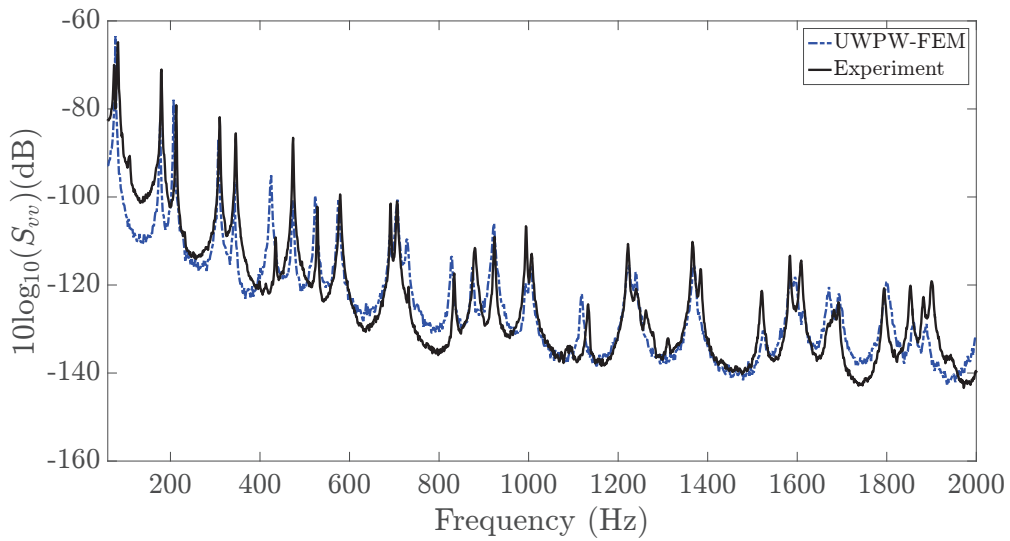


Figure 11: Velocity spectra predicted numerically using the UWPW-FEM technique for different number of realizations, as well as predicted analytically, for a flow speed of 40 m/s (dB ref. 1 (m/s)²/Hz)



(a)



(b)

Figure 12: Predicted and measured velocity spectra for a simply supported plate for a flow speed of (a) 20 m/s and (b) 40 m/s (dB ref. 1 (m/s)²/Hz)

Table 2. The first ten natural frequencies of the panels in case studies A and B

Case Study A			Case Study B		
Mode	FEM (Hz)	Experiment (Hz)	Mode	FEM (Hz)	Experiment (Hz)
(1,1)	78	82	(1,1)	84	86
(2,1)	177	180	(2,1)	144	148
(1,2)	208	213	(1,2)	192	192
(2,2)	308	310	(3,1)	244	244
(3,1)	344	346	(2,2)	248	246
(1,3)	426	435	(3,2)	344	345
(3,2)	474	474	(1,3)	358	356
(2,3)	526	528	(4,1)	380	378
(4,1)	578	579	(2,3)	412	389
(3,3)	692	692	(4,2)	476	413

