



HAL
open science

Low achiever's mathematical thinking: The case study of Maya

Hanna Viitala

► **To cite this version:**

Hanna Viitala. Low achiever's mathematical thinking: The case study of Maya. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02410266

HAL Id: hal-02410266

<https://hal.science/hal-02410266>

Submitted on 13 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Low achiever's mathematical thinking: The case study of Maya

Hanna Viitala

Luleå University of Technology, Sweden; hanna.viitala.edu@gmail.com

In this paper, one low achiever's, Maya's, mathematical thinking is reported. Mathematical thinking is studied through problem solving and view of mathematics. The results indicate that Maya has severe gaps in mathematical knowledge and problem solving. Furthermore, affect hinders her learning and activities in mathematics. Despite the challenges, Maya wants to learn mathematics and succeeds in giving an answer to every mathematical problem in the study. In addition to further support in mathematics and affect, she could benefit from learning to use different representations in mathematics and picturing the tasks in real life.

Keywords: View of mathematics, problem solving, metacognition, affect, meta-affect.

Introduction

Finnish pupils' success in mathematics has been acknowledged in international studies such as PISA and TIMSS (see e.g. Välijärvi, 2014) and national assessments show that Finnish pupils' performance is at a satisfactory level (e.g. Hirvonen, 2012). However, the most recent studies indicate that Finnish pupils' performance is descending (ibid.; Rautopuro, 2013), showing also the diminishing number of high achievers and the growing share of low achievers in Finland (Kupari & Nissinen, 2015). Furthermore, pupils' affect towards mathematics is alarmingly low at the end of lower secondary school (Tuohilampi & Hannula, 2013).

To understand the recent development, we need to know more about Finnish pupils' mathematical thinking. What is it like at the end of comprehensive school? Previously, Viitala (2017a) has reported on four Finnish high achieving pupils' mathematical thinking. These pupils all liked mathematics, were successful problem solvers and enjoyed learning mathematics. However, a deeper look into their problem solving and view of mathematics revealed very different skills and competences in mathematics. Nevertheless, whatever their strengths and weaknesses were, their strengths compensated their weaknesses in mathematics. How is the situation different with low achievers? What is it that separates low achievers from high achievers? In addition to differences in mathematical knowledge, what might be the possible reasons for the low performance in mathematics?

In this paper, one low achiever's, Maya's, mathematical thinking is reported. The data is collected through problem solving and view of mathematics. The purpose is to study Maya's strengths and weaknesses in mathematics that go beyond her mathematical knowledge. The purpose is to answer the question 'What characterizes Maya's mathematical thinking at the end of lower secondary school?'

Theoretical framework

Following the recent theories on affect, mathematical thinking can be viewed through two temporally different aspects; state and trait (cf. Hannula, 2011; 2012). While thinking is always situational and contextual, a state, it is also influenced by more stable constructs such as beliefs and

attitudes, traits (cf. *ibid.*). These two temporally different sources of data together can reveal the dynamic and complex thinking processes that pupils go through while thinking mathematically (for an example, see e.g. Viitala, 2017b).

In this study, the state aspect is studied through problem solving. The pupils' actions and explanations during problem solving are interpreted as visible signs and expressions of their mathematical thinking. In addition to the mathematical knowledge and heuristics in problem solving, pupils' problem-solving behaviour is influenced by their metacognition, affect and meta-affect that occur in a problem-solving situation (cf. Schoenfeld, 1992).

The successful application of problem-solving activities at the correct moment is a result of metacognitive skilfulness (e.g. Schoenfeld, 1987; Flavell, 1979). These skills include for instance monitoring one's actions and directing resources in a problem-solving situation (Schoenfeld, 1987). Affective state influences problem-solving activities for instance through the feeling of confidence, and meta-affect transforms individuals' feelings (DeBellis & Goldin, 2006) directing pupils' problem-solving behaviour (Carlson & Bloom, 2005).

Problem-solving processes can show patterns of thought that can be interpreted as signs of more stable ways of thinking. These patterns, together with pupils' view of mathematics, represent the traits in this study. Pupils' view of mathematics is studied through five components: mathematics as a science and as a school subject, oneself as a learner and user of mathematics, learning mathematics, teaching mathematics (Pehkonen, 1995), and view of mathematical thinking.