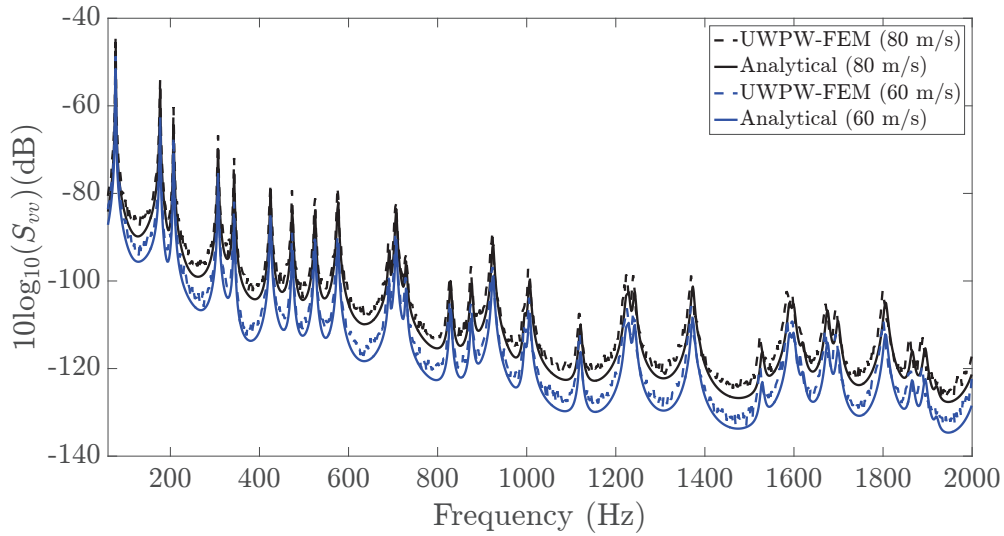
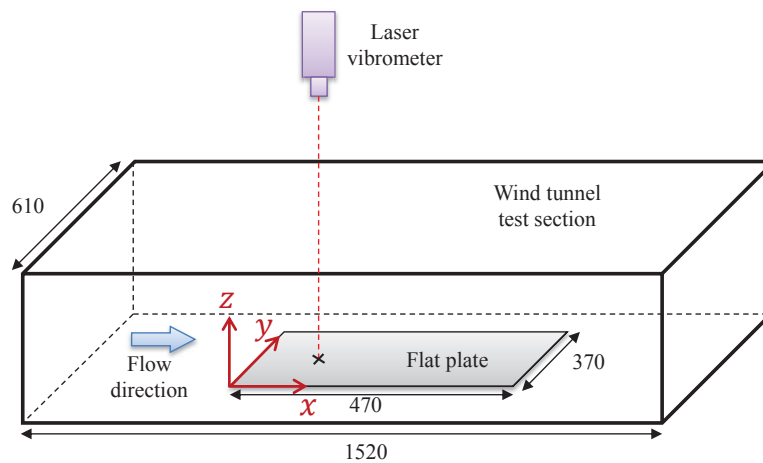


Figure 13: Predicted spectral velocity for a simply supported plate predicted numerically using the UWPW-FEM technique and analytically, for flow speeds of 60 m/s and 80 m/s (dB ref. 1 (m/s)²/Hz)

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1240 266 *3.2. Case study B - Clamped plate*

1241 267 In the second case study we examined a steel flat plate with clamped
1242 268 boundary conditions along its edges excited by turbulent flow. The plate
1243 269 was used in the experiment conducted by Han et al. (1999) in a wind tunnel
1244 270 at Purdue University. Figure 14 shows a schematic diagram of the wind
1245 271 tunnel test section and plate. The plate vibration was measured using a
1246 272 scanning laser Doppler vibrometer at a point 150 mm from the plate's left
1247 273 edge and 120 mm from its bottom edge. The experiment was carried out at
1248 274 flow speeds of 35.8 m/s and 44.7 m/s (Han et al., 1999; Hambric et al., 2004).
1249 275 The displacement thickness was measured for these flow speeds to be 2.9 mm
1250 276 and 2.4 mm, respectively. These values were used in equations (20)-(22) to
1251 277 calculate the input TBL parameters for the CSD function of the WPF.



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1270 Figure 14: Schematic diagram showing the experimental set-up in the wind tunnel at
1271 Purdue University, (dimensions in mm)

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1273 278 Figure 15 presents predicted and measured velocity spectra for the clamped
1274 279 plate at a flow speed of 44.7 m/s. The UWPW-FEM results are shown using
1275 280 the Corcos, generalized Corcos and Mellen models. A similar trend to Fig-
1276 281 ure 4 was observed indicating that using the Mellen model for the normalized
1277 282 CSD function of the pressure field provides the closest prediction to the ex-
1278 283 perimental data. Similar to the first case study, there is a small discrepancy
1279 284 between the predicted resonant frequencies and those obtained experimen-
1280 285 tally, whereby the first ten resonances are compared in Table 2 using the

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286 Mellen model. In the experiment, the plate was bolted along its edges to a
287 fixture to simulate clamped boundary conditions. However, the experimen-
288 tal boundary conditions may not exactly correspond to zero displacement
289 and zero rotation along all edges of the plate, as modelled using the FEM.
290 Further, there may be small differences in the plate material properties in
291 the experiment and those used in the FEM model.

292 For the second case study, the aerodynamic coincidence frequency is 47 Hz
293 and 74 Hz for flow speeds of 35.8 m/s and 44.7 m/s, respectively. The first
294 two modes of the clamped plate occur at frequencies close to the aerodynamic
295 coincidence frequency. The three TBL models have almost the same levels
296 of magnitude as shown at the convection peak in Figure C1. Hence, at low
297 frequencies, the velocity spectra generated using the three TBL models are
298 almost identical. At higher frequencies, the predicted velocity using the three
299 models are different from each other which is consistent with the behaviour
300 of the TBL models in the wavenumber domain in Figure C1. Comparison
301 between predicted spectral velocity using the Mellen model and experimental
302 data for a flow speed of 35.8 m/s is presented in Figure 16, showing excellent
303 agreement. It is worth noting that the maximum flow parameter $\rho_f U_\infty^2 L_y^3 / D$
304 for the cases studied here is approximately 2.8 for case study A at the highest
305 considered flow speed of 80 m/s, and 1.7 for case study B at 44.7 m/s. These
306 values for the flow parameter are well below the critical values at which static
307 instability (divergence) or dynamic instability (flutter) occurs (Ellen, 1973;
308 Bochkarev et al., 2016)

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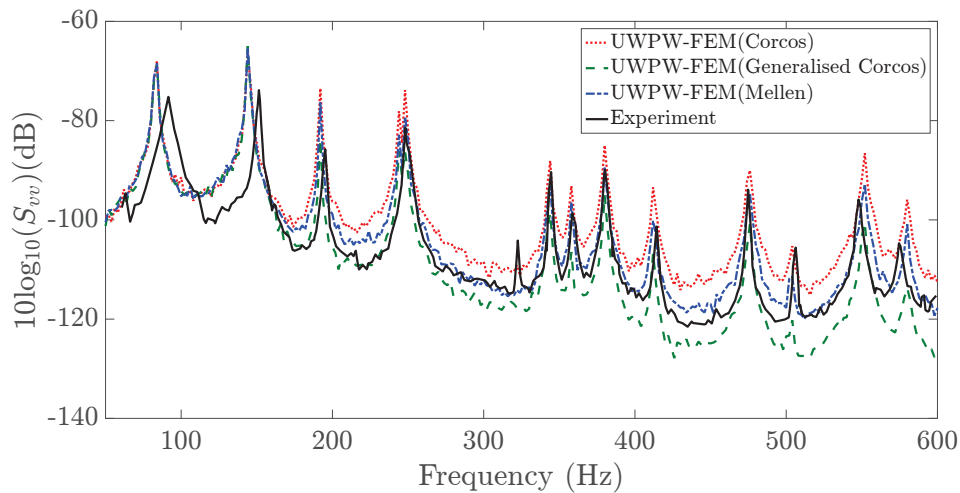


Figure 15: Predicted and measured velocity spectra for the clamped plate for a flow speed of 44.7 m/s (dB ref. 1 (m/s)²/Hz)