The structure of pupils' view of mathematics is derived from belief-research that has traditionally been studied as traits (see e.g. Op't Eynde, de Corte, & Verschaffel, 2002). However, in separation from many earlier studies, in this study pupils' view of mathematics is considered as psychological phenomenon that is a mixture of cognitive, motivational and emotional processes (Hannula, 2011; 2012). In addition to beliefs, these processes include also for instance attitudes, values, feelings and motivation (*ibid.*; thus the word 'view', see Rösken, Hannula, & Pehkonen, 2011).

Methods

Data collection

At the time of data collection, Maya was 15 years old. She was in 9th and final year of Finnish comprehensive school. She was a low achiever in mathematics (mathematics grades between 5 and 6 on a whole number scale of 4 to 10).

The data was collected in three cycles over the course of three months. In each cycle, 1-2 mathematical problems (PISA tasks) were solved in an ordinary classroom situation. In class, Maya solved the tasks individually, but she could talk about the tasks with her friends and ask help from the teacher. In the third cycle, Maya was absent from class and solved the problems in the interview. Maya was video recorded while she solved the tasks and her solutions on paper were collected.

Here is an example of a PISA task used in data collection: 'This problem is about planning the best route for a holiday. Figures A and B show a map of the area and the distances between towns [in a

table]. [...] Calculate the shortest distance by road between Nuben and Kado.’ (OECD, 2006, pp. 77-78; for a part of the task, see also Figure 1).

Maya was interviewed three times, on the same day or on the next day after each of the problem-solving lessons. The interviews were semi-structured and contained two parts. The first part concentrated on Maya’s view of mathematics. It followed the categorisation introduced in theoretical framework: background, mathematics as a school subject and as a science, oneself as a learner of mathematics, learning mathematics, teaching mathematics (following Pehkonen, 1995; cf. Op’t Eynde et al., 2002), and view of mathematical thinking (for example questions, see Table 1).

Theme	Example question
Background	Tell me about your family.
Mathematics	Does mathematics exist outside of school? Where?
Oneself and mathematics	Is mathematics important to you?
Mathematics learning	How do you learn mathematics?
Teaching mathematics	Does teaching matter to your learning? How?
View of mathematical thinking	How do you recognise mathematical thinking?

Table 1: Interview themes and example questions

The second part concentrated on problem solving. The data from class was used as stimuli when Maya’s problem-solving processes were discussed. She was asked to explain her thinking and actions during the problem-solving situation. Also supporting questions were asked (e.g. What are you thinking now? Why are you doing so?).

Finally, in each of the three interviews, Maya was asked to assess her confidence before, during and after solving the problems, as well as to assess her confidence in school mathematics using a 10 cm line segment (scale from ‘I couldn’t do it at all’ to ‘I could do it perfectly’). All interviews were video recorded.

Data analysis

The analysis was divided into two sections: problem solving (state) and view of mathematics (trait). First, problem-solving processes were analysed through four problem-solving phases: orienting, planning, executing and checking (Carlson & Bloom, 2005). Then, the results were complemented with the metacognitive activities of orienting, planning, evaluating and elaborating (see examples in van der Stel, Veenman, Deelen, & Haenen, 2010), affect (cognitive, emotional and motivational states and traits; Hannula, 2011; 2012) and meta-affect (DeBellis & Goldin, 2006) emerging in the problem-solving processes. Finally, Maya’s confidence to solve the problems were analysed.

Second, Maya’s view of mathematics was analysed following the themes of data collection (Pehkonen, 1995). After data condensation, a pupil profile was created. Pupil profile is a short description of the pupil that is used as background information. It is based on Maya’s mathematics grade, motivation to learn mathematics, and ability, success, difficulty, and enjoyment of mathematics also known as the core of herself as a learner of mathematics (Rösken et al., 2011).

In the end, the results from problem solving (state) and view of mathematics (trait) were compared and combined to get a holistic view of Maya's mathematical thinking.

Results

Pupil profile: Maya is a low-achieving, very unsure and quite anxious mathematics learner. She does not value mathematics as important, but would still like to learn it. Her low level of mathematical knowledge and skills go hand in hand with her low level of affect and meta-affective skills in mathematics.

The most prominent result of the study is the low level of affect that Maya has in and towards mathematics. When we begin discussing about mathematics, Maya starts by saying that she is very bad at mathematics, has always been. She then continues to express her low level of affect throughout the study: She explains that she has no idea what mathematics is as a science because she is very bad at mathematics; Mathematics is not important because she does not really know mathematics; She does not feel confident in mathematics; She feels anxious in mathematics classes and especially so in tests; She feels that she has never succeeded in a mathematics test; Or simply, mathematics is too difficult for her.

Maya has great difficulties also in solving problems. She felt stressed in problem-solving situations and the tasks felt difficult. Before she could start solving the problems, she needed confirmation

Kaavio 2. Kaupunkien välimatkataulukko, lyhin ajoreitti kilometreinä

Angaz						
Kado	550					
Lapat	500	300				
Megal	300	850	550			
Nuben	500	1450	1300	450		
Piras	300	850	800	600	250	
	Angaz	Kado	Lapat	Megal	Nuben	Piras

Tehtävä 1: LOMAMATKA

Laske Nubenin ja Kadon välimatka lyhintä reittiä pitkin.

Välimatka: 1450 kilometriä.

Nuben - Angaz = 500 km

Angaz - kado = 850 km

550 km

$$\begin{array}{r} 850 \\ + 500 \\ \hline 1350 \\ + 850 \\ \hline 2200 \\ + 1405 \\ \hline 3605 \end{array}$$

$$\begin{array}{r} 550 \\ 500 \\ \hline 1050 \end{array}$$

from the teacher (or from the researcher in the interviews) that she had understood the problem

correctly. Unfortunately, Maya is not initiative in asking help in class. She feels afraid of making mistakes, she

Figure 1: A partial caption of the task ‘Holiday’ (OECD, 2006, p. 77) and Maya’s interview notes

is afraid that the teacher asks questions or gives critique, and she is afraid that her classmates notice her problems. This mismatch between the need for help and not asking for it, and consequently staying stuck, is just one example of the low level of meta-affective skills that Maya has in mathematics. Fortunately, though, when the teacher approaches her, she usually gets the help she needs to continue with the problem.

In addition to the low affect and meta-affective skills in mathematics, Maya’s mathematical skills are quite limited. For example, in the problem introduced above, Maya had difficulties in reading the table and in adding. First, she did not understand what the numbers in the table represent before the teacher explained the table to her. Even then, she took the distances she needed from the first cell next to the town names (500 next to Nuben and 850 above Kado; see Figure 1). Maya explained that she did not understand the table until the interview.

After getting the distances she needed from the table, Maya had to add the numbers together. She explained that she could not add the numbers (500 and 850) mentally, so she calculated them on paper (for a reproduction of the calculation, see Figure 1). Maya’s calculation was a mixture of adding and multiplying rules and the result was incorrect. In the interview, Maya explained that she took the first and last two numbers as the result and ignored the third and fourth numbers. Even though the method seems absurd, there is something positive: she had a realistic view of the magnitude of the answer.

There are severe problems in Maya’s knowledge base and affect. Metacognitive actions are almost non-existent, also meta-affective actions, and she needs very much and very concrete help to solve the problems. However, there too is something positive: She does want to learn mathematics. She recognizes that for her own learning, she needs to follow teaching in mathematics classes and ask questions. She has noticed that when the teacher relates new knowledge to old knowledge, it helps her to learn mathematics. Even though she has emotional obstacles in communicating with her teacher, she seeks and gets more mathematical help from their special needs teacher. She emphasizes the importance of doing homework alone and undisturbed. And, she recognizes that she needs mathematics in future studies and jobs.