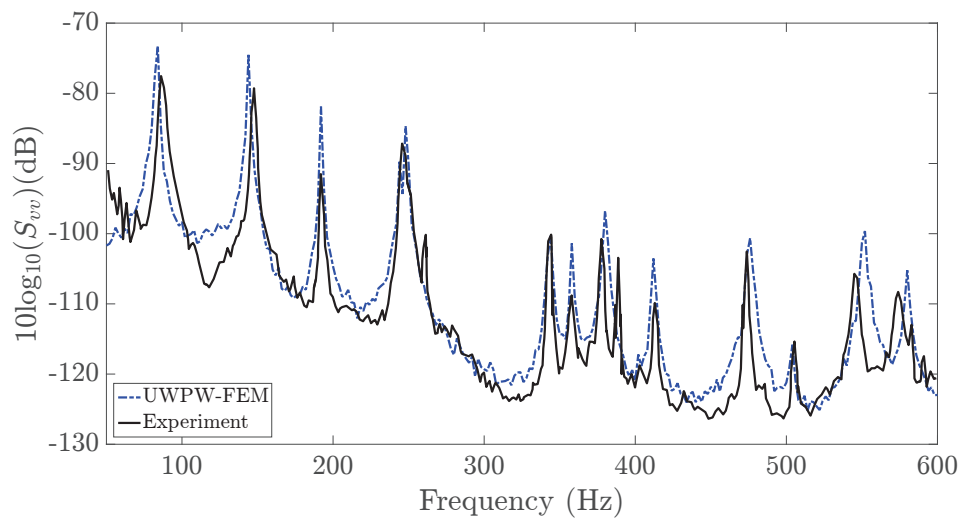


Figure 16: Predicted and measured velocity spectra for the clamped plate for a flow speed of 35.8 m/s (dB ref. 1 (m/s)²/Hz).

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310 An uncorrelated wall plane wave technique was employed to determin-
311 istically synthesize the wall pressure field underneath a turbulent boundary
312 layer. The pressure field was then used as an input to an FEM model of
313 a panel to predict its vibrational response. One of the main advantages of
314 using the UWPW technique is that the deterministic WPF is calculated at
315 each FEM nodal point for a small number of realizations, and can be applied
316 as an input to the FEM or any other element-based approach to evaluate the
317 panel structural response. An analytical method based on a sensitivity func-
318 tion was employed to verify the numerical method for the case of a simply
319 supported plate. Experimental data for panels with different material prop-
320 erties, flow conditions and boundary conditions were also used to validate the
321 proposed approach. It was shown that the hybrid UWPW-FEM technique
322 can be confidently used to predict the structural responses of panels excited
323 by turbulent flow. Among the three TBL models considered in this work,
324 the Mellen model provided better estimation of the WPF compared with
325 experimental data. It has been shown that except at low frequencies, for a
326 flat plate under low Mach number TBL excitation, theoretical expressions
327 from literature can be used to estimate the input TBL parameters required
328 to compute the CSD function of the pressure field. Whilst the case studies
329 presented here comprise simple panels with simply supported or clamped
330 boundary conditions, the proposed method can be applied to study the vi-
331 brations of complex panels under TBL excitation. Further, in the presence
332 of complex flow conditions, a RANS simulation can be performed for more
333 accurate calculation of the TBL parameters.

334 **Acknowledgements**

335 This research was supported by the Australian Government through the
336 Australian Research Council's Discovery Early Career Project funding scheme
337 (project DE190101412).

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1464 **338 Appendices**

1465 **339 Appendix A: Auto spectrum density function**

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1467 *340 The Goody model*

1468 The empirical model of the auto spectrum density function of the pressure
1469 field is given by Goody (2004)
1470

$$1471 \Psi_{pp}(\omega) = \frac{3\tau_w^2 \delta \left(\frac{\omega\delta}{U_e}\right)^2}{1472 U_e \left(0.5 + \left(\frac{\omega\delta}{U_e}\right)^{0.75}\right)^{3.7} \left(1.1R_T^{-0.57} \left(\frac{\omega\delta}{U_e}\right)\right)^7} \quad (1)$$

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1479 where $R_T = U_\tau^2 \delta / U_e \nu$ and U_e is the velocity at the boundary layer edge.
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1481 **342 Appendix B: Normalized cross spectrum density function**

1482
1483 *343 The Corcos model*

1484 The Corcos normalized wavevector-frequency spectrum is given by Corcos
1485 (1963)
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$$1487 \tilde{\phi}_{pp}(k_x, k_y, \omega) = \frac{4\alpha_x \alpha_y}{1488 \left(\alpha_x^2 + \left(\frac{k_x}{k_c} - 1\right)^2\right) \left(\alpha_y^2 + \left(\frac{k_y}{k_c}\right)^2\right)} \quad (2)$$

1489
1490 where $k_c = \omega / U_c$. The exponential decay coefficients in the normalized CSD
1491 function in the streamwise and spanwise directions are respectively $\alpha_x = 0.1$
1492 and $\alpha_y = 0.77$ (Graham, 1997).
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1497 *349 The generalized Corcos model*

1498 The normalized wavevector-frequency spectrum of the generalized Corcos
1499 model is given by Caiazzo et al. (2016)
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$$1501 \tilde{\phi}_{pp}(k_x, k_y, \omega) = \frac{\omega^2}{1502 U_c^2} \frac{4B_n(k_x)B_m(k_y)}{\left(\frac{-A_{k_x}A_{k_y}\alpha\omega\beta\omega}{nm}\right) \sum_{j=0}^{n-1} e^{-i\theta_j} \sum_{j=0}^{m-1} e^{-i\theta_j}} \quad (3)$$

$$B_n(k_x) = \frac{A_{k_x}}{1 + \left(\frac{k_x - k_c}{\alpha_\omega}\right)^{2n}} \quad (.4)$$

$$B_n(k_y) = \frac{A_{k_y}}{1 + \left(\frac{k_y}{\beta_\omega}\right)^{2m}} \quad (.5)$$

$$A_{k_x} = \frac{n \sin\left(\frac{\pi}{2n}\right)}{\pi \alpha_\omega}; \quad A_{k_y} = \frac{m \sin\left(\frac{\pi}{2m}\right)}{\pi \beta_\omega} \quad (.6)$$

$$\alpha_\omega = k_c \alpha_x; \quad \beta_\omega = k_c \alpha_y; \quad (.7)$$

$$\theta_j = \frac{\pi}{2n}(1 + 2j) \quad (.8)$$

352 *The Mellen model*

353 The Mellen normalized wavenumber-frequency model is given by Mellen
354 (1994)

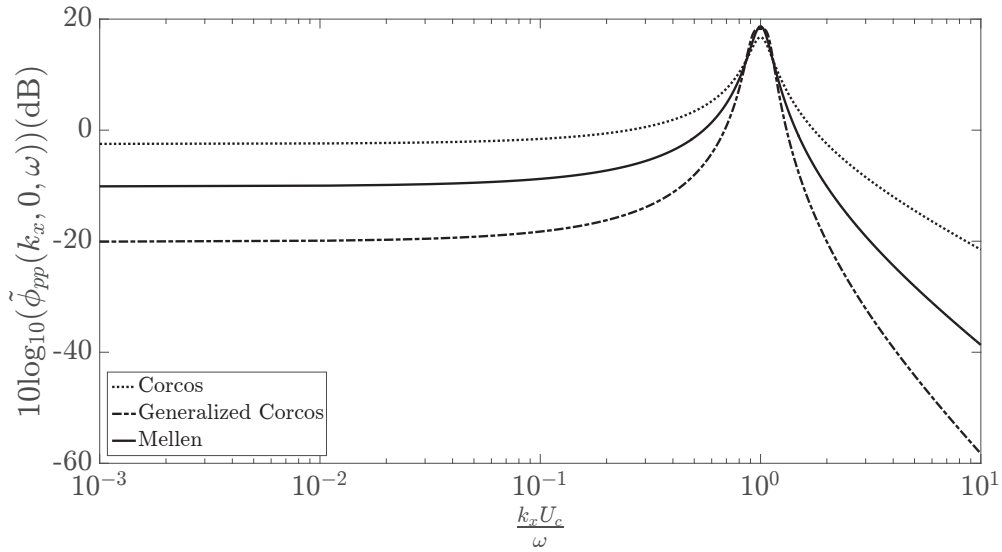
$$\tilde{\phi}_{pp}(k_x, k_y, \omega) = \frac{2\pi(\alpha_x \alpha_y)^2 k_\omega^3}{((\alpha_x \alpha_y k_c)^2 + (\alpha_x k_y)^2 + \alpha_y^2 (k_x - k_c)^2)^{3/2}} \quad (.9)$$

355 where $\alpha_x = 0.1$ and $\alpha_y = 0.77$.