In mathematics learning, Maya needs help in developing her conceptual and procedural skills, and support to overcome many emotional issues. However, in connection to problem solving, there is one concrete thing that rose above other issues. When Maya got stuck in problem solving, she understood the problem best with a new representation. Moving the task to real life or drawing a picture was enough to help Maya to understand the problem. Moving between different representations in mathematics could help her to improve her problem-solving skills and her conceptual knowledge.

Discussion

This paper aimed at answering the question ‘What characterizes Maya’s mathematical thinking at the end of lower secondary school?’ Maya is best characterized through the low level of mathematical knowledge and skills, and through the low level of affect towards mathematics. More specifically, the very negative view of herself in mathematics seem to hinder her learning and solving mathematical problems. This negative trait connected with the negative state reactions to the problems and problem-solving situations, as well as nearly non-existent meta-affective skills, almost prevented her to solve the problems in the study. The low level of mathematical and metacognitive skills support the situation.

In addition to the low level of mathematical knowledge, the low level of meta-affective and metacognitive skills seem to be what separates Maya most from the high-achieving students in the research project (see the high-achieving students’ results in Viitala, 2017a). Being unsure and having negative affect towards mathematics are features that students in all achievement levels experience. In fact, the most recent national studies show that Finnish pupils’ affect towards mathematics is unnecessary negative at the end of comprehensive school (Tuohilampi & Hannula, 2013; Hirvonen, 2012). However, while Maya seem to paralyse when experiencing difficulties in solving mathematical tasks, the high-achievers use their meta-affective and metacognitive skills to overcome them (see e.g. Viitala, 2017a). What all students in the research project have in common, though, is the positive feeling they get when they succeed in mathematics, and the will to learn mathematics, on their own levels and on their own terms.

The national studies also show that pupils start school with positive attitudes, but their attitudes towards mathematics as well as self-efficacy beliefs decrease consistently throughout comprehensive school, and pupils’ anxiety in mathematics increase in lower secondary school (Tuohilampi & Hannula, 2013; Metsämuuronen & Tuohilampi, 2014). Maya’s affect fits well with the results of Finnish large-scale studies at the end of comprehensive school. However, the results indicate that the situation has not been positive even at the beginning of her school years. For instance, Maya states that she has always been bad at mathematics and she cannot remember ever succeeding in a mathematics test. Having a different mathematics teacher in every grade might also been a factor in Maya’s learning of mathematics: While Maya is shy and very unsure of herself in mathematics, it takes time for the teacher to get to know her thinking so that the teacher could have the best tools to help Maya to develop mathematically.

So, what can be done to help pupils like Maya first to create and then to maintain more positive attitude towards mathematics, and to build mathematical knowledge and skills at their own rate? One solution could be to increase the amount of problem solving in mathematics classes. Even one monthly problem-solving lesson has shown to slow down the decreasing trend in girls’ attitudes towards mathematics (Tuohilampi, Näveri, & Laine, 2015). Problem solving is also an effective tool for the teacher to learn about pupils’ mathematical knowledge and thinking. Additionally, referring to real-life situation helped Maya to solve problems. Problem solving and applying mathematics to real-life situations are written as learning goals also in the Finnish curriculum (see FNBE, 2014).

One way of supporting Maya's growth in mathematics could also be setting long-term goals for her learning. In Maya's case, the long-term goal could be referring to real life and learning to use different representations in mathematics and problem solving (cf. Heinze, Star, & Verschaffel, 2009). Setting and evaluating the long-term goals in learning discussions, using written self-evaluation and supporting the development through formative assessment in mathematics classes, have shown promising results for students learning in a small pilot study implemented during spring semester 2018 (see more in Viitala, 2018). However, how and if these activities help pupils in a longer period of time, is a matter of a long-term continuation study with a larger number of pupils.