356 **Appendix C**

357 Discrepancies in the results for the velocity spectra in Figure 4 predicted
358 by the three TBL models are herein examined. The normalized cross spec-
359 trum of the wall pressure field as a function of non-dimensionalized wavenum-
360 ber $k_x U_c / \omega$ for the three TBL models at $k_y = 0$ are compared in Figure C1.
361 Close to the convective peak ($k_x U_c / \omega = 1$), all three models have similar
362 magnitudes. However, at higher frequencies or lower wavenumbers, the Cor-
363 cos model provides a higher spectral level than that of the Mellen model.
364 The generalized Corcos model with filter orders of $m = 1$ and $n = 2$ esti-
365 mates a lower spectral level compared to the Mellen model prediction. This
366 is consistent with the trend observed in Figure 4. It was also previously

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1576 demonstrated that the convective peak in the Mellen model expressed in the
1577 wavenumber domain has an oval shape, which is in better agreement with
1578 the measurements, in contrast to the model of Corcos and, by extension,
1579 the generalized Corcos which has a diamond-like shape (Mellen, 1990; Miller
1580 et al., 2012).
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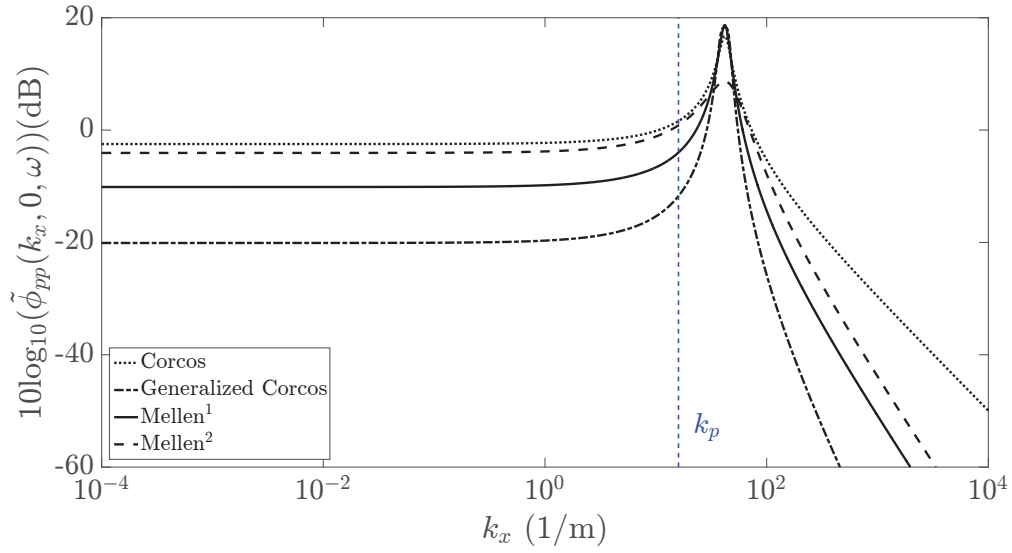


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1600 Figure C1: Normalized cross spectrum models of the wall pressure field at $k_y = 0$
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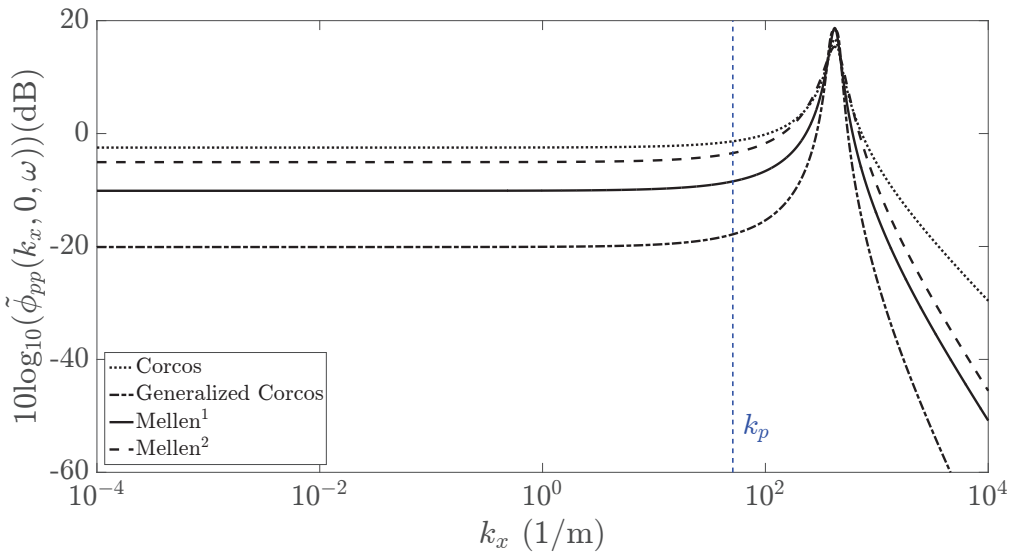
1602 The panel filters out the pressure waves with high wavenumbers and the
1603 panel response is essentially dominated by the wavenumbers below or close to
1604 the flexural wavenumber of the panel. As such, only the wavenumber region
1605 of the normalized CSD below the plate flexural wavenumber is of interest.
1606 The normalized CSD of the wall pressure field using the Corcos model, gen-
1607 eralized Corcos model, the standard Mellen model (Mellen¹) and the exper-
1608 imentally fitted Mellen model (Mellen²) as a function of wavenumber in the
1609 streamwise direction, k_x , are compared in Figure C2, at two discrete frequen-
1610 cies corresponding to 200 Hz (Figure C2(a)) and 2000 Hz (Figure C2(b)). At
1611 200 Hz and 2000 Hz, the flexural wavenumbers k_p are respectively 16 (1/m)
1612 and 51 (1/m) which are below convected wavenumber as shown in Figure C2.
1613 At 200 Hz (Figure C2(a)), it can be observed that the experimentally fitted
1614 Mellen model produces a higher spectral level of almost 6 dB than that of the
1615 standard Mellen model. Further, the experimental ASD function at 200 Hz
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386 in Figure 6 is approximately 7 dB above the predicted values by the Goody
387 model. Since the CSD of the WPF is the product of the normalized CDS
388 function and ASD function of the WPF as given by equation (1), a differ-
389 ence of 13 dB between the standard and experimental CSD functions can
390 be observed. This explains the reason why results at 200 Hz in Figure 7 for
391 the velocity spectra obtained using the standard Mellen and Goody models
392 (Analytical¹) are lower by almost 13 dB than the velocity spectra obtained
393 using the measured pressure spectra and experimentally fitted Mellen model
394 (Analytical²). A similar interpretation can be provided for the discrepancy
395 between the two sets of analytical results at 2000 Hz in Figure 7. At this
396 frequency, the Goody model over-predicts the pressure spectrum by almost
397 6 dB. Hence, when multiplied by the standard Mellen model (Mellen¹) which
398 is almost 5 dB lower than the experimentally fitted Mellen model (Mellen²),
399 a small difference between the standard and experimental CSD functions of
400 the wall pressure field occurs. Consequently, the velocity spectra at 2 kHz in
401 Figure 7 represented by Analytical¹ only slightly differs from results obtained
402 using Analytical².



(a)



(b)

Figure C2: Normalized CSD of the wall pressure field at $k_y = 0$ using the standard Mellen model (Mellen¹), the experimentally fitted Mellen model (Mellen²), the Corcos model and the generalized Corcos model at (a) 200 Hz and (b) 2000 Hz (dB ref. 1 Pa²/Hz). k_p indicates the panel flexural wavenumber

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