References

- Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, 58(1), 45–75.
- DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics*, 63(2), 131–147.
- Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, 34, 906–911.
- FNBE. (2014). *Perusopetuksen opetussuunnitelman perusteet 2014* [National core curriculum for basic education 2014]. Määräykset ja ohjeet 2014: 96. Helsinki, Finland: Finnish National Board of Education.
- Hannula, M. S. (2011). The structure and dynamics of affect in mathematical thinking and learning. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (pp. 34–60). Rzeszów, Poland: University of Rzeszów.
- Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education*, 14(2), 137–161.
- Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM Mathematics Education*, 41(5), 535–540.
- Hirvonen, K. (2012). *Onko laskutaito laskussa? Matematiikan oppimistulokset peruskoulun päättövaiheessa 2011* [Are calculating skills declining? Mathematics learning outcomes at the end of the comprehensive school in 2011]. Koulutuksen seurantaraportit 2012:4. Helsinki, Finland: FNBE.
- Kupari, P., & Nissinen, K. (2015). Matematiikan osaamisen taustatekijät [The background for mathematical performance]. In J. Välijärvi & P. Kupari (Eds.), *Millä eväillä uuteen nousuun?: PISA 2012 tutkimustuloksia* (pp. 10–27). Julkaisuja 2015:6. Helsinki, Finland: Ministry of Education.
- Metsämuuronen, J., & Tuohilampi, L. (2014). Changes in achievement in and attitude toward mathematics of the Finnish children from grade 0 to 9: A longitudinal study. *Journal of Educational and Developmental Psychology*, 4(2), 145–169.

- OECD. (2006). *Assessing scientific, reading and mathematical literacy: A framework for PISA 2006*. Retrieved from http://www.oecd-ilibrary.org/education/assessing-scientific-reading-and-mathematical-literacy_9789264026407-en
- Op't Eynde, P., de Corte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 13–37). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Pehkonen, E. (1995). *Pupils' view of mathematics: Initial report for an international comparison project*. Research report 152. Helsinki, Finland: University of Helsinki, Department of teacher education.
- Rautopuro, J. (Ed.). (2013). *Hyödyllinen pakkolasku. Matematiikan oppimistulokset peruskoulun päättövaiheessa 2012* [Useful forced landing. Mathematics learning outcomes at the end of the comprehensive school in 2012]. Koulutuksen seurantaraportit 2013:3. Helsinki, Finland: FNBE.
- Rösken, B., Hannula, M. S., & Pehkonen, E. (2011). Dimensions of students' views of themselves as learners of mathematics. *ZDM Mathematics Education*, 43(4), 497–506.
- Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189–215). Hillsdale, NJ: Lawrence Erlbaum.
- Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York, NY: Macmillan.
- Tuohilampi, L., & Hannula, M.S. (2013). Matematiikkaan liittyvien asenteiden kehitys sekä asenteiden ja osaamisen välinen vuorovaikutus 3., 6. ja 9. luokalla [The development of mathematics related attitudes and the interplay between attitudes and performance in 3rd, 6th and 9th grades]. In J. Metsämuuronen (Ed.), *Perusopetuksen matematiikan oppimistulosten pitkäjäsenarviointi vuosina 2005-2012*. Helsinki, Finland: FNBE.
- Tuohilampi, L., Näveri, L., & Laine, A. (2015). The restricted yet crucial impact of an intervention on pupil's mathematics-related affect. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1287–1293). Prague, Czech Republic: Charles University and ERME.
- van der Stel, M., Veenman, M. V. J., Deelen, K., & Haenen, J. (2010). The increasing role of metacognitive skills in math: A cross-sectional study from a developmental perspective. *ZDM Mathematics Education*, 42(2), 219–299.
- Viitala, H. (2017a). A case study on Finnish pupils' mathematical thinking: Problem solving and view of mathematics. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 1218–1225). Dublin, Ireland: DCU Institute of Education and ERME.
- Viitala, H. (2017b). A tool for understanding pupils' mathematical thinking. *Nordic Studies in Mathematics Education*, 22(2), 5–31.

- Viitala, H. (2018). Using formative assessment to support student development in lower secondary school mathematics. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, p. 184). Umeå, Sweden: PME.
- Väljörvi, J. (2014). Osaaminen kestäväällä perustalla: Suomen PISA-tulosten kehitys vuosina 2000-2009. Tilannekatsaus helmikuu 2014. [Know-how on solid ground: The development of Finnish PISA results in 2000-2009. Review in February 2014]. Muistiot 2014:1. Helsinki, Finland: FNBE